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On The Study of Slow and Fast Ion-Acoustic Solitary Waves In An Electron-Positron-Plasma

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Abstract

The propagation of slow and fast ion-acoustic solitary wave (IASW) in an electron-ion-positron (e-i-p) plasma having isothermal electrons, isothermal positrons and warm streaming positive ions is studied analytically. Some simple necessary and sufficient conditions are derived for the existence of IASW in the e-i-p plasma. The expressions of critical values of positron density, positron temperature and ion temperature for the existence of IASW have been obtained and discussed graphically. The Limiting values of soliton- speed for slow and fast ion-acoustic solitary wave have also been calculated and graphically discussed. It is seen that slow and fast IASW would be excited in the e-i-p plasma in presence of the ion-stream. The profiles of slow and fast IASW for different values of ion-stream velocity, positron density and positron temperature are drawn and discussed.

Keywords: Electron-ion-positron plasma, Slow and Fast ion-acoustic solitary wave, Necessary and sufficient conditions, Critical values of phase velocity for slow and fast wave.

1. Introduction

A lot of works on theoretical and experimental investigation of the solitary waves and double layers have been done in last few years by various authors considering two-component(electrons-ions) plasma[1-3], three-component plasma (electrons, positive and negative ions) [4-7] and dusty plasma (electrons, positive ions and charged dust particles[8-13]. But, in recent years, propagation of waves in astrophysical plasma are being investigated in presence of positrons together with electrons and ions [14-17]. It is believed that positron in the plasma has its origin in the early stage of universe [18-20] and this has important contribution on the physical processes in galactic nuclei [21], pulsar magnetosphere [22], polar caps of neutron star [23]. It has been observed that nonlinear waves in electron-ion-positron (e-i-p) plasma behave differently from the plasma consisting of electrons and ions[24]. Some works of ion acoustic solitons in an e-i-p plasma have been carried out by Pillay and Bharuthram [25]. It is found that both compressive and rarefactive solitons are being excited in e-i-p plasma. Later, Popel et al [26] have shown that the presence of positron in a multispecies plasma can result in the reduction of the ion-acoustic soliton amplitudes. Subsequently, Shukla et al.[27], Alinejad[28], Ghosh and

Bharuthram [29],Masud et al.[30] have obtained interesting results on the nonlinear propagation of ion acoustic waves in an e-i-p plasma with trapped electrons. Recently, Pakzad [31] has theoretically investigated IASW in a weakly `relativistic plasma with nonthermal electrons, positrons and warm ions. It is shown that relativistic effect has considerable contribution on the excitation of solitary waves in presence of stream velocity of the ions which was first shown by Das and Paul [32].

On the other hand , it is known that in a non-collisional plasma Landau damping is very much important on the propagation of waves in plasma [33-36]. Roychowdhury et al. [37], Bandyopadhyay and Das [38] have studied IASW in a Landau damped plasma. Recently, Ghosh and Bharuthram [39] have theoretically investigated the ion-acoustic solitary wave in e-i-p plasma considering Landau damping and neglecting the particle trapping effect in e-i-p plasma using the reductive perturbation method. It is found that Landau damping causes the solitary amplitude to decay with time. Moreover, phase velocity of linear wave and soliton amplitude are increased with the increase of positron density in the absence of Landau damping, whereas these are increased with the increase of positron temperature in such plasma.

It is to be mentioned that the influence of drift motion of the ions on the IASW in e-i-p plasma has not been explored critically. Earlier authors have used the reductive perturbation method [1] for their studies of IASW in e-i-p plasma. However, pseudo potential method [40] has been used by many authors namely Popel et al. [26], Pakzad [41,42] and other authors [43-47] for theoretical investigation of IASW in e-i-p plasma.

In the present paper, propagation of slow and fast IASW in an e-i-p plasma having isothermal electrons, isothermal positrons and warm streaming positive ions is analytically studied. Some necessary and sufficient conditions are derived for the existence of IASW in the e-i-p plasma. The expressions of critical values of positron density, positron temperature and ion temperature for the existence of IASW have been obtained and discussed graphically. The Limiting values of soliton- speed for slow and fast ion-acoustic solitary wave have also been calculated and graphically discussed. It is seen that slow and fast IASW would be excited in the e-i-p plasma in presence of the ion-stream. The profiles of slow and fast IASW for different values of ion-stream velocity, positron density and positron temperature are drawn and discussed.

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2. Basic Equations

We consider collisionless non-relativistic e-i-p plasma consisting of isothermal electrons, isothermal positrons and warm positive ions with streaming motion. The system of basic equations governing the one-dimensional propagation of ion-acoustic waves in such plasmas are given by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0 \qquad (1)$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}+\frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}=-\frac{\partial \phi}{\partial x} \quad (2)$$

$$\frac{\partial p_i}{\partial t}+v_i\frac{\partial p_i}{\partial x}+3p_i\frac{\partial v_i}{\partial x}=0 \quad (3)$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_i-n_p \quad (4)$$

where

$$n_e=\exp(\varphi), n_p=\chi\exp(-\sigma_p\varphi), \chi=n_{p0}/n_{e0}=1-n_{i0},$$
$$\sigma_p=T_e/T_p, \sigma_i=T_i/T_p \quad (5)$$

n_e, n_i, n_p is the number density of electrons, ions and positrons normalized by the equilibrium density n_0 ; v_i is the ion fluid velocity normalized by the ion-acoustic speed $(K_BT_e/m_i)^{1/2}$ in which K_B is the Boltzmann constant and m_i is ion mass ; p_i is the ion pressure normalized by $n_0K_BT_i$; ϕ is the electrostatic potential normalized by K_BT_e/e , e being the magnitude of electronic charge ; the space variable x is normalized by the Debye length $(K_BT_e/4\pi n_0e^2)^{1/2}$ and time t by the ion plasma period $(m_i/4\pi n_0e^2)^{1/2}$.

3. Formulation and Analytical Study

To obtain a solution of IASW, we introduce the single independent variable η defined by the relation $\eta=x-Vt$, where V is the velocity of the solitary wave.

Now, using the boundary conditions

$$n_i\to n_{i0}, v_i\to v_{io}, p_i\to p_{i0}, \phi\to 0 \quad \text{at} \quad |\eta|\to\propto \quad (6)$$

and the charge neutrality condition $\chi+n_{io}=1$, Eqs. (1) – (4) in the stationary frame can be integrated to give,

$$n_i=\frac{n_{i0}(V-v_{i0})}{(V-v_i)} \quad (7a)$$

$$p_i=\frac{p_{i0}(V-v_{i0})^3}{(V-v_i)^3} \quad (7b)$$

$$\phi(v_i)=V(v_i-v_{i0})-\frac{1}{2}(v_i^2-v_{i0}^2)-\frac{3\sigma_i p_{i0}(V-v_{i0})^2}{2n_{i0}(V-v_i)^2}[\frac{(V-v_{i0})^2}{(V-v_i)^2}-1] \quad (7c)$$

$$n_i = \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}}\left[\left\{\left(V - v_{i0} + \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2 - 2\varphi\right\}^{1/2} - \left\{\left(V - v_{i0} - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2 - 2\varphi\right\}^{1/2}\right] \quad (8)$$

For real value of n_i the restriction on φ from Eq.(8) is

$$0 < \varphi < \frac{1}{2}\left(V - v_{i0} + \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2$$

Using.(6) , (7a) and (7b) in Eq.(4) we obtain,

$$\frac{d^2\phi}{d\eta^2} = P(v_i) \quad (9)$$

where

$$P(v_i) = \exp(\varphi) - \frac{n_{i0}(V - v_{i0})}{(V - v_i)} - \chi\exp(-\sigma_p\varphi) \quad (10a)$$

Eq.7(c) leads to

$$\frac{d\phi}{du_i} = (V - v_i) - \frac{3\sigma_i p_{i0}(V - v_{i0})^2}{n_{i0}(V - v_i)^3} \quad (10b)$$

Integrating (9), one gets

$$\left(\frac{d\varphi}{dv_i}\right)^2 . \left(\frac{dv_i}{d\eta}\right)^2 = Q(v_i) - K \quad (11)$$

where,

$$Q(v_i) = 2\int P(v_i).\frac{d\varphi}{dv_i}dv_i$$

i.e.

$$Q(v_i) = 2\left[\exp(\varphi) - \left\{n_{i0}(V - v_{i0})v_i - \sigma_i p_{i0}\left(\frac{V - v_{i0}}{V - v_i}\right)^3\right\} + \frac{\chi}{\sigma_p}\exp(-\sigma_p\varphi)\right] \quad (12)$$

and K is constant of integration.
For cold ion, when $\sigma_i = 0$,

$$Q(v_i) = 2\left[\exp(\varphi) - n_{i0}(V - v_{i0})v_i + \frac{\chi}{\sigma_p}\exp(-\sigma_p\varphi)\right]$$

However, in view of (9) and (12) one has to ensure that

i). $\left(\frac{d\varphi}{dv_i}\right)^2 . \left(\frac{dv_i}{d\eta}\right)^2 > 0$

ii). v_i and $\left(\frac{dv_i}{d\eta}\right)$ must be bounded

iii). $P(v_i) \to \pm\infty$ when $v_i \to V$

iv). For solitary wave solution of Eq.(9), $\left(\frac{d\varphi}{dv_i}\right) > 0$ for $v_{i0} < v_i < v_i'$ where v_i' is given by

$$\left.\frac{d\varphi}{dv_i}\right|_{v_i = v_i'} = 0$$

To obtain physically admissible solution of Eq.(9) for the ion-acoustic solitary wave in e-i-p plasma it is easy to make the following observations :

Observation **1:**

$$\varphi(v_i)\big|_{v_i = v_{i0}} = 0$$

and $\varphi(v_i)\big|_{v_i = V} = \infty$

Observation **2:**

$$P(v_i)\big|_{v_i = v_{i0}} = 1 - n_{i0} - \chi = 0$$

and $P(v_i)\big|_{v_i = V} = \infty$

Observation **3:**

$$Q(v_i)\big|_{v_i = v_{i0}} = 2[1 - n_{i0}(V - v_{i0}) - \sigma_i p_{i0} + \frac{\chi}{\sigma_p}]$$

and $Q(v_i)\big|_{v_i = V} = \infty$

Observation **4:**

The Function $P(v_i)$ vanishes at most once between two values of v_i, viz. v_{i0} and V.

Proof: Let us assume that $P(v_i)$ vanishes at two different values of v_i, say v_1 and v_2 where either

a) $v_{i0} < v_1 < v_2 < V$,

or

b) $V < v_1 < v_2 < v_{i0}$

From (10a) one can obtain $P(v_{i0}) = 0$

Now, by Rolle's Theorem there exists two values of v_i, say v_3 and v_4 such that

$$P'(v_3) = P'(v_4) = 0 \tag{13}$$

Where, $P'(v_i)=\dfrac{dP}{dv_i}$ and either a) $v_{i0}<v_1<v_3<v_2<v_4<V$, and b) $V<v_1<v_3<v_2<v_4<v_{i0}$

From Eq.(10) and (13) we get

$Q(v_i)=0$ at $v_i=v_3,v_4$

where,

$$Q(v_i)=[\exp(\varphi)+\chi\sigma_p\exp(-\sigma_p\varphi)\left[(V-v_i)-\frac{3\sigma_i p_{i0}}{n_{i0}}\left\{\frac{(V-v_{i0})^2}{(V-v_i)^3}\right\}-n_{i0}\frac{(V-v_{i0})^2}{(V-v_i)^3}\right]$$

Since,

$Q(v_i)\big|_{v_i=v_3}=0=Q(v_i)\big|_{v_i=v_4}$, we have again by Rolle's Theorem obtain that there exists a value of v_i, between v_3 and v_4 such that $\left.\dfrac{dQ}{dv_i}\right|_{v_i=v_5}=0$, so that

$$[\exp(\varphi)-\chi\sigma_p^2\exp(-\sigma_p\varphi)]\left[(V-v_i)-\frac{3\sigma_i p_{i0}}{n_{i0}}\left\{\frac{(V-v_{i0})^2}{(V-v_i)^3}\right\}-n_{i0}\frac{(V-v_{i0})^2}{(V-v_i)^3}\right]=$$

$$[\exp(\varphi)+\chi\sigma_p^2\exp(-\sigma_p\varphi)]\left[1+\frac{9\sigma_i p_{i0}}{n_{i0}}\left\{\frac{(V-v_{i0})^2}{(V-v_i)^4}\right\}-\frac{2n_{i0}(V-v_{i0})}{(V-v_i)^3}\right]\ \text{for } v_i=v_5 \qquad (14)$$

From Eq.(14) one can say that the right hand side is definite positive for either the case $v_{i0}<v_5<V$ or $V<v_5<v_{i0}$.

Hence the left hand side to be positive , the only possibility is

$[\exp(\varphi)-\chi\sigma_p^2\exp(-\sigma_p\varphi)>0$ for $V=v_5$

Since

$\left[(V-v_i)-\dfrac{3\sigma_i p_{i0}}{n_{i0}}\left\{\dfrac{(V-v_{i0})^2}{(V-v_i)^3}\right\}\right]^2$ is always is positive for either $v_{i0}<v_5<V$ or $V<v_5<v_{i0}$.

Now, from

$[\exp(\varphi)-\chi\sigma_p^2\exp(-\sigma_p\varphi)]$ for $V=v_5$ we get $\chi<[\dfrac{\exp(1+\sigma_p)\varphi}{\sigma_p^2})$ for $V=v_5$.

This is impossible. Hence $P(v_i)$ vanishes at most once between v_{i0} and V.

Observation **5:**

For solitary wave solution of Eq.(9) [or Eq.(11)], $\dfrac{d\varphi}{dv_i}>0$ for $v_{i0}<v_i<v_i'<V$ where v_i' is given by $\left.\dfrac{d\varphi}{dv_i}\right|_{v_i=v_i'}=0$. Also $\varphi(v_i)$ is maximum at $v_i=v_i'$.

Now from Eq.(10b), we get $\left.\dfrac{d\varphi}{dv_i}\right|_{v_i=v_i'}=0$. This gives finally

$$v_i' = V - \sqrt{V - v_{i0}} \cdot \sqrt[4]{\frac{3\sigma_i p_{i0}}{n_{i0}}}) = -\frac{1}{2}\left[(V - v_i)^2 - \frac{3\sigma_i p_{i0}}{(1-\chi)} \left\{ 1 - \left(\frac{V - v_{i0}}{V - v_i} \right)^3 \right\} \right] \quad (15)$$

For which $\varphi''(v_i') < 0$ and

$$\varphi_{\max} = -\frac{1}{2}\left[(V - v_i)^2 - \frac{3\sigma_i p_{i0}}{(1-\chi)} \left\{ 1 - \left(\frac{V - v_{i0}}{V - v_i} \right)^3 \right\} \right]$$

We now establish the following Theorem:
Statement:
The Eq.(9) will admit the real and bounded solution if and only if

i). $V > v_{i0} + \left(\frac{3\sigma_i p_{i0}}{n_{i0}} + \frac{1-\chi}{1+\chi\sigma_p} \right)^{1/2}$ for $V > v_{io}$

Or, $V < v_{i0} + \left(\frac{3\sigma_i p_{i0}}{n_{i0}} + \frac{1-\chi}{1+\chi\sigma_p} \right)^{1/2}$ for $V < v_{io}$

ii). $Q(v_i') - Q(v_{i0}) < 0$ for $v_{i0} < v_i < v_i' < V$ and v_i' is defined by $\left.\frac{d\varphi}{dv_i}\right|_{v_i = v_i'} = 0$.

Proof.
The condition i) is necessary , if $Q(v_i) \geq K$ for $v_{i0} < v_i < v_{\max}$ where $v_{\max}$ exists in such a way that $Q(v_{i0}) = Q(v_{\max}) = K$ for $v_{i0} < v_{\max} < v_i'$, then one may write $Q(v_i) > Q(v_{i0})$ for $v_i = v_{i0} + \lambda$, $\lambda (> 0)$ however small, an arbitrary number.
Thus we may say that
$Q''(v_{i0}) > 0$
Or,

$$P'(v_{i0}).\varphi'(v_{i0}) > 0 \quad (16)$$

From (16) we may write $P'(v_{i0}) > 0$ when $\varphi'(v_{i0}) > 0$
Thus, $P'(v_{i0})$ gives

$$\left[(V - v_i) - \frac{3\sigma_i p_{i0}}{n_{i0}(V - v_{i0})} \right].(1 + \chi\sigma_p) > \frac{n_{i0}}{(V - v_{i0})} \quad (17a)$$

Or, $V > v_{i0} + \left(\frac{3\sigma_i p_{i0}}{n_{i0}(V - v_{i0})} + \frac{1-\chi}{1+\chi\sigma_p} \right)^{1/2}$ for $V > v_{io}$

Again, from (16) we may also write

$$V < v_{i0} - \left(\frac{3\sigma_i p_{i0}}{n_{i0}(V - v_{i0})} + \frac{1-\chi}{1+\chi\sigma_p} \right)^{1/2} \quad \text{for } V < v_{io} \tag{17b}$$

The critical phase velocities for the existence of IASW are

$$V_S = v_{i0} - \left(\frac{3\sigma_i p_{i0}}{n_{i0}(V - v_{i0})} + \frac{1-\chi}{1+\chi\sigma_p} \right)^{1/2} \quad \text{for } V < u_{io} \tag{18a}$$

$$V_F = v_{i0} + \left(\frac{3\sigma_i p_{i0}}{n_{i0}(V - v_{i0})} + \frac{1-\chi}{1+\chi\sigma_p} \right)^{1/2} \quad \text{for } V > v_{io} \tag{18b}$$

V_S and V_F are the phase velocity of Slow-mode and Fast- mode of the ion-acoustic wave.
From Eq.(18b),the critical values of the positron density (χ_C), positron temperature (σ_{pC}), ion temperature (σ_{iC}) and ion density (n_{i0C}) may be obtained for which IAS will exist in plasma in presence of positrons. The critical values of these parameters are :

$$\chi_C = \frac{1}{2(1+\sigma_p M_i^2)}[\gamma \pm \{\gamma^2 - 4(1+\sigma_p M_i^2)(1+3\sigma_i p_{i0} - M_i^2)\}^{1/2}] \tag{19a}$$

$$\sigma_{pC} = \frac{1}{\chi}[\frac{(1-\chi)^2 - M_i^2(1-\chi) + 3\sigma_i p_{i0}}{M_i^2(1-\chi) - 3\sigma_i p_{i0}}] \tag{19b}$$

$$\sigma_{iC} = \frac{(1-\chi)}{3p_{i0}}[M_i^2 - \frac{(1-\chi)}{(1+\chi\sigma_p)}] \tag{19c}$$

$$n_{i0C} = \frac{1}{2}[M_i^2(1+\chi\sigma_p) \pm \{M_i^4(1+\chi\sigma_p)^2 - 12\sigma_i p_{i0}(1+\chi\sigma_p)\}^{1/2}] \tag{19d}$$

Where,

$$\gamma = 2 - (1-\sigma_p)M_i^2 - 3\sigma_p\sigma_i p_{i0},\ M_i = V - u_{i0},\ V > u_{i0}$$

4. Sagdeev Potential an solution of IASW

In previous section , we have discussed the conditions for the existence of IASW in terms of ion fluid velocity (v_i) instead of electrostatic potential ϕ or number density (n_i) of ions. However, expressing the term Q in Eq.(12) as a sole function of ϕ and integrating the resulting equation one can find out the Sagdeev potential. We can use the Sagdeev potential the properties of IASW structures in e-i-p plasma.

Now, using Eqs.(1),(2) and(9) we obtain

$$\frac{\partial^2\varphi}{\partial\eta^2} = \exp(\varphi) + \chi\exp(-\sigma_p\varphi) + \frac{n_{i0}^{3/2}}{2\sqrt{3\sigma_i p_{i0}}}[(A_1 - A_2 - 2\varphi)^{1/2} - (A_1 + A_2 - 2\varphi)^{1/2}] \tag{20}$$

Where

$$A_1 = (V - v_{i0})^2 + \frac{3\sigma_i p_{i0}}{n_{i0}} \tag{21a}$$

$$A_2^2 = \frac{12\sigma_i p_{i0}}{n_{i0}}(V - u_{i0})^2 \tag{21b}$$

Eq.(20) is now written as

$$\frac{d^2\varphi}{d\eta^2} = -\frac{d\psi}{d\varphi} \tag{22}$$

where $\psi(\varphi)$ is the Sagdeev potential [40]

Now, using the boundary conditions (6) we get from Eq.(22) after integration

$$\frac{1}{2}(\frac{d\varphi}{d\eta})^2 + \psi(\varphi) = 0 \tag{23}$$

where

$$\psi(\varphi) = \psi_e(\varphi) + \psi_p(\varphi) + \psi_i(\varphi) \tag{24a}$$

$$\psi_e(\varphi) = 1 - \exp(\varphi) \tag{24b}$$

$$\psi_p(\varphi) = \frac{\chi}{\sigma_p}[1 - \exp(-\sigma_p \varphi)] \tag{24c}$$

$$\psi_i(\varphi) = \frac{n_{i0}^{3/2}}{6\sqrt{3\sigma_i p_{i0}}}[(A_1 - A_2 - 2\varphi)^{3/2} - (A_1 + A_2 - 2\varphi)^{3/2} - (A_1 - A_2)^{3/2} + (A_1 + A_2)^{3/2}] \tag{24d}$$

The form of the pseudo-potential $\psi(\varphi)$ would determine whether a solitary wave like solution of Eq. (22) will exist or not. The conditions for the existence of solitary wave solution are:

i) $\psi(\varphi) = 0$ at $\varphi = 0$ and $\varphi = \varphi_c$

(25a)

ii) $\frac{d\psi}{d\varphi} = 0$ at $\varphi = 0$

(25b)

and

iii) $\frac{d^2\psi}{d\varphi^2} < 0$ at $\varphi = 0$

(25c)

An analytical solution of Eq.(20) for the IASW can be obtained by expanding the right hand side in terms of φ keeping the terms up to second- order ,third- order or even next higher orders of φ. After a straight forward calculation one can get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 + R\varphi^3 + \cdots\cdots \tag{26}$$

where,

$$P = 1 + \frac{(1-\chi)^{3/2}}{2\sqrt{3\sigma_i}}\left(\frac{1}{G_1^{1/2}} - \frac{1}{G_2^{1/2}}\right) + \chi\sigma_p \tag{27a}$$

$$Q=-\frac{1}{2}[1+\frac{(1-\chi)^{3/2}}{2\sqrt{3\sigma_i}}(\frac{1}{G_1^{3/2}}-\frac{1}{G_2^{3/2}})-\chi\sigma_p^2] \quad (27b)$$

$$R=\frac{1}{2}[\frac{1}{3}+\frac{(1-\chi)^{3/2}}{2\sqrt{3\sigma_i}}(\frac{1}{G_1^{5/2}}-\frac{1}{G_2^{5/2}})+\frac{1}{3}\chi\sigma_p^3] \quad (27c)$$

$$G_1=A_1+A_2,\ G_2=A_1-A_2,\ A_1=(V-v_{i0})^2+\frac{3\sigma_i}{(1-\chi)},\ A_2^2=\frac{12\sigma_i}{(1-\chi)}(V-v_{i0})^2. \quad (28)$$

For Slow-mode and Fast-mode of the IASW, the velocity V in (21a) and (21b) is replaced by V_S and V_F of (27d) .

So, we obtain

$$A_{1S,1F}=(V_{S,F}-u_{i0})^2+\frac{3\sigma_i p_{i0}}{n_{i0}} \quad \text{and} \quad A^2_{2F,2S}=\frac{12\sigma_i p_{i0}}{n_{i0}}(V_{F,S}-v_{i0})^2 \quad (29a)$$

$$.\ G_{1S,1F}=A_{1S,1F}+A_{2S,2F} \quad \text{and} \quad G_{2S,2F}=A_{1S,1F}-A_{2S,2F} \quad (29b)$$

Now, for small amplitude IASW , the terms up to φ^2 in Eq.(24) are taken into consideration, neglecting next higher -order terms of φ i.e. φ^3, φ^4 etc. So, we get for Slow and Fast mode of small amplitude IASW Eq.(26) gives

$$\frac{d^2\varphi_{S,F}}{d\eta^2}=P_{S,F}\varphi_{S,F}-Q_{S,F}\varphi_{S,F}^2 \quad (30)$$

Using the standard mathematical technique, the solution of Eq.(30) for the small amplitude IASW for Slow and Fast- mode we obtain:

$$\varphi_{1S}=\frac{3P_S}{2Q_S}\sec h^2\theta_S,\quad \varphi_{1F}=\frac{3P_F}{2Q_F}\sec h^2\theta_F \quad (31)$$

where, $\theta_{S,F}=[(P_{S,F}/4)^{1/2}\eta]$.

The amplitude and width for the Small amplitude IAS are:

$$\varphi_{01S} = \frac{3P_S}{2Q_S} \quad , \qquad \varphi_{01F} = \frac{3P_F}{2Q_F} \tag{32}$$

And

$$\delta_{1S} = \frac{2}{\sqrt{P_S}}, \; \delta_{1F} = \frac{2}{\sqrt{P_F}} \tag{33}$$

5.Results and Discussion

5.1.Profiles of Critical Values of the Phase Velocity

The critical value of the phase velocity for the excitation of IAS in e-i-p plasma is obtained from the expression (18a) and (18b) give two values of the phase velocity, one is Slow- mode (V_S) and the other is Fast- mode (V_F) given by (18a) and (18b). The variations of V_S and V_F with the positron density (χ) and ion drift velocity (v_{i0}) are shown in Fig.1(a) and Fig.1(b).

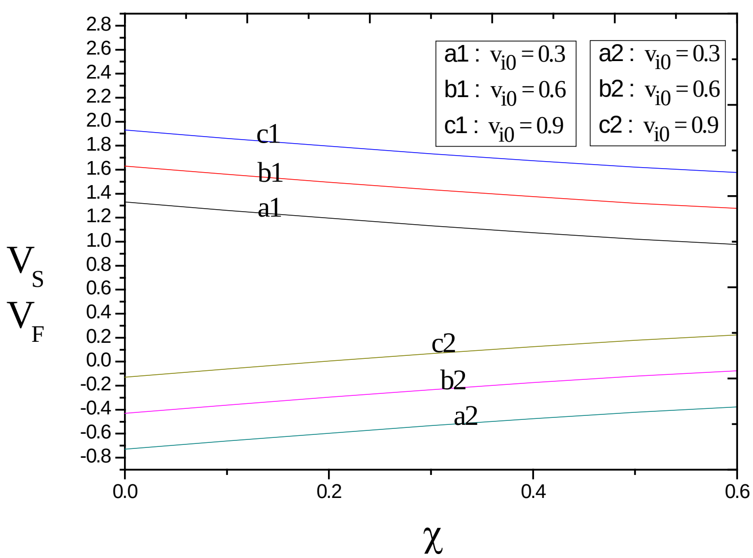


Fig. 1(a). Variation of phase velocity of wave with positron density (χ) for different values of ion stream velocity (u_{i0}) in e-i-p plasma having σ_i = 0.02 σ_p = 0.5, p =1 . The lines a1,b1, c1 represent fast-mode (V_F) and the lines a2, b2, c2 represent slow mode (V_S) of phase velocity. Curves a1, b1, c1 and a2, b2, c2 represent v_{i0} = 0.3, 0.6 and 0.9 respectively.

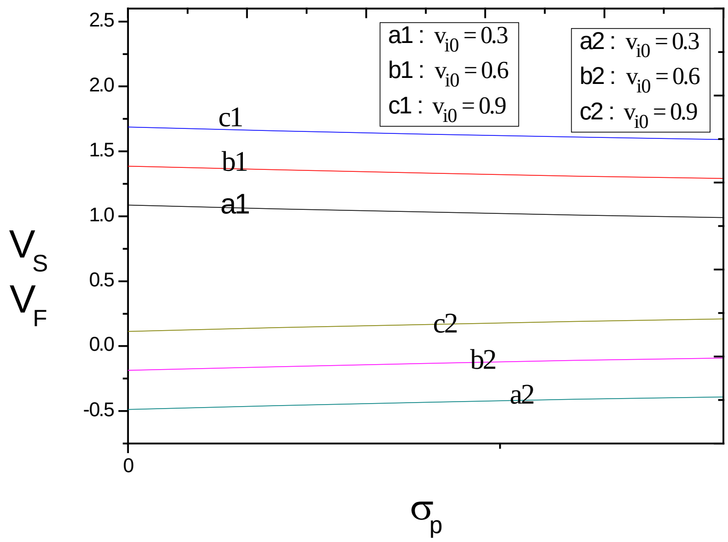
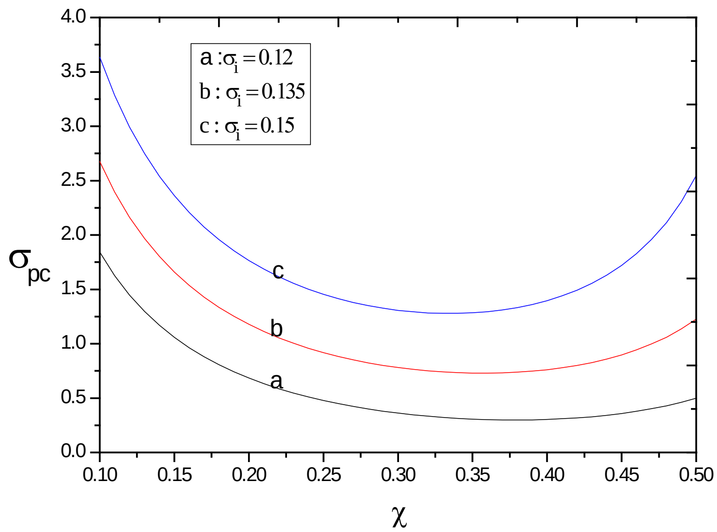


Fig.1(b) Variation of phase velocity of wave with positron temperature (σ_p) for different values of ion stream velocity (u_{i0}) in e-i-p plasma having σ_i =0.02, χ = 0.3, p =1 . The lines a1,b1, c1 represent Fast-mode (V_F) and the lines a2, b2,, c2 Slow mode (V_S) of phase velocity. Curves a1, b1, c1 and a2, b2, c2 represent v_{i0} = 0.3, 0.6 and 0.9 respectively.

In an e-i-p plasma having the ion drift velocity (v_{i0}) = 0.3 , 0.6 and 0.9; σ_i =0.02; σ_p = 0.5; V=1.6 and p_{i0}=1, Fig.1(a) shows that V_F is always positive and it decreases with the increase of χ but V_S is always negative and it increases with the increase of χ. Similarly, the variations of V_F and V_S with the positron temperature (σ_p) and ion drift velocity (v_{i0}) are numerically estimated and are shown graphically in Fig. 1(b). It is found that V_F is always positive and it decreases with the increase of σ_p. But, V_S is negative it increases with the increase of σ_p and v_{i0} . The positive value of V_F indicates that the fast-mode of the wave is propagating in e-i-p plasma.. But, the negative value of V_S for v_{i0} = 0.3 to 0.6 means the Slow-mode of the wave is non-propagating in the e-i-p plasma, whereas positive V_S for v_{i0} = 0.9 indicates that the slow-mode is propagating through the plasma. So, to study nonlinear propagation of IAW in e-i-p plasma for fast mode (V_F) and slow mode (V_S), the value of ion-stream must be taken as $v_{i0} \geq 0.9$.

5.2. Profiles of Critical Values of Plasma Parameters

It is to be noted that IAS will be excited in e-i-p plasma only for certain restricted regions of plasma parameters. The critical values of various plasma parameters for the excitation of IAS in e-i-p plasma may be obtained by using the expressions (19a) , (19b), (19c), (19d) and following which the condition for the existence IAS solution would be obtained. Fig.2(a) shows the variation of critical positron temperature (σ_p) with positron density for different ion temperature (σ_i) in plasma having V=1.2, u_{i0} =0.1 and p =1; and Fig.2(b) shows the variation of critical density of positrons (χ_c) with ion stream velocity (v_{i0}) for different value of ion temperature (σ_i) in plasma having σ_p =0.1 , V =1.5 and p=1.



Fig,2(a).Variation of critical positron temperature (σ_{pC}) with positron density (χ) for different ion temperature (σ_i) in e-i-p plasma having V=1.2, v_{i0} = 0.1 and p=1. Curves a, b and c represent σ_i =0.12, 0.135 and 0.15 respectively.

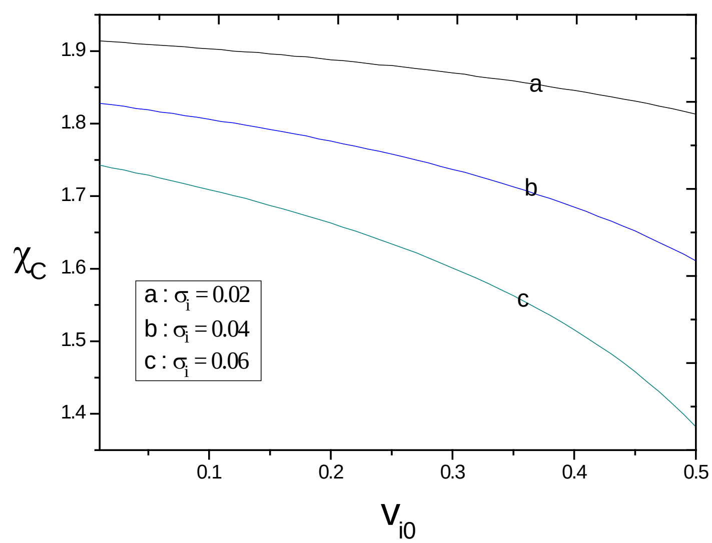


Fig.2(b). Variation of critical density of positrons (χ_c) with ion stream velocity (v_{i0}) for different value of ion temperature (σ_i) in plasma having σ_p =0.1 , V =1.5 and p=1. Curves a, b and c represent σ_i =0.02, 0.04 and 0.06 respectively.

Fig. 2(a) shows that critical positron temperature (σ_{pC}) decreases with positron density and reaches to a minimum value at certain positron density and it starts to increase. Moreover, σ_{pC} increases with the increase of ion temperature. On the other hand, it is seen from Fig. 2(b) that the critical positron density (χ_C) decreases with the increase of ion stream velocity (v_{i0}) and ion temperature (σ_i) in the e-i-p plasma.

5.3. Profiles of IASW for the Phase Velocities of Slow-Mode and Fast-Mode

From Eq.(18a) and Eq.(18b) we see that ion-acoustic wave propagates with two modes in presence of an ion stream in e-i-p plasma., one is Slow mode (V_S) and the other is Fast- mode (V_F). The variations of V_S and V_F with positron density and positron temperature for different values of ion-stream velocity on the Slow-mode and Fast-mode of the wave are shown in Fig. 1(a) and Fig. 1(b). So, to see the real nature of IAS in an e-i-p plasma in presence of an ion stream, the values of the phase velocities V_S and V_F should be used for drawing the profiles of the IASW. Using V_S and V_F for our model e-i-p plasma, the profiles of small amplitude IAS for the fast-mode and slow-mode are drawn for different values of the parameters of e-i-p plasma, e.g. positron density, positron temperature, ion stream velocity etc.

i). Effect of ion stream velocity

The main factor of the excitation of Slow-mode and Fast- mode of IAS in e-i-p plasma is the stream velocity of ions. To see the effects of ion stream velocity on the Slow-mode and Fast-mode of the IASW , we assume a model e-i-p plasma and the profiles of IAS are drawn as shown in Fig. 4(a) and Fig. 4(b).

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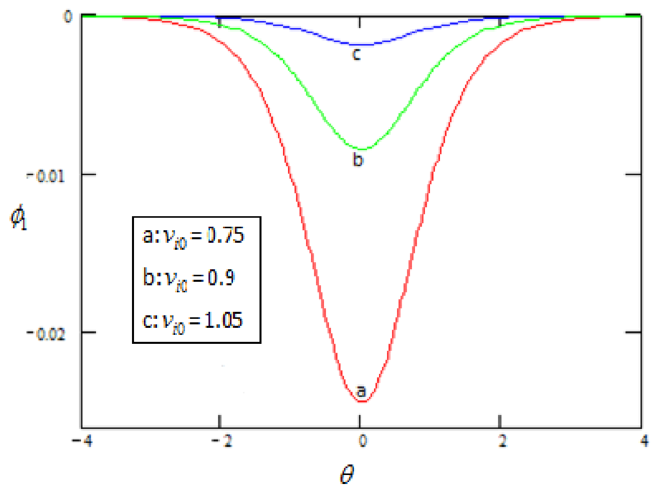

Fig.3(a). Profiles of Small amplitude IASW of Slow-mode for different value of ion stream (v_{i0}) in e-i-p plasma having σ_i =0.001, σ_p =0.5, χ =0.1, $p=1$. Graphs a, b and c represent v_{i0} = 0.75,0.9 and 1.05 respectively.

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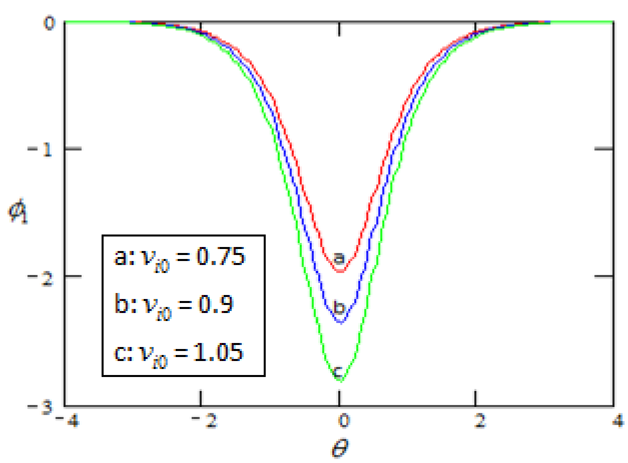

Fig. 3(b). Profiles of Small amplitude IASW of Fast- mode for different value of ion stream (v_{i0}) in e-i-p plasma having σ_i =0.001, σ_p =0.5, χ =0.1, $p=1$. Graphs a, b and c represent v_{i0} = 0.75,0.9 and 1.05 respectively

It is observed from Fig.3(a) and Fig.3(b) that small amplitude IASW for Slow-mode and Fast-mode are rarefactive. for Slow-mode, the decrease of ion stream velocity increases the amplitude of IASW. But, for Fast-mode, the amplitude of IASW increases with the increase of ion stream velocity.

ii). Effect of positron density

Taking a model e-i-p plasma having different values of plasma parameter, the profiles of Small-amplitude IAS for different values of positron density for Slow-mode and Fast- mode of the waves are shown in Figs.4(a) and 4(b).

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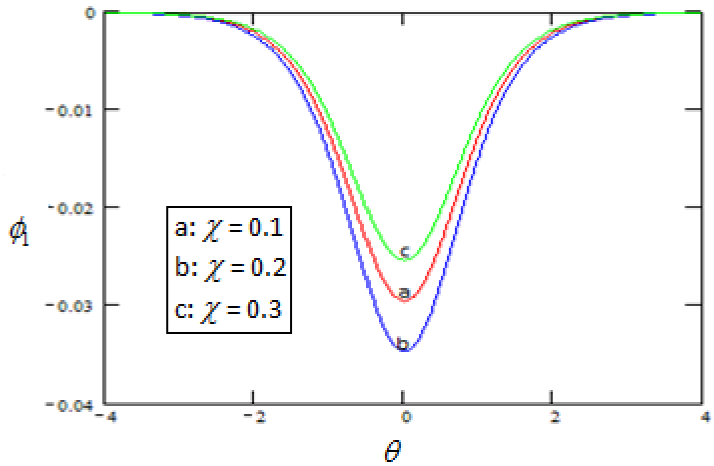

Fig.4(a). Profiles of Small amplitude IASW of Slow-mode for different value of positron density in e-i-p plasma having v_{i0} =0.9, σ_i =0.01, σ_p =0.5, $p=1$. Graphs a, b and c represent χ = 0.1 ,0.2 and 0.3 respectively.

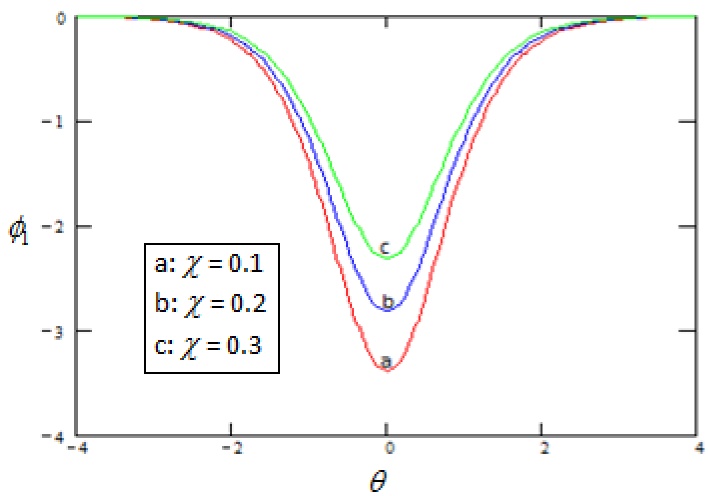

Fig.4(b). Profiles of Small amplitude IASW of Fast- mode for different value of positron density in e-i-p plasma having v_{i0} = 0.9, σ_i = 0.01, σ_p = 0.5, $p=1$. Graphs a, b and c represent χ = 0.1 , 0.2 and 0.3 respectively.

Fig. 4(a) and 4(b) show that Small amplitude IASW for both Slow-mode and Fast-mode are rarefactive. In both modes, the amplitude of IAS increases with the decrease of positron density. The amplitude of IAS for Slow-mode is much smaller than that of the Fast-mode of IASW.

iii). Effect of positron temperature

Similarly, the profiles of small amplitude and large amplitude IASW in an e-i-p plasma for different values of positron temperature for Slow-mode and Fast-mode of IASW are shown in Figs. 5(a) and 5(b).

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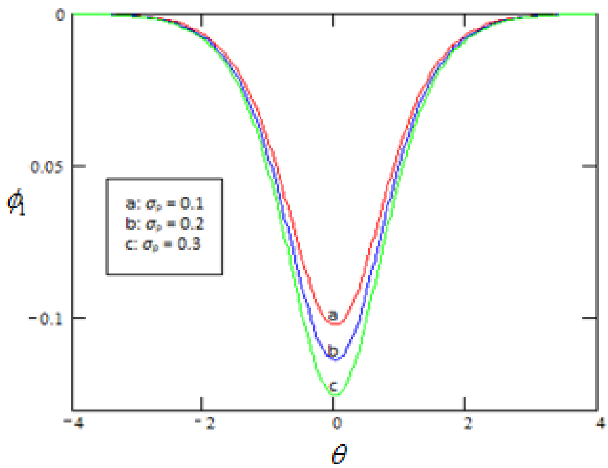

Fig.5(a). Profiles of Small amplitude IASW of Slow- mode for different value of positron temperature in e-i-p plasma for fixed values of v_{i0} = 1.2, σ_i = 0.01, χ = 0.5, $p = 1$. Graphs a, b and c represent σ_p = 0.1 , 0.2 and 0.3 respectively.

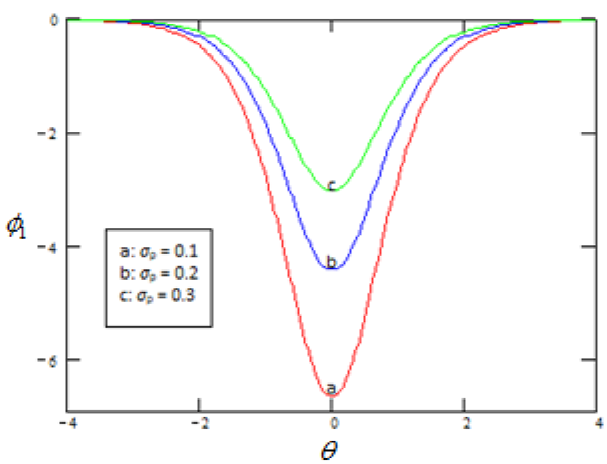

Fig.5(b). Profiles of Small amplitude IASW for Fast- mode for different value of positron temperature in e-i-p plasma for fixed values of v_{i0} = 1.2, σ_i = 0.01, χ = 0.5, $p = 1$. Graphs a, b and c represent σ_p = 0.1 , 0.2 and 0.3 respectively.

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It is observed from Figs.5(a) and 5(b) that Small amplitude IASW for different values of positron temperature for Slow-mode and Fast-mode of the IASW are rarefactive in e-i-p plasma.. For Slow-mode, the amplitude of IAS increases due to decrease of positron temperature. But, For Fast-mode, the amplitude of IASW decreases with the increase of positron temperature. Similarly, the effects of ion temperature on the IASW for the Slow- and Fast- mode can be studied in e-i-p plasma having the stream of ions. In this case, some interesting results would also be obtained which may be important in the context of nonlinear wave processes in drifting plasma.

6. Summary and Conclusion

We have studied Slow-mode and Fast-mode of IASW in a collisionless, unmagnetized electron-ion-positron plasma consisting of warm streaming ions , isothermal electrons and isothermal positrons. and nonthermal electrons using an analytical method. Some necessary and sufficient conditions have been derived for Slow and Fast IASW in an e-i-p plasma. Moreover, the expressions for the critical values phase velocity, positron density, positron temperature and ion temperature are obtained for the existence of Slow and Fast IASW structure. The dependence of these quantities on the IASW has been depicted graphically and discussed. An important observation is that the presence of streaming ions gives rise to excite the Slow and Fast IASW waves in the electron-ion-positron plasma. The effects of positrons, stream velocity of ions on the structure of Slow and Fast IASW are studied graphically. Finally, it is to be pointed out that the analysis presented in this paper can be applied to study both electro-static and ion-acoustic solitons in a multi-species plasma having nonthermal electrons, superthermal electrons, negative ions, positrons, streaming of ions and electrons which are found in space plasmas and cometary plasmas.

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