Quantization of the Rest Mass and Energy Using Schrodinger Equation Based on the Relativistic Energy Non Zero Rest Mass Relation

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Abstract:
The ordinary conventional Schrodinger equation is based on the Einstein energy - momentum relativistic relation for zero ret mass. This motivates searching for a new quantum Schrodinger equation based on non-zero rest mass energy - momentum relativistic relation. The new equation consists of an additional mass term and reduced to the conventional one for zero ret mass. It predicted a stationary time independent wave solution. For the case of a particle in a box it shows that the mass is quantized and is strongly dependent on the box size.

Key words: Schrodinger equation, rest mass, energy momentum relativistic relation, mass quantization

Introduction:
In spite of the great success which is done by the Schrodinger equation [1, 7], it cannot describe the relativistic phenomena because it is not contain a term which is responsible for that. In this research, the Schrodinger equation based on the new momentum relativistic operator [2, 3], which affects the kinetic energy as well as the total energy of the particle, the total energy is written as the sum of the kinetic and potential energies. The derivation of the Schrodinger equation from a new momentum relativistic operator, adds a new important term to the Schrodinger equation. This term is called the energy mass and leads to mass and energy equivalence [4], which shows agreement between relativistic mechanics and quantum mechanics. The discovery of mass energy equivalence proved crucial improvement to the development of theories of atomic fusion and fission reaction. [5] This new Schrodinger equation is solved for a one dimensional particle in an infinitely box [6, 8]. The solution is in agreement with the ordinary one of the normal Schrodinger equation. The potential energy is constant within the box and it is equal to zero in this region. Since the walls are infinitely high, the potential energy is assumed to be infinite. So the particle cannot penetrate the walls and exist outside the box. Consequently the wave function must be zero outside the box and at the walls. It is only exists inside the box or in the well [9, 10]. The wave functions which are calculated from this new Schrodinger equation contain terms in which speed of light appears [2, 3].

Schrodinger Equation Based On the New Momentum Relativistic Operator:
According to plank hypothesis the energy E can be expressed in terms of the angular frequency w and the wave number K to be in the form.

$$E = \hbar\omega = \hbar ck(1)$$
$$E = \hbar ck = \hbar\omega$$
$$E^2 = c^2P^2 + m_0^2c^4(2)$$
$$c^2P^2 = E^2 - m_0^2c^4(3)$$
$$c^2P^2 = c^2\hbar^2k^2 - m_0^2c^4$$
$$P^2 = \hbar^2k^2 - m_0^2c^2(4)$$

$$\psi = e^{i(kx-\omega t)}$$
$$\frac{\partial\psi}{\partial t} = -i\omega\psi$$
$$i\hbar\frac{\partial\psi}{\partial t} = \hbar\omega\psi = E\psi$$

$$\frac{\partial\psi}{\partial x} = ik\psi(5)$$
$$\frac{\partial^2\psi}{\partial x^2} = (ik)^2\psi = -k^2\psi(6)$$
$$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = \hbar^2k^2\psi(7)$$
$$E = \frac{P^2}{2m} + V(8)$$
$$E\psi = \frac{P^2}{2m}\psi + V\psi(9)$$
$$E\psi = \frac{\hbar^2k^2}{2m} - \frac{mo^2c^2}{2m}\psi + V\psi(10)$$

$$m = mo(11)$$
$$E\psi = \frac{\hbar^2k^2}{2mo} - \frac{mo^2c^2}{2m}\psi + V\psi(12)$$
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2mo}\frac{\partial^2\psi}{\partial x^2} - \frac{mo}{2}c^2\psi + V\psi(13)$$

Solution for particle in a box

$$V = o(14)$$
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2\nabla^2}{2mo}\psi - \frac{moc^2}{2}\psi(15)$$
$$\psi = Ae^{-i\omega t-\propto x}(16)$$
$$\frac{\partial\psi}{\partial t} = -i\omega\psi$$
$$\nabla\psi = \frac{\partial\psi}{\partial x} = -\propto\psi$$
$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} = \propto^2\psi(17)$$
$$-i^2\hbar\omega\psi = \frac{-\hbar^2\propto^2}{2mo}\psi - \frac{moc^2}{2}\psi(18)$$
$$E = \hbar\omega(19)$$

$$-i^2\hbar\omega\psi = \frac{-\hbar^2 \propto^2}{2mo}\psi - \frac{moc^2}{2}\psi (20)$$

$$\frac{\hbar^2 \propto^2}{2mo} = -\left(E + \frac{moc^2}{2}\right)$$

$$\propto^2 = \frac{-2mo}{\hbar^2}\left(E + \frac{moc^2}{2}\right)(21)$$

$$\propto = \frac{i}{\hbar}\sqrt{2moE + m_o^2c^2}(22)$$

Consider the case when

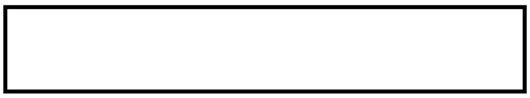
$$w = 0$$

$$E = 0(23)$$

$$\propto = \pm\frac{i}{\hbar}moc = \pm i \propto_0 \ (24)$$

$$\psi = Ae^{\pm\frac{imocx}{\hbar}} = Ae^{i\propto_0 x}(25)$$

Assume a box having dimension



$x = o$ $x = L$

Since just outside the box to the left the particle does not exist thus

$$\psi(x = o) = \psi(o) = o(26)$$

According to equation (25) this means that

$$|\psi(o)| = \left|Ae^{io}\right| = |A|^2 = o(27)$$

$$A = 0(28)$$

This means that the particle wave function vanishes everywhere according to equations (24) and (28) equation (16) becomes

$$\psi(x) = Ae^{-iwt}e^{i\propto_0 x} = 0 \ (29)$$

Which is not physically acceptable?

Now consider the boundary condition at $x = L$. The particle just exists. This means that

$$|\psi(x = L)|^2 = |\psi(L)|^2 = o$$

$$\psi(x = L) = \psi(L) = 0 \ (30)$$

According to equation (23) this means that

$$\psi(L) = Ae^{i\propto_0 L} = Acos \propto_0 L + iAsin \propto_0 L = 0 \ (31)$$

$$A \ cos \propto_0 L = o$$

$$A \ sin \propto_0 L = 0 \ (32)$$

If one assumes in equation (31) that

$$A \neq o$$

$$cos \propto_0 L = 0 \ (33)$$

$$\propto_0 L = \frac{n\pi}{2} \ (34)$$

$$n = 1, 2, 3 \ldots \ldots$$

Thus equation (31) gives

$$A \sin\frac{n\pi}{2} = 0 \ (35)$$

$$A = 0 \ (36)$$

One can try the other alternative by assuming

$$A \neq 0 \ (37)$$
$$sin \propto_0 L = 0 \ (38)$$

This means that

$$\propto_0 L = n\pi$$
$$n = 1, 2, 3 \ldots \ldots (39)$$

Thus equation (31) gives

$$A \cos n\pi = 0 \ (40)$$
$$A = 0 \ (41)$$

Thus in all cases

$$A = 0 \ (42)$$

This means that (see equation (25))

$$\psi(x) = \psi = Ae^{i\propto_0 x} = 0 (43)$$

Now consider another boundary condition by taking into account the fact that the particle exists inside the box. This means that just to the right of $x = o$
The probability of its existence isPo^2, i.e

$$|\psi(x = o)|^2 = Po^2 \ (44)$$
$$\psi(x = o) = Ae^o = A = Po(45)$$

The particle also exists just to the left of the point $(x = L)$ inside the box with probability P_l^2.his means that (see eqn (31))

$$|\psi(x = L)|^2 = P_l^2$$
$$\psi(x = L) = Ae^{i\propto_0 L} = P_l (46)$$
$$A \ cos \propto_0 L + isin \propto_0 L = P_l (47)$$

Equating the real and imaginary parts

$$A \ cos \propto_o L = P_l$$
$$Asin \propto_o L = 0$$

A ≠ 0
So

$$sin \propto_o L = 0 \ (48)$$

$$\propto_o L = n\pi$$
$$n = 2, 4, 6, 8 \ldots \ldots (49)$$

According to equation (47) thus

$$A \sin n\pi = P_l$$
$$A = P_l (50)$$

Thus according to equations (45) and (50)

$$A = P_0 = P_l \ (51)$$

In view of equations (49)and(24)

$$moc = \hbar \propto_o = \frac{n\pi\hbar}{L}$$
$$moc = \frac{n\pi h}{2\pi L} = \frac{nh}{2L}$$
$$mo = \frac{nh}{2cL} (52)$$

The probability of finding the particle in a box

$$|\psi|^2 = \left|Ae^{i\propto_o x}\right|^2 = |A|^2 = P_0^2 = P_l^2 \ (53)$$

Consider

$$\psi = A \ sin \ \propto_o x + B \ cos \propto_o x (54)$$

$$\frac{\partial\psi}{\partial x} = \propto_o A\cos \propto_o x - \propto_o B \sin \propto_o x$$

$$\frac{\partial^2\psi}{\partial x^2} = -\propto^2 A\sin \propto x - \propto^2 B \cos \propto x$$

$$= -{\propto_o}^2 (A\sin \propto_o x + B \cos \propto_o x) = -{\propto_o}^2 \psi (55)$$

A direct substitution of equation (5) and equation (55) in equation (15) gives

$$E\,\psi = -\frac{\hbar^2}{2mo}\nabla^2\psi - \frac{moc^2}{2}\psi$$

$$E\,\psi = +\frac{\hbar^2\propto^2}{2mo}\psi - \frac{moc^2}{2}\psi \ (56)$$

$${\propto_o}^2 = \frac{2mo}{\hbar^2}\left(E + \frac{moc^2}{2}\right)$$

$$\propto_o = \pm\frac{\sqrt{2mo}}{\hbar}\sqrt{\left(E + \frac{moc^2}{2}\right)}(57)$$

Applying the boundary conditions for equation (54) outside the box, gives

$$\psi(x = o) = \psi(o) = A\sin o + B \cos o = 0 \ (58)$$

Thus

$$B = 0 \ (59)$$

And eqaution (54) becomes

$$\psi = A \sin \propto_o x \ (60)$$

At

$$x = L$$

$$\psi(L) = A\sin \propto_o L = 0 \ (61)$$

Thus

$$\propto_o L = \frac{n}{2}\pi$$

$$n = 1, 3, 5, 7 \ldots\ldots (62)$$

$$\propto_o = \frac{n\pi}{2L} \ (65)$$

Conclusion & Discussion:

A new expression of the Schrodinger equation was found using the special relativistic energy - momentum relation. According to equation (1) the energy was found first in terms of the wave number K using Plank hypothesis. In equation (2) special relatively was used to find the momentum P in terms of the energy E. Then the momentum was expressed in terms of the wave number K in equation (4).The energy operator is a differential time operator as shown by equation (5).However equation (7) indicates that the spatial differential operator is related to the wave number K directly. Using these arguments Schrodinger rest mass equation was found in equation (13). This new equation reduces to the conventional one for zero ret mass. This equation was examined for particle in a box, by assuming that the potential vanishes inside the box according to equation (14).For zero frequency the Schrodinger equation still represents a time independent wave oscillating spatially as shown by equations (16) and (25). For particle in a box the exponential solution shows that the rest mass is quantized as equation (52) indicates. The rest mass is dependent on the box dimensions and is. Inversely proportional to it. If one consider a sine- cosine solution the particle energy is quantized as shown in equations (57l and (62)The Schrodinger equation which is based on non-zero rest mass energy - momentum relativistic relation consists of an additional mass term It is reduced to the ordinary equation for zero ret mass. It predicted an stationary time independent wave solution. For the case of a particle in a box it shows that the mass is quantized and dependents on the box size.

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