Scattering Potential Dependent Special Relativity Theory with Rest Mass Dependent Cross Section

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Abstract:

Potential dependent special relativity theory as used to drive new relativistic quantum equation. This new quantum equation consist of an additional rest mass term. This equation reduces to the conventional one in the absence of the rest mass term. A new expression of the scattering flux and scattering cross section was obtained. These new expressions consist of an additional term standing for rest mass. The two equations reduce to the conventional one for zero rest mass. The additional rest mass term indicates that the scattering flux and cross section is different for scattered particles having different rest masses which agrees with experimental observations and common sense.

Keywords: scattering, cross section, rest mass, flux, potential dependent special relativity.

Introduction:

Electromagnetic waves are very important role for people life. They are widely used in telecommunicaction and sensings [1].Newton's laws are the oldest laws used to describe their nature. The corpiscular theory was suggested by Newton to describe the nature of the electromagnetic waves. According to this theory light is treated as consisting of a small tiny particles. Newton's particle theory successfully describe the light reflection as well as the refraction of light [2]. This theory was completed by Huygen who formulate his well-known wave theory. According to his principle light behave as waves. This wave nature was confirmed by the Maxwell's equations [2].The wave nature of light can explain easily the laws of reflection, refraction and interference as well as diffraction [3] .Unfortunately the wave theory failed in explaining the black body radiation [4]. This encourages Max Plank to suggest that the light and electromagnetic fields behave as a discrete quanta or a particles [4].This means that they have a dual nature. This dual nature lead to the formation of quantum mechanical laws [4, 5].The oldest widely used quantum equation was made by Schrodinger [5].Schrodinger equation is mainly based on the classical Newton energy momentum relation. Later on Klein and Gordon beside Dirac formulated quantum equations based on the special relativity energy momentum relation [6].One of the most known issues that quantum laws are the interaction of electromagnetic radiation with bulk matter. The most popular one is the scattering of electromagnetic radiation with bulk matter [7].This processes can be described either by the wave model using Maxwell's equations or by the quantum model using quantum laws. The two models can describe the behavior of electromagnetic waves inside bulk matter [8, 9].This needed to understand the behavior and nature of photons and electromagnetic waves .The ordinary conventional scattering theories suffer from explaining some scattering processes concerning the neutrons and elementary particles.

General Scattering Theory:

In many cases the laws of conservation of momentum and energy alone can be used to obtain important results concerning the properties of various mechanical processes. It should be noted that these properties are independent of the particular type of interaction between the particles involved [26].

The energy according to the GSR given by

$$E = \frac{m_\circ c^2}{\sqrt{g_{oo} - v^2/c^2}} \tag{1}$$

Thus

$$E^2 = \frac{m_\circ^2 c^4}{\frac{g_{oo} m^2 c^4 - m^2 v^2 c^2}{m^2 c^4}} \tag{2}$$

But $$E = mc^2 \tag{3}$$

Inserting equation (3) in equation (2) yields

$$g_{oo} E^2 = p^2 c^2 + m_\circ^2 c^4 \tag{4}$$

Where $$g_{oo} = 1 + \frac{2\emptyset}{c^2} = 1 + \frac{2m\emptyset}{mc^2}$$

$$g_{oo} = 1 + \frac{2V}{E} \tag{5}$$

Substituting equation (5) in equation (4) yields

$$\left(1 + \frac{2V}{E}\right) E^2 = p^2 c^2 + m_\circ^2 c^4 \tag{6}$$

Rearranging equation (6) gives

$$E^2 + 2VE = p^2 c^2 + m_\circ^2 c^4 \tag{7}$$

Multiplying both side of this equation by ψ gives

$$E^2 \psi + 2VE\psi = p^2 c^2 \psi + m_\circ^2 c^4 \psi \tag{8}$$

According to the wave nature of particles

$$\psi = A e^{\frac{i}{\hbar}(px - Et)} \tag{9}$$

Differentiating equation (9) with respect to space and time yields

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi$$

$$\frac{-\hbar}{i} \frac{\partial \psi}{\partial t} = E\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = E^2 \psi$$

$$\frac{\partial \psi}{\partial x} = \frac{ip}{\hbar} \psi \quad and \quad \frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2} \psi \quad \Rightarrow \quad p^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \tag{10}$$

Substituting equation (10) in equation (9) yields

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} + 2V\left(i\hbar \frac{\partial \psi}{\partial t}\right) = -\hbar^2 c^2 \nabla^2 \psi + m_\circ^2 c^4 \psi \tag{11}$$

From this equation by suggesting a solution

$$\psi = u(r)e^{\frac{-i}{\hbar}Et} \quad (12)$$

$$\frac{\partial\psi}{\partial t} = \frac{-iE}{\hbar}\psi \quad \Rightarrow \quad \frac{\partial^2\psi}{\partial t^2} = -\frac{E^2}{\hbar^2}\psi \quad (13)$$

A direct substitution equation (13) in equation (11) yields

$$E^2u(r)e^{\frac{-i}{\hbar}Et} + 2VEu(r)e^{\frac{-i}{\hbar}Et} = -\hbar^2c^2\,\nabla^2\psi + m_\circ^2c^4\psi \quad (14)$$

$$E^2\psi + 2VE\psi = -\hbar^2c^2\,\nabla^2\psi + m_\circ^2c^4\psi \quad (15)$$

This equation can be written as

$$\nabla^2\psi + \frac{E^2}{\hbar^2c^2}\psi = \frac{(m_\circ^2c^4 - 2EV)}{\hbar^2c^2}\psi \quad (16)$$

Where:

$$\frac{E^2}{\hbar^2c^2} = k^2 \quad (17)$$

$$\frac{(m_\circ^2c^4 - 2EV)}{\hbar^2c^2} = U \quad (18)$$

Substituting (17)(18) in (16) yields

$$\nabla^2\psi + k^2\psi = U\psi \quad (19)$$

This equation represent scattered particles equation.

$$\nabla^2(GA) + k^2(GA) = A\delta_{rr'} \quad (20)$$

Integrating both sides yields

$$\int \nabla^2(GA)dr' + k^2\int(GA)dr' = \int A(r')\delta_{rr'}\,dr' = A(r) \quad (21)$$

Comparing this equation with equation (19) gives

$$A(r) = U(r)\psi(r) \quad (22)$$

$$\psi = \int(GA)dr' = \int G(r,r')A(r,r')dr' \quad (23)$$

$$\psi(r) = \int G(r,r')U(r')\psi(r')dr' \quad (24)$$

Thus the general equation to equation (19) gives:

$$\psi(r) = e^{ikr} + \int G(r,r')U(r')\psi(r')dr' \quad (25)$$

Where

$$G(r,r') = \frac{-1}{4\pi}\frac{e^{ikr}}{r}e^{-ikr'} \quad (26)$$

$$U = \frac{(m_\circ^2 c^4 - 2EV)}{\hbar^2 c^2} \qquad (27)$$

Substituting (26) (27) in equation (25) yields

$$\psi(r) = \frac{-1}{4\pi} \frac{e^{ikr}}{r\hbar^2 c^2} \int e^{-ik.r'} \psi(r')(m_\circ^2 c^4 - 2EV(r'))dr' \qquad (28)$$

This equation can be written as

$$\psi(r) = \frac{-1}{4\pi} \frac{e^{ikr}}{r\hbar^2 c^2} \left[m_\circ^2 c^4 \int e^{-ik.r'} \psi(r')dr' - 2E \int e^{-ik.r'} \psi(r')V(r')dr' \right] \qquad (29)$$

For simplification consider

$$Q(\theta, \varphi) = \frac{-m_\circ^2 c^2}{4\pi\hbar^2} \int e^{-ikr' cos\alpha} \psi(r')dr' \qquad (30)$$

$$f(\theta, \varphi) = \frac{E}{2\pi\hbar^2 c^2} \int e^{-ikr' cos\alpha}\, \psi(r')V(r')dr' \qquad (31)$$

Thus equation (25) becomes

$$\psi(r) = e^{ikr} + Q(\theta, \varphi)\frac{e^{ikr}}{r} + f(\theta, \varphi)\frac{e^{ikr}}{r} \qquad (32)$$

Where

$$\psi_{in}(r) = e^{ikr} \qquad (33)$$

$$\psi_{sc}(r) = \frac{e^{ikr}}{r}(Q + f) = \frac{e^{ikr}}{r} D(\theta, \varphi) \qquad (34)$$

Where

$$Q + f = D(\theta, \varphi) \qquad (35)$$

Using the

$$S_{sc} = \frac{i\hbar}{2m}[\psi_{sc}(r)\nabla\overline{\psi_{sc}}(r) - \overline{\psi_{sc}}(r)\nabla\psi_{sc}(r)] \qquad (36)$$

Where:

$$\psi_{sc}(r) = D(\theta, \varphi)\frac{e^{ikr}}{r} \quad \Rightarrow \quad \overline{\psi_{sc}}(r) = \overline{D}(\theta, \varphi)\frac{e^{-ikr}}{r} \qquad (37)$$

$$\nabla\overline{\psi_{sc}}(r) = \frac{\partial\overline{\psi_{sc}}}{\partial r} = \overline{D}(\theta, \varphi)\left[\frac{-ik}{r} - \frac{1}{r^2}\right] e^{-ikr} \qquad (38)$$

$$\nabla\overline{\psi_{sc}}(r) = -\overline{D}(\theta, \varphi)\left[\frac{ik}{r} + \frac{1}{r^2}\right] e^{-ikr} = \frac{-\overline{D}(\theta, \varphi)}{r^2} e^{-ikr}[ikr + 1]$$

$$\nabla\psi_{sc}(r) = \frac{\partial\psi_{sc}}{\partial r} = D(\theta, \varphi)\left[\frac{ik}{r} - \frac{1}{r^2}\right] e^{ikr}$$

$$\nabla\psi_{sc}(r) = \frac{D(\theta,\varphi)}{r^2} e^{ikr}[ikr - 1] \qquad (39)$$

A direct substitution equation (39) in equation (36) gives

$$S_{sc} = \frac{i\hbar}{2m}\frac{|D^2|}{r^3}[-(ikr + 1) - (ikr - 1)] \qquad (40)$$

$$S_{sc} = \frac{i\hbar}{2m}\frac{|D^2|}{r^3}(-2ikr)$$

$$S_{sc} = \frac{\hbar k}{mr^2}|D^2| \qquad (41)$$

but $p = mv$

$$= \hbar k \qquad (42)$$

Thus the scattering flux is given by:

$$S_{sc} = \frac{mv}{mr^2}|D^2| = S_{sc} = \frac{v}{r^2}|D^2| = (43)$$

According to the definition of the scattering cross section $S_{sc} = \frac{v}{r^2}\sigma$

Thus the scattering cross section σ is given by:

$$\sigma = |D^2| \qquad (44)$$

Maxwell's equation describe the behavior of moving and static changes as well as electromagnetic waves (emw), the electric field intensity E for a medium with electric permittivity and conductivity is given by

$$\nabla^2 E + \epsilon\mu\frac{\partial^2 E}{\partial t^2} - \mu\sigma\frac{\partial E}{\partial t} = 0 \qquad (45)$$

Consider a solution in the form

$$E = E_\circ e^{i(kx-\omega t)} \qquad (46)$$

Differentiating equation (46) with respect to space and time yields

$$\nabla^2 E = -k^2 E \quad , \quad \frac{\partial E}{\partial t} = -i\omega E$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E \quad ,$$

$$and\ let \quad \epsilon\mu = \frac{1}{c^2} \qquad (47)$$

Inserting equation (47) in (45) gives

$$-k^2 E + \frac{\omega^2}{c^2}E + i\mu\sigma\omega E = 0 \qquad (48)$$

Multiplying both side of equation (48) by $\frac{\hbar^2 c^2}{E}$ gives

$$-\hbar^2 k^2 c^2 + \hbar^2\omega^2 + i\mu\sigma\omega\hbar^2 c^2 = 0 \qquad (49)$$

Rearranging equation (49) gives

$$\hbar^2\omega^2 = \hbar^2k^2c^2 - i\mu\sigma\omega\hbar^2c^2 \qquad (50)$$

Where

$$E = \hbar\omega \qquad and \qquad p = \hbar k \qquad (51)$$

Thus equation (50) becomes:

$$E^2 = p^2c^2 - i\mu\sigma E\hbar c^2 \qquad (52)$$

Comparing this equation with the Einstein energy momentum relation

$$E^2 = p^2c^2 + m_\circ^2c^4 \qquad (53)$$

Gives:

$$m_\circ^2c^4 = -i\mu\sigma E\hbar c^2 \qquad (54)$$

Using the definition of the conductivity ($\sigma = \sigma_1 + i\sigma_2$) thus equation (54) becomes:

$$m_\circ^2c^4 = -i\mu(\sigma_1 + i\sigma_2)E\hbar c^2 \qquad (55)$$

But this term ($m_\circ^2c^4$) represent the real part:

$$m_\circ^2c^4 = \mu\sigma_2E\hbar c^2 \qquad (56)$$

To find the conductivity of a certain medium consider a particle moving with velocity v in a resistive medium of coefficient γ under the action of the electric field E .the equation of motion of the particle is given by:

$$m\frac{dv}{dt} = -k_\circ x + \gamma v + eE \qquad (57)$$

Assume the solution

$$x = x_\circ e^{i\omega t} \qquad (58)$$

Where:

$$v = \frac{dx}{dt} = i\omega x \quad , \quad x = \frac{v}{i\omega}$$

$$\frac{dv}{dt} = (i\omega)(i\omega x) = -\omega^2x$$

$$k_\circ = m\omega_\circ^2 \qquad (59)$$

Substituting (59) in (57) yields:

$$-m\omega^2x = -m\omega_\circ^2x + i\gamma\omega x + eE \qquad (60)$$

Rearranging equation (60) gives:

$$eE = m(\omega_\circ^2 - \omega^2)x - i\gamma\omega x \qquad (61)$$

Thus:

$$x = \frac{eE}{m(\omega_\circ^2 - \omega^2) - i\gamma\omega} \qquad (62)$$

Multiplying both side of equation (62) by $m(\omega_\circ^2 - \omega^2) + i\gamma\omega$ gives:

$$x = \frac{eE[m(\omega_\circ^2 - \omega^2) + i\gamma\omega]}{m^2(\omega_\circ^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad (63)$$

Where:

$$x = \frac{v}{i\omega} \quad \Rightarrow \quad v = i\omega x \qquad (64)$$

Thus:

$$v = \frac{\omega eE[-\gamma\omega + im(\omega_\circ^2 - \omega^2)]}{m^2(\omega_\circ^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad (65)$$

From equation (65) one gets:

$$v = v_1 + iv_2 \quad \Rightarrow \quad \sigma = \sigma_1 + i\sigma_2 \qquad (66)$$

Using the definition of current density j and the conductivity σ one gets:

$$j = nev = \sigma E \qquad (67)$$

Substituting (65) in (67) yields:

$$\sigma = \frac{n\omega e^2[-\gamma\omega + im(\omega_\circ^2 - \omega^2)]}{m^2(\omega_\circ^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad (68)$$

Thus the real part $\sigma_1$and the imaginary part $\sigma_2$of the conductivity are given by:

$$\sigma_1 = \frac{n\omega e^2[-\gamma\omega]}{m^2(\omega_\circ^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad (69)$$

$$\sigma_2 = \frac{n\omega e^2[m(\omega_\circ^2 - \omega^2)]}{m^2(\omega_\circ^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad (70)$$

Hence from (70) equation (56) becomes:

$$m_\circ^2 c^4 = \frac{n\mu e^2 E^2 c^2 m(\omega_\circ^2 - \omega^2)}{m^2(\omega_\circ^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad (71)$$

While the imaginary part in equation (55) represent the energy of frictional medium E_f
Thus For frictional medium

$$E^2 = p^2c^2 + m_\circ^2 c^4 + iE_f \qquad (72)$$

From equation (55) yields E_f

$$m_\circ^2 c^4 = -i\mu\sigma E\hbar c^2$$
$$= -i\mu E\hbar c^2(\sigma_1 + i\sigma_2) \qquad (73)$$

Hence:

$$E_f = -i\mu E\hbar c^2\sigma_1 \qquad (74)$$

$$E_f = \frac{in\mu e^2 E^2 c^2 \gamma\omega}{m^2(\omega_\circ^2 - \omega^2)^2 + \gamma^2\omega^2} \qquad (75)$$

Let:

$$V = 0 \ \text{[Thus see equation (31)]} \ \ f(\theta,\varphi) = o$$

Thus according to equation (35)

$$D(\theta,\varphi) = Q(\theta,\varphi) \qquad (76)$$

Thus equation (27) becomes

$$U = \frac{m_\circ^2 c^4}{\hbar^2 c^2} \qquad (77)$$

For simplification equation (71) consider

$$m^2(\omega_\circ^2 - \omega^2)^2 \ll \gamma^2\omega^2 \quad \text{and using } E^2 = \hbar^2\omega^2 \qquad (78)$$

Therefor equation (71) becomes:

$$m_\circ^2 c^4 = \frac{n\mu e^2 \hbar^2 c^2 m(\omega_\circ^2 - \omega^2)}{\gamma^2} \qquad (79)$$

Substituting equation (79) in equation (77) yields:

$$U = \frac{n\mu e^2 m(\omega_\circ^2 - \omega^2)}{\gamma^2} \qquad (80)$$

In this case equation (28) becomes:

$$\psi_{sc}(r) = \frac{-1}{4\pi}\frac{e^{ikr}}{r}\left[\frac{n\mu e^2 m(\omega_\circ^2 - \omega^2)}{\gamma^2}\int e^{-ik.r'}\psi(r')dr'\right] \qquad (81)$$

$$\psi_{sc}(r) = \frac{Q(\theta,\varphi)}{4\pi}\frac{e^{ikr}}{r} \qquad (82)$$

Where:

$$Q(\theta,\varphi) = \left[-\frac{n\mu e^2 m(\omega_\circ^2 - \omega^2)}{\gamma^2}\int e^{-ik.r'}\psi(r')dr'\right] \qquad (83)$$

Conclusion & Discussion:

Using the GSR and PDSR at useful expression relating energy, momentum and potential energy has been found in equation (7)with the aid of the of the wave equation (9) a new PDSR and GSR quantum was found in equation (11).For time independent potential $V(r)$ a time dependent solution was suggested in equation (12) the new quantum equation which depend on the potential V, energy E and the spatial variation of the wave function , has been exhibited in equations (16)and (19).

Greens equation and function in equation (20)(26) and (27)was used to solve the quantum equation (19) as shown by equations (28) and (32). According to equations (30) (31) (32) (34)and (44) the scattering cross section σ consist of two terms f and . The term f is the ordinary well known term, while the new term Q consists of rest mass energy $m_\circ c^2$ instead of the potential$V$. Using Maxwell's equation(45), beside Max Plank

and De Broglie hypothesis a relativistic energy – momentum relation consisting of an imaginary term dependent on the conductivity σ was found as shown by equation(52). This term is related to the rest mass $m_\circ$ according to equation (54), when comparing (52). with the ordinary SR energy – momentum relation (53). Considering σ in a complex form (see equation (55)) an additional imaginary term standing for friction as shown by equation (74). Using the equation (57), which describes the freely vibrating particle moving in a frictional medium under the action of a travelling electromagnetic field, a useful expression for conductivity was found in equations (69) and (70). This enables expressing the rest mass term in terms of electromagnetic wave frequency w. This enables describing the scattering proses of electromagnetic waves which propagate inside a medium in which particles vibrate with natural frequency $w_\circ$.
Restricting ourselves to high resistive medium as shown by equation (78) the scattering cross section $\sigma = Q(\theta, \varphi)$ is frequency dependent as shown by equation (83). According to plank theory $E = \hbar w$ the scattering cross section depends on particles energy which agrees with observations.

The new quantum PDSR (GSR) equation for scattering process shows some interesting results. It indicates that the scattering cross section and flux reduces to the ordinary one. It consists also of an additional term standing for rest mass energy and is frequency dependent.

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