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Derivation of Special Relativistic Relations Using Space-time Four Dimensional Coordinates Distance Force and Energy

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Abstract
The invariance of space-time infinitesimal distance is used to derive special relativistic time dilation and length contraction. The concept of force in 4 dimensional space-time is defined in terms of momentum and potential. This shows that the relativistic energy is a complex quantity when defined in terms of this new definition of the force. This new energy relation successfully gives the ordinary special relativistic energy-momentum relation. This complex energy relation resembles that of Ac circuit.

Keywords: Special relativity, force, space-time coordinate, energy.

Introduction

Einstein's special theory of relativity SR is well known to provide an accurate description of physical phenomena and has enjoyed spectacular success as a mathematical construct and in terms of the experiments to which it has been subjected. But it has been the subject of much criticism even to the present epoch [1, 2, and 3].

The SR is all about what's relative and what's absolute about time, space and motion. Some of Einstein's conclusions are rather sup rising. They are nonetheless correct, as numerous physics experiments have shown [4, 5, and 6].In this paper we derive by means of the space-time interval the length contraction and time dilation. Possible vulnerabilities of SR will be explored that break the symmetry of reciprocal observation of length, time and mass. And we will examine the theoretical foundation of SR for this paper's approach is not like any previous ones. It is a sympathetic position that can broaden rather than narrow SR. the aim herein this work is to find the expression of is the rest mass energy of the same body. When a transformation was made from a frame in which the photon source is at rest too one in which the photon source moving. To this assume that the force is invariant .in the sense that it take the same value for all frames.

The Rest Mass Energy in the Special Relativity

The space time interval and if you are dealing with infinitesimals', you have the space time line elementds,

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 \tag{1}$$

Where s dose not depend upon which inertial frame one is dealing with, in depends on the observer, it is invariant.

Thus

$$\frac{t_0}{t} = \frac{t_0 - 0}{t - 0} = \frac{\Delta t_0}{\Delta t} = \frac{dt_0}{dt} = \frac{d\tau}{dt} \quad (2)$$

Then

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{1}{c^2}\left(\frac{dx}{dt}\right)^2 = 1 - \frac{v^2}{c^2} \quad (3)$$

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}} = \beta = \gamma^{-1} \quad (4)$$

$$\frac{t_0}{t} = \sqrt{1 - \frac{v^2}{c^2}} \quad (5)$$

The invariance of the proper time lead to the well-known phenomenon of time dilation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (6)$$

To find length contraction consider an observer moving with a clock on a rod of length $l_0$with speed v. For an observer moving with the clock the length of the rod is l. Thus the speed v w.r.t is

$$v = \frac{l}{t_0} = \frac{l - 0}{t_0 - 0} = \frac{\Delta l}{\Delta t_0} = \frac{dl}{dt_0} \quad (7)$$

For the observer at rest w.r.t to the rod the length, l_0, while he sods the moving clock interval ist_0 , t:

$$v = \frac{l_0}{t} = \frac{l_0 - 0}{t - 0} = \frac{\Delta l_0}{\Delta t} = \frac{dl_0}{dt} \quad (8)$$

Thus

$$v = \frac{dl}{dt_0} = \frac{dl_0}{dt} \quad (9)$$

Then

$$\frac{d\tau}{dt} = \frac{dt_0}{dt} \quad (10)$$

$$\frac{d\tau}{dt} = \frac{dl}{dl_0} = \frac{l}{l_0} \quad (11)$$

Where

$$\left.\begin{array}{c} l = l - 0 = \Delta l = dl \\ dl_0 = \Delta l_0 = l_0 - 0 = l_0 \end{array}\right\} \quad (12)$$

Thus using equations (4) and (11)

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}} \quad (13)$$

Using equation (4) we find relativistic mass to by:

$$\frac{d\tau}{dt} = \frac{m_0}{m} \quad (14)$$

Where

$$\frac{m_0}{m} = \sqrt{1 - \frac{v^2}{c^2}} \quad (15)$$

Let us put in explicit form the ratio m/m_0 of equation (15) a function of the speed v of the body that moves:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{16}$$

Inδr the conventional definition of space time intervalds, requires

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{17}$$

One can first define the vector$d\underline{s}$ to be

$$d\underline{s} = cd\underline{t} = cdt\hat{e}_0 - dx\hat{e}_1 - dy\hat{e}_2 - dz\hat{e}_3 \tag{18}$$

Squaring both sides yields

$$ds^2 = d\underline{s}\,.d\underline{s} = c^2dt^2 + dx^2 + dy^2 + dz^2 \tag{19}$$

This version is in direct conflict with the definition ofδr. This conflict can be removed by defining or redefining $d\underline{s}$ to be:

$$d\underline{s} = \eta_{00}^{\frac{1}{2}}cdt + \eta_{11}^{\frac{1}{2}}dx\hat{e}_1 + \eta_{22}^{\frac{1}{2}}dy\hat{e}_2 + \eta_{33}^{\frac{1}{2}}dz\hat{e}_3 \tag{20}$$

To get

$$ds^2 = \eta_{oo}c^2dt^2 + \eta_{11}dx^2 + \eta_{22}dy^2 + \eta_{33}dz^2 \tag{21}$$

But the space- time interval in δr satisfies:

$$ds^2 = c^2d\tau^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{22}$$

Comparing equations (20) and (21) requires

$$\eta_{oo} = 1 \;,\quad \eta_{11} = -1 \;,\quad \eta_{22} = -1 \;,\quad \eta_{33} = -1 \tag{23}$$

$$\eta_{00}^{\frac{1}{2}} = 1 \;,\quad \eta_{11}^{\frac{1}{2}} = i \;,\quad \eta_{22} = i \;,\quad \eta_{33} = i \tag{24}$$

Thus

$$d\underline{s} = cdt\hat{e}_0 + idx\hat{e}_1 + idy\hat{e}_2 + idz\hat{e}_3$$
$$= cdt\hat{e}_0 + idr\hat{e} = cdt\hat{e}_0 + idr\hat{r} \tag{25}$$

The force of $\underline{F_s}$ by

$$\underline{F_s} = \frac{dm\underline{v}}{dt} - \nabla V = \frac{dmv}{dt}\hat{e}_0 - \vec{\nabla}V \tag{26}$$

Then

$$E_s = \int \underline{F_s}\,\underline{ds} = \int \frac{dmv}{dt}cdt - i\int \nabla V.\underline{dr}$$
$$= c\int dp - i\int dV \tag{27}$$

$$E_s = cp - iV + c_0 \tag{28}$$

For

$$c_0 = 0 \tag{29}$$
$$E_s = cp - iV \tag{30}$$
$$E_s^2 = c^2p^2 + V^2 \tag{31}$$

This resembles δrexpression

$$E^2 = c^2p^2 + m_0^2c^4 \tag{32}$$

With

$$V = m_0c^2 \tag{33}$$

One can also sat

$$V = 0 \;,\quad c_0 = ic_1 = im_0c^2 \tag{34}$$

In equation (28) to get

$$E_s = cp + ic_1 \quad (35)$$

Thus

$$E_S^2 = c^2p^2 + c_1^2 = c^2p^2 + m_0^2c^4 \quad (36)$$

To find the expression of E_s, consider the case of a particle moving in free space , where no potential exist. In this case equation (35) be comes

$$E_s = cp + c_0 = cmv + c_0 \quad (37)$$

But this expression consists of two variable parameters which are mend$v$ .To make E_s depends on one variable only consider the motion of a photon, which is characterized by constant speed. In this case

$$v = c \quad (38)$$

And

$$E_s = mc^2 + c_0 \quad (39)$$

Thus the aim here is to find the expression of E_s when atrans formation was made from a frame in which the photon source is at rest to one in which the photon source move with speed$v$. To do is assume that the force is invariant. In the sense that it takes the same value for all frames. This means that

$$F_0 = \frac{dE_{s_0}}{cd\tau} = F = \frac{dE_s}{cdt} \quad (40)$$

Where F_0 is the force in a rest frame, while $F$is the force in a frame moving with speed$v$. Thus

$$\frac{dE_{s_0}}{d\tau} = \frac{dE_s}{dt} \quad (41)$$

$$\frac{d(m_0c^2)}{d\tau} + \frac{dc_0}{d\tau} = \frac{d(mc^2)}{dt} + \frac{dc_0}{dt} \quad (42)$$

$$\frac{d(m_0c^2)}{d\tau} = \frac{dmc^2}{dt} \quad (43)$$

$$dmc^2 = \left(\frac{dt}{d\tau}\right) d(m_0c^2) \quad (44)$$

But according to equation

$$dmc^2 = \gamma dm_0c^2 \quad (45)$$

Thus

$$\int dmc^2 = \gamma \int dm_0c^2 \quad (46)$$

$$mc^2 = \gamma m_0c^2 + c_1 \quad (47)$$

$$mc^2 = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} + c_1 \quad (48)$$

To find c_1, one knows that , when

$$v = 0 \qquad , \qquad m = m_0 \quad (49)$$

Thus

$$m_0c^2 = m_0c^2 + c_1 \quad (50)$$

Thus

$$c_1 = 0 \quad (51)$$

Hence equation (48) be comes

$$mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 c^2 \qquad (52)$$

Inserting equation (53) in (28) for

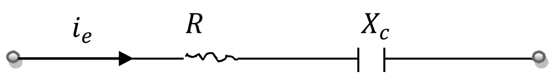
$$V = V_0 \quad , \qquad c_0 = 0 \qquad (53)$$

$$E_s = cp - iV_0 \qquad (54)$$

Also from equations (39) and (51)

$$E_s = mc^2 = \gamma\, m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (55)$$

To find V_0, consider the model of the electric circuit which consists of a resister of resistance R and a capacitor of capacitance C, with effective current



The total energy of the system is

$$E = i_e^2 Z = i_e^2 \sqrt{R^2 + X_c^2} \qquad (56)$$

This energy expression comes from the complex representation of impedance and energy which are written in the form:

$$\underline{Z} = R + iX_c \qquad (57)$$

$$\underline{E} = i_e^2 \underline{Z} = i_e^2 R + i i_e^2 X_c = E_R + iE_c \qquad (58)$$

$$E = |\underline{E}| = i_e^2 |\underline{Z}| = i_e^2 \sqrt{R^2 + X_c^2} \qquad (59)$$

$$E = \sqrt{E_R^2 + E_c^2} = \sqrt{i_e^4 (R^2 + X_c^2)} = i_e^2 \sqrt{R^2 + X_c^2} \qquad (60)$$

Here the real part which is the resistor liberate and gives energy to the surrounding medium whereas the capacitor which represent the imaginary part stores energy in the form of charges. In view of this model the first term in equation (54) is the momentum which reflect the motion of the particle that can collide with the medium molecules to give energy to the surrounding. Thus this term should correspond to the real part. When the particle is at rest

$$v = 0 \qquad , \qquad p = 0 \qquad (61)$$

The energy E_s to is equal to the rest mass energy which is the energy steered which the particle, thus it represent the imaginary part corresponding to the capacitor. Thus in this case

$$E_s = im_0 c^2 \qquad (62)$$

Thus in view of equations (54), (61) and (62), one gets

$$im_0 c^2 = -iV_0 \qquad (63)$$

As a result

$$V_0 = -\, m_0 c^2 \qquad (64)$$

Hence equation (54) be comes

$$E = cp + im_0 c^2 \qquad (65)$$

Therefore the relativistic relation between the total energy, momentum and rest mass:

$$E^2 = c^2 p^2 + m_0^2 c^4 \qquad (66)$$

Conclusion and Discussion:

Using relation (1) for infinitesimal space-time distance equations (5), (13) and (19) gives the SR expressions for time, length and mass. Defining the force in the space-time 4 dimensions by equation (26) and defining the corresponding energy in the 4 dimensions space-time coordinate by equation (27) the SR energy-momentum relation was derived. Definition (27) shows that the energy is in a complex form. This resembles the Ac circuit complex energy shows in equation (56).

The new definition of force and energy in 4 dimensional space-time coordinates enables expressing energy in a complex form and deriving SR energy-momentum relation.

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