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Derivation of the Schwarzschild Radius Formula on the Basis Generalized Special Relativity

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Abstract

In this work an alternative method of deriving the Schwarzschild radius and short range energy of a black hole has been used. The method uses short range repulsive gravitational force beside long range attractive gravity force. The critical radius of the star requires minimizing the total energy and can be found by using the conditions for minimum value within generalized special relativity. This critical value was typical to that of general relativity for black hole. This model indicates finite ness of mass and energy and shows that the mass is in the form of a hollow sphere.

Keywords: Schwarzschild metric, critical radius, black hole radius, generalized special relativity, short range repulsive, long range attractive, short range energy.

Introduction

If we want to obtain a correct derivation of the Schwarzschild radius formula, we should use either general relativity, quantum gravity or the Planck units. In 2014 the work of R. A. Frino, published an article entitled, the special quantum gravitational theory of black holes [1] where derived: (a) a general formula for the temperature of a black hole. (b) The formula for the black hole entropy. In that formulation used the uncertainty principle [2]. As the cornerstone of the theory along with the Schwarzschild radius formula discovered by Schwarzschild in 1915. And also In 2015 published a paper entitled derivation of the Schwarzschild radius without general relativity [3], he was in this work he present method of deriving the Schwarzschild radius of a black hole, the method uses three of the Planck units formulas: the Planck mass, the Planck momentum and the Planck length. In this paper we have chosen a new method of deriving the Schwarzschild radius without general relativity, quantum gravity or the Planck units. One will discuss in this work derivation of the Schwarzschild radius of a black hole and also the string self-energy based on the generalized special relativity.

Generalized Special Relativity

Generalization of Schwarzschild metric which is describing the space time of the gravitational fields is given as [4]:

$$ds^2 = g_{00}dt^2 - g_{00}^{-1}dt^2 - r^2(d\theta^2 + sin^2\theta d\emptyset^2) \qquad (1)$$

Where

$$g_{00} = \left(1 + \frac{\varphi}{c^2}\right)^2 = \left(1 - \frac{GM}{c^2 r}\right)^2 \quad (2)$$

Where φ the Newtonian potential takes the form:

$$\varphi = -\frac{GM}{r} \quad (3)$$

Then:

$$g_{00} = 1 - \frac{2GM}{c^2 r} + \frac{G^2 M^2}{c^4 r^2} \quad (4)$$

In the case of weak gravitational field where the term $\left(\frac{G^2 M^2}{c^4 r^2}\right)$ is neglected.

$$g_{00} = 1 - \frac{2GM}{c^2 r} \quad (5)$$

Can be obtained the critical radius, using the following energy for generalized special relativity equilibrium condition by minimizing energy E with respect to radius star:

$$E = \frac{m_0 c^2 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \quad (6)$$

$$E = \frac{m_0 c^2 \left(1 - \frac{2GM}{c^2 r}\right)}{\sqrt{1 - \frac{2GM}{c^2 r} - \frac{v^2}{c^2}}}$$

$$E = m_0 c^2 \left(1 - \frac{2GM}{c^2 r}\right)\left(1 - \frac{2GM}{c^2 r} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad (7)$$

$$E = m_0 c^2 \left(1 - \frac{2c_1}{c^2} r^{-1}\right)\left(1 - \frac{2c_1}{c^2} r^{-1} - \frac{v^2}{c^2}\right)^{-1/2} \quad (8)$$

Where

$$c_1 = GM$$

$$E = m_0 c^2 (1 + c_2 r^{-1})\left(1 + c_2 r^{-1} - \frac{v^2}{c^2}\right)^{-1/2} \quad (9)$$

Where

$$c_2 = -\frac{2c_1}{c^2} = -\frac{2GM}{c^2}$$

The critical radius of the star requires minimizing the total energy E and can be found by using the conditions for minimum value, i.e.

$$\frac{dE}{dr} = \frac{-m_0 c^2 c_2 r^{-2}}{\left(1 + c_2 r^{-1} - \frac{v^2}{c^2}\right)^{1/2}} + \frac{\frac{1}{2} m_0 (1 + c_2 r^{-1})(c^2 c_2 r^{-2})}{\left(1 + c_2 r^{-1} - \frac{v^2}{c^2}\right)^{3/2}} = 0$$

$$\frac{-m_0 c^2 c_2 r^{-2}\left(1 + c_2 r^{-1} - \frac{v^2}{c^2}\right) + \frac{1}{2} m_0 (1 + c_2 r^{-1})(c^2 c_2 r^{-2})}{\left(1 + c_2 r^{-1} - \frac{v^2}{c^2}\right)^{3/2}} = 0$$

$$-m_0 c^2 c_2 r^{-2}\left(1 + c_2 r^{-1} - \frac{v^2}{c^2}\right) + \frac{1}{2} m_0 (1 + c_2 r^{-1})(c^2 c_2 r^{-2}) = 0$$

$$2m_0c^2c_2r^{-2}\left(1 + c_2r^{-1} - \frac{v^2}{c^2}\right) = m_0c^2c_2r^{-2}(1 + c_2r^{-1})$$

$$2 + 2c_2r^{-1} - \frac{2v^2}{c^2} = 1 + c_2r^{-1}$$

$$2c_2r^{-1} - c_2r^{-1} = \frac{2v^2}{c^2} - 1$$

$$c_2r^{-1} = \frac{2v^2}{c^2} - 1 \quad (10)$$

For particle at rest when speed is neglected, i.e. when $(v = 0)$ one gets

$$c_2r^{-1} = -1$$

$$r = -c_2 \quad (11)$$

From equation (11) we to get:

$$r = \frac{2GM}{c^2} \quad (12)$$

(r, the minimum radius of a collapsed star or black hole)

Thus the radius which makes E minimum is given by

$$r_c = \frac{2GM}{c^2} \quad (13)$$

Generalized special relativistic energy (7) expression, when the stars particles speed are small compared to speed of light

$$\frac{v^2}{c^2} \ll 1$$

Thus

$$E = m_0c^2\left(1 - \frac{2GM}{c^2r}\right)\left(1 - \frac{2GM}{c^2r}\right)^{-1/2} = m_0c^2\left(1 - \frac{2GM}{c^2r}\right)^{1/2} \quad (14)$$

Minimum energy

$$\frac{dE}{dr} = m_0c^2\left(\frac{2MG}{r^2c^2}\right)\left(\frac{1}{2}\right)\left(1 - \frac{2MG}{rc^2}\right)^{-1/2} = \frac{m_0c^2\left(\frac{MG}{r^2c^2}\right)\left(1 - \frac{2MG}{rc^2}\right)}{\left(1 - \frac{2MG}{rc^2}\right)^{3/2}} = 0$$

Thus the radius which makes E minimum is given by

$$1 - \frac{2MG}{rc^2} = 0$$

$$\frac{2MG}{rc^2} = 1$$

Critical radius

$$r_c = \frac{2MG}{c^2} \quad (15)$$

M Was, originally, the mass of a star that collapsed into a black hole. r is radius equivalent to Schwarzschild radius. Thus, we found the formula for the Schwarzschild radius of a black hole as it is generally written in the literature review [1,2,3]:

$$r_s = \frac{2GM}{c^2} \quad (16)$$

According to generalized special relativity there is a short range repulsive gravitational force given by [5]:

$$\varphi_s = \frac{GM}{r} e^{-\frac{r_c}{r}} \quad (17)$$

$$\varphi_l = -\frac{GM}{r} \tag{18}$$

Find the mass of the star, consider particles with energy E:

$$dW = \frac{dmc^2}{2} = Fdr \tag{19}$$

Then

$$dmc^2 = 2Fdr \tag{20}$$

$$dmc^2 = 2\frac{GMm}{r^2}dr$$

$$\frac{dm}{m} = \frac{2GM}{r^2c^2}dr \tag{21}$$

Integrate on both sides of the equation:

$$Ln\,m = -\frac{2GM}{rc^2} + c_0$$

$$m = e^{\frac{-2GM}{rc^2} + c_0} = e^{c_0}\,e^{\frac{-2GM}{rc^2}}$$

There

$$e^{c_0} = c_3$$

$$m = c_3 e^{-\frac{2GM}{rc^2}} \tag{22}$$

Where, c_3 : is a constant.

Suppose the rest mass of the star in the absence of gravitation is m_0 . Then at infinity we have $m = m_0$, which leads to $c_3 = m_0$ and critical radius $r = r_c$. From equation (22) we find:

$$m = m_0 e^{-\frac{2GM}{rc^2}} \tag{23}$$

The energy in this case:

$$E_{r_c} = m_0 c^2 e^{-\frac{r_c}{r}} \tag{24}$$

For small radius r or strictly speaking small $\frac{r_c}{r}$:

$$e^{-\frac{r_c}{r}} = 1 - \frac{r_c}{r} = \left(1 - \frac{2GM}{rc^2}\right) \tag{25}$$

On the other hand, we find from equations (24) and (25):

$$E_{r_c} = m_0 c^2\left(1 - \frac{2GM}{rc^2}\right) \tag{26}$$

But

$$M = m_0 \quad , \quad R = r \tag{27}$$

For attractive force

$$E = Mc^2 - \frac{2GM^2}{R} \tag{28}$$

The critical mass of the star requires minimizing the total energy E and can be found by using the conditions for minimum value, i.e.

$$\frac{dE}{dM} = c^2 - \frac{4GM}{R} = 0$$

$$c^2 = \frac{4GM}{R} \tag{29}$$

$$M = \frac{Rc^2}{4G} \tag{30}$$

$$M = \frac{R}{2G}c_a^2 = \frac{R}{2G}\left(\frac{c_m}{\sqrt{2}}\right)^2 = \frac{R}{2G}c^2 \tag{31}$$

For

$$c \to c_a = \frac{c_m}{\sqrt{2}} = \frac{c}{\sqrt{2}} \tag{32}$$

The mass at which E is minimum is given conforms with that of black hole to general relativity. Therefore radius of the black hole we find:

$$R = \frac{2GM}{c^2} \tag{33}$$

But the critical mass is given by equation (33), i.e.

$$r_c = \frac{2Gm_c}{c^2} \tag{34}$$

$$m_c = \frac{c^2 r_c}{2G}$$

For

$$\frac{m_c}{r_c} = \frac{c^2}{2G} = c_4 \tag{35}$$

One can know the critical density σ_c of the material "when we take the hollow particle model [6]". In this case the surface density is given by:

$$\sigma = \frac{m_c}{A} = \frac{m_c}{4\pi r_c^2} \quad , \quad A = 4\pi {r_c}^2 \tag{36}$$

The critical density satisfies

$$\sigma_c = \frac{m_c}{r_c^2} \tag{37}$$

Where

$$\sigma_c = 4\pi\sigma \tag{38}$$

Also, from equation (37) we find:

$$\sigma_c r_c = \frac{m_c}{r_c} = \frac{c^2}{2G} = c_4 \tag{39}$$

Wherec_4: is a constant.

Conclusion and Discussion

Critical a useful expressions for black hole radius, mass and energy were derived using generalized special relativistic energy expression and the star evolution model [5]. In the first approach short range repulsive beside long range attractive gravity force (see equations (17) and (18)) were used. A useful expression for finite energy and mass were found in equations (24). It shows that the star critical radius was that of general relativity black hole radius as in the equations (13) and (16), (34). An equation (23) and (24) indicates finite mass and energy with energy in the form of a hollow sphere. This is since equation (23) shows that $m \rightarrow 0$ at $r \rightarrow 0$, m is maximum at $r \rightarrow \infty$.

This mathematical model was simple compared to general relativity model. Using this model, one derives the Schwarzschild radius. The mass and energy are shown to be finite with mass in the form of a hollow sphere.

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