Parity Combination Cordial Labeling for Some Standard Graph

Soni Devyani Vinodkumar(1) , Dr.Ritu Khanna(2) , Dr. P.H. Bhathawala(3)
(1)Research Scholar at Pacific University, Udaipur
(2)Professor,Department of Mathematics,Pacific University, Udaipur.
(3)Professor & Former Head,Department of Mathematics, V.N.S.G. University, Surat.

Abstract:

In this paper we investigate parity combination cordial labeling for some graphs obtained by duplication of graph elements and also we drive some results for$K_{1,n}$.and $K_{2,n}$.

Keywords:

Graph labeling, parity combination cordial labeling, parity combination cordial graph, duplication.

1.Introduction

All graph in this paper are finite, simple, undirected graph $G = (V, E)$, With the vertex set V and the edge set E If the vertices of the graph are assigned values subject to certain conditions then it is known as graph labeling. Throughout this work $K_{2,n}$ denotes the bipartite graph in which $M = \{u_1, u_2\}$ and $N = \{v_1, v_2, ..., v_n\}$ are two partite sets of $K_{2,n}$ such that each edge has one end in M and the other end in N, $K_{1,n}$ denotes the bipartite graph in which $M = \{v_0\}$ and $N = \{v_1, v_2, ..., v_n\}$ are two partite sets of $K_{1,n}$ such that each edge has one end in M and the other end in N, C_n denotes the cycle with n vertices and P_n denotes the path on n vertices.

The notion of parity combination cordial labeling was introduced by R. Ponraj, S. Narayanan and Ramasamy [9].In this paper we investigate parity combination cordial labelings for a duplication of graph elements in $K_{1,n}$.and $K_{2,n}$

Definition 1.1: let G be a (p, q) graph. Let f be an injective map from $V(G)V(G)$ to $\{1, 2, 3,..., P\}$. For each edge xy, assign the label $\frac{x}{y}$ or $\frac{y}{x}$ according as $x > y$ or $y > x$, f is called a parity combination cordial labeling (PCClabeling) if f is a one to one map and

$\left|e_f(0) - e_f(1)\right| \leq 1$, where $e_f(0)$ and $e_f(1)$ denote the number of edges labeled with an even number and odd number respectively. A graph with a parity combination cordial labeling is called a parity combination cordial graph (PCC- graph).

Definition 1.2: Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.

Definition 1.3: Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produced a new graph G' such that $N\left(v'_k\right) = \left\{v_k, v''_k\right\}$and $N\left(v''_k\right) = \left\{v_k, v'_k\right\}$.

Definition 1.4: Duplication of edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.

Definition 1.5: Duplication of an edge $e = uv$ of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that$N(u') = N(u) \cup \{v'\} - \{v\}$ and
$N(v') = N(v) \cup \{u'\} - \{u\}$.

2. Main results

Duplication of graph elements in $K_{2.n}$

Throughout this work $K_{2,n}$ denotes the bipartite graph in which $M = \left\{u_1, u_2\right\}$ and $N = \left\{v_1, v_2, ..., v_n\right\}$ are two partite sets of $K_{2,n}$ such that each edge has one end in M and the other end in N.

Theorem 2.1 The graph obtained by duplication of a vertex from N in $K_{2,n}$ is a parity combination cordial graph where $n \not\equiv 0 (mod\ 4)$.

Proof The result is obvious for $n = 1$ as when we duplicate v_1, the resulting graph will be a cycle C_4, which is a parity combination cordial graph. .

Let u_1, u_2, v_1, v_2, v_3... v_n be the consecutive vertices of $K_{2,n}$ and G be the graph obtained by duplication of the vertex v_j by a vertex v'_j. Then G is a graph with $n + 3$ vertices and $2(n + 1)$ edge.

$$|V(G)| = n + 3; | E(G)| = 2(n + 1)$$

Then define $f: V(G) \rightarrow \{1, 2, 3,..., n + 3\}$ as follow

$$f(u_1) = 1$$

$$f(u_2) = 2$$

$$f(v_j) = j + 2; \forall j = 1, 2,..., n$$

$$f(v'_j) = n + 3$$

Then we get $\left|e_f(0) - e_f(1)\right| = 0$.

Hence, G is a PCC-graph.

Illustration: A parity combination cordial labeling of the graph obtained by duplication of a vertex from N in $K_{2,5}$ is shown in Figure .

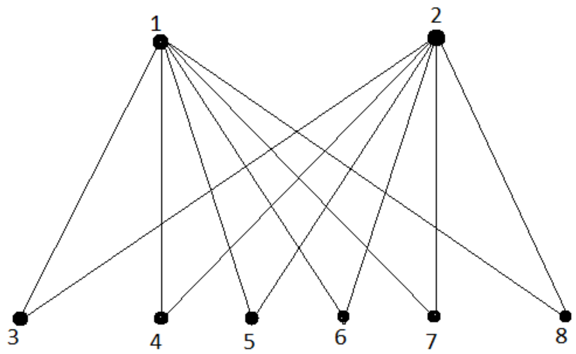


Figure 2.1: A PCC-labeling of the graph obtained by duplication of a vertex from N in $K_{2,5}$

Theorem 2.2 The graph obtained by duplication of a vertex by an edge from M in $K_{2,n}$ is a parity combination cordial graph.

Proof : Let G be a graph obtained by duplication of one of the vertices from M in $K_{2,n}$ by an edge $e = u'_1 u"_1$. Without loss of generality we duplicate u_1 by an edge $e = u'_1 u"_1$.Then the resultant graph G will have $n + 4$ vertices and $2n + 3$ edges.

$$|V(G)| = n + 4; \ |E(G)| = 2n + 3$$

We have the following cases

Case (i): For $n = 1, 5$

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 4\}$ as follows

$$f(u_1) = 1;$$

$$f(u_2) = 3;$$

$$f(v_1) = 2;$$

$$f(v_j) = j + 2; \ \forall j = 2, 3,..., n;$$

$$f(u'_1) = n + 3;$$

$$f(u"_1) = n + 4$$

Then we get $\left|e_f(0) - e_f(1)\right| = 1$

Hence, G is a PCC-graph.

Case (ii): F or $n \neq 1, 5$

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 4\}$ as follows

$$f(u_1) = 1;$$

$$f(u_2) = 2;$$

$$f(v_j) = j + 2; \ \forall j = 1, 2, 3,..., n;$$

$$f(u'_1) = n + 3;$$

$$f(u"_1) = n + 4$$

Then we get $\left|e_f(0) - e_f(1)\right| = 1$

Hence, G is a PCC-graph.

Illustration. A parity combination cordial labeling of the graph obtained by duplication of a vertex by an edge $e = u'_1 u"_1$ from M in $K_{2,5}$ is shown in Figure .

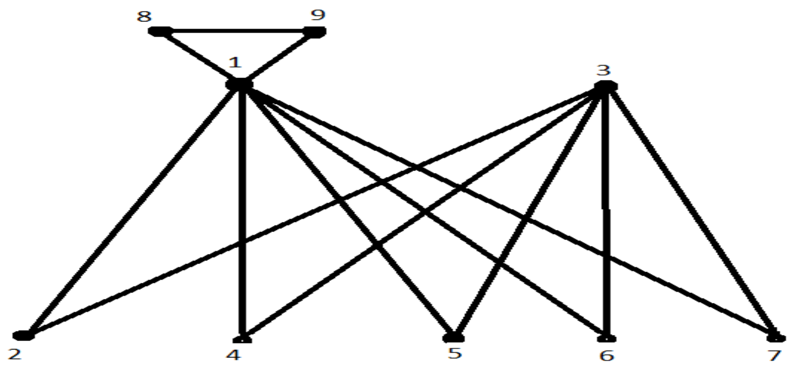


Figure 2.2: A PCC-labeling of the graph obtained by duplication of a vertex by an from M in $K_{2,5}$

Illustration. A parity combination cordial labeling of the graph obtained by duplication of a vertex by an edge $e = u'_1 u''_1$ from M in $K_{2,4}$ is shown in Figure

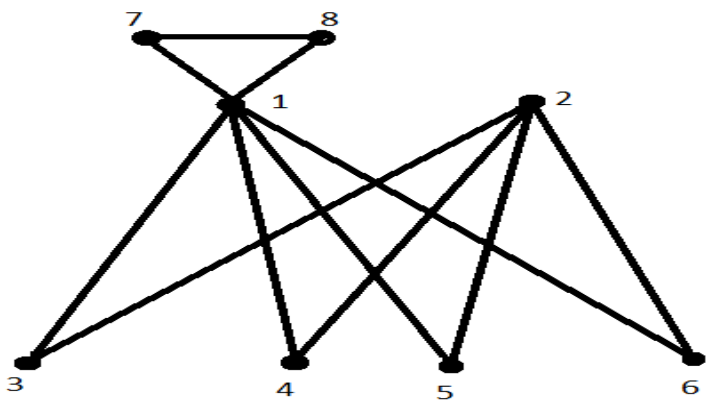


Figure 2.3: A PCC-labeling of the graph obtained by duplication of a vertex by an edge from M in $K_{2,4}$

Theorem 2.3 The graph obtained by duplication of a vertex by an edge from N in $K_{2,n}$ is a parity combination cordial graph.

Proof : Let G be a graph obtained by duplication of one of the vertices from N in $K_{2,n}$ by an edge $e = u'_1 u''_1$. Without loss of generality we duplicate v_1 by an edge $e = v'_1 v''_1$.Then the resultant graph G will have $n + 4$ vertices and $2n + 3$ edge.

$$|V(G)| = n + 4; |E(G)| = 2n + 3$$

We have the following cases :

Case (i): For $n + 2 \not\equiv 1(mod\ 4)$

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 4\}$ as follows

$$f(u_1) = 1;$$

$$f(u_2) = 2;$$

$$f(v_1) = 3;$$

$$f(v_j) = j + 4;\ \forall j = 2, 3,..., n;$$

$$f(v'_1) = 4;$$

$$f(v''_1) = 5$$

Then we get $|e_f(0) - e_f(1)| = 1$

Hence, G is a PCC-graph.

Case (ii): For $n + 2 \equiv 1(mod\ 4)$

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 4\}$ as follows

$$f(u_1) = 1;$$

$$f(u_2) = 2;$$

$$f(v_3) = 6;$$

$$f(v_j) = j + 2;\ j = 1, 2;$$

$$f(v_j) = j + 4;\ j \geq 4;$$

$$f(v'_1) = 5;$$

$$f(v''_1) = 7$$

Then we get $|e_f(0) - e_f(1)| = 1$

Hence, G is a PCC-graph.

Illustration. A parity combination cordial labeling of the graph obtained by duplication of a vertex by an edge $e = u'_1u''_1$ from N in $K_{2,3}$ is shown in Figure .

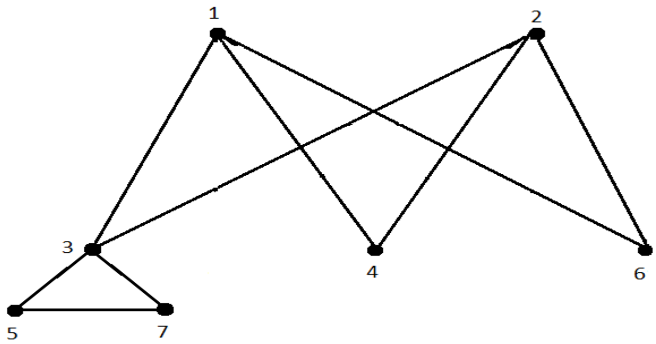


Figure 2.4: A PCC-labeling of the graph obtained by duplication of a vertex by an edge from N in $K_{2,3}$

Illustration. A parity combination cordial labeling of the graph obtained by duplication of a vertex by an edge $e = u'_1u"_1$ from N in $K_{2,5}$ is shown in Figure .

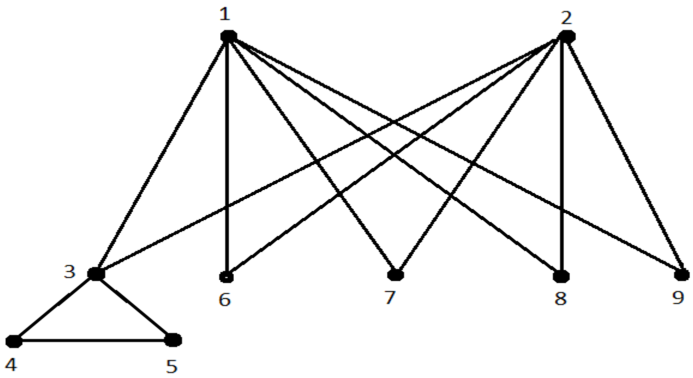


Figure 2.5: A PCC-labeling of the graph obtained by duplication of a vertex by an edge from N in $K_{2,5}$

Theorem 2.4. The graph obtained by duplication of an edge by a vertex in $K_{2,n}$ is a parity combination cordial graph.

Proof Let G be a graph obtained by duplication of an edge by a vertex. Without loss of generality we duplicate an edge $e = u_1v_1$ by a vertex w. Then the resultant graph G will have $n + 3$ vertices and $2n + 2$ edges.

$$|V(G)| = n + 3; |E(G)| = 2n + 2$$

Case (i): For $n \not\equiv 0(mod\ 4)$

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 3\}$ as follows

$$f(u_1) = 1;$$

$$f(u_2) = 2;$$

$$f(v_1) = 3;$$

$$f(w) = 4;$$

$$f(v_j) = j + 3;\ 2 \le j \le n$$

Then we get $\left|e_f(0) - e_f(1)\right| = 0$

Hence, G is a PCC-graph.

Case (ii): For $n \equiv 0(mod\ 4)$

We define $f: V(G) \to \{1, 2, 3,..., n + 3\}$ as follows

$$f(u_1) = 1;$$

$$f(u_2) = 2;$$

$$f(w) = 3;$$

$$f(v_j) = j + 3;\ 1 \le j \le n$$

Then we get $\left|e_f(0) - e_f(1)\right| = 0$

Hence, G is a PCC-graph.

Illustration. A parity combination cordial labeling of the graph obtained by duplication of an edge by a vertex in $K_{2,5}$ is shown in Figure .

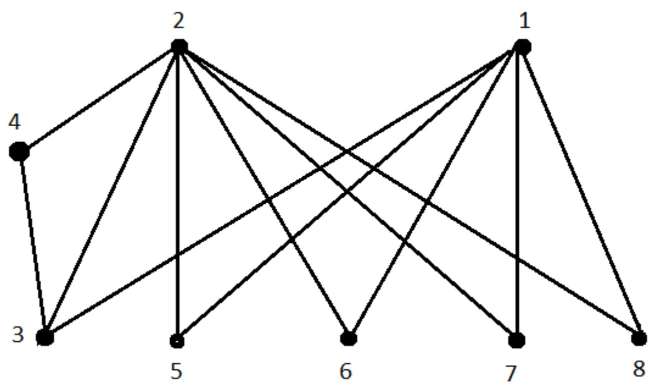


Figure 2.6: A PCC-labeling of the graph obtained by duplication of an edge by a vertex in $K_{2,5}$

Illustration. A parity combination cordial labeling of the graph obtained by duplication of an edge by a vertex in $K_{2,4}$ is shown in Figure .

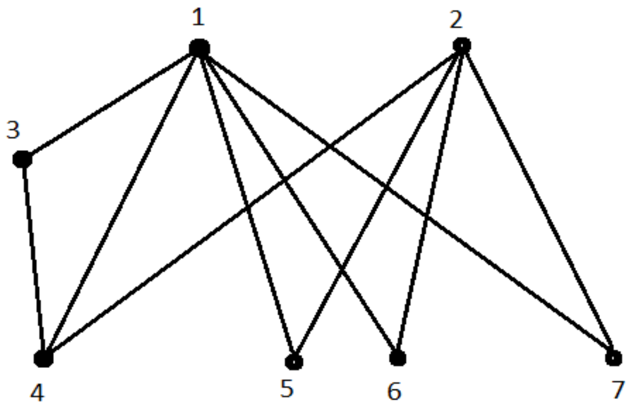
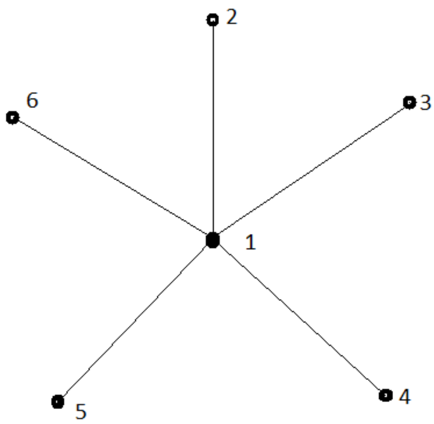


Figure 2.7: A PCC-labeling of the graph obtained by duplication of an edge by a vertex in $K_{2,4}$.

Duplication of graph elements in $K_{1.n}$

Throughout this work $K_{1,n}$ denotes the bipartite graph in which $M = \{u_0\}$ and $N = \{v_1, v_2, ..., v_n\}$are two partite sets of $K_{1,n}$ such that each edge has one end in M and the other end in N.



Theorem 3.1: The graph obtained by duplication of a vertex in $K_{1,n}$ is a parity combination cordial graph where $n \not\equiv 0(mod\ 4)$.

Proof Let v_0 be the apex vertex and $v_1, v_2, v_3 ... v_n$ are pendant vertices of $K_{1,n}$. Let G denote the graph obtained by duplication of any vertex v_j by a vertex v'_j in $K_{1,n}$

Depending upon the $deg(v_j)$ in $K_{1,n}$.

We have the following two cases.

Case (i): Duplication of apex vertex.

The graph obtain by duplication of apex vertex v_0 in $K_{1,n}$, which is the complete bipartite graph $K_{2,n}$. Hence it is a parity combination cordial graph for $n \not\equiv 0 (mod\ 4)$ as proved in theorem 2.1

Case (2): Duplication of pendant vertex.

The graph obtained by duplication of any pendant vertex in $K_{1,n}$, which is again a star graph $K_{1,n+1}$, Hence it is a parity combination cordial graph.

Theorem 3.2: The graph obtained by duplication of an edge in $K_{1,n}$ is a parity combination cordial graph.

Proof Let G be a graph obtained by duplication of the edge $e = v_0 v_n$ by a new edge $e = v'_0 v'_n$ in $K_{1,n}$

Hence in G, $deg(v_0) = n$, $deg(v'_0) = n$,$deg(v_n) = 1$,$deg(v'_n) = 1$ and $deg(v_i) = 2\ \forall i \in \{1, 2,..., n - 1\}$.

Then the resultant graph G will have $n + 3$ vertices and $2n$ edges.

$$|V(G)| = n + 3;\ |E(G)| = n + 3$$

We consider the following cases:

Case (i) : For$n \not\equiv 0(mod\ 4)$

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 3\}$ as follows

$$f(v_0) = 1;$$

$$f(v_n) = 3;$$

$$f(v'_0) = 2;$$

$$f(v'_n) = 4;$$

$$f(v_j) = j + 4;\ 2 \le j \le n - 1$$

Then we get $|e_f(0) - e_f(1)| = 0$

Hence, G is a PCC-graph.

Case (ii) : For$n \equiv 0(mod\ 4)$

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 3\}$ as follows

$$f(v_0) = 1;$$
$$f(v_n) = 3;$$
$$f(v'_0) = 2;$$
$$f(v'_n) = 7;$$
$$f(v_j) = j + 3;\ j = 1, 2, 3;$$
$$f(v_j) = j + 4;\ 4 \le j \le n - 1$$

Then we get $|e_f(0) - e_f(1)| = 0$

Hence, G is a PCC-graph.

Illustration. A parity combination cordial labeling of the graph obtained by duplication of an edge in $K_{1,5}$ is shown in Figure .

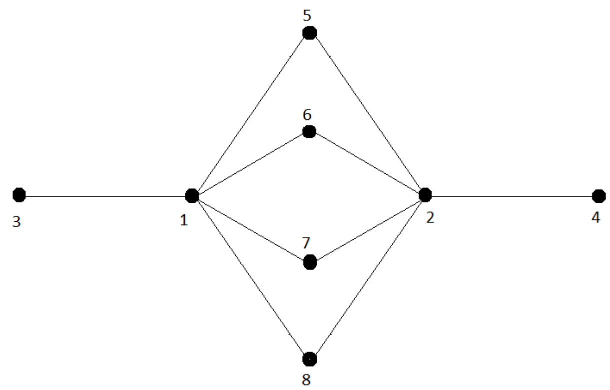


Figure 3.1: A PCC-labeling of the graph obtained by duplication of an edge in $K_{1,5}$.

Illustration. A parity combination cordial labeling of the graph obtained by duplication of an edge in $K_{1,7}$ is shown in Figure .

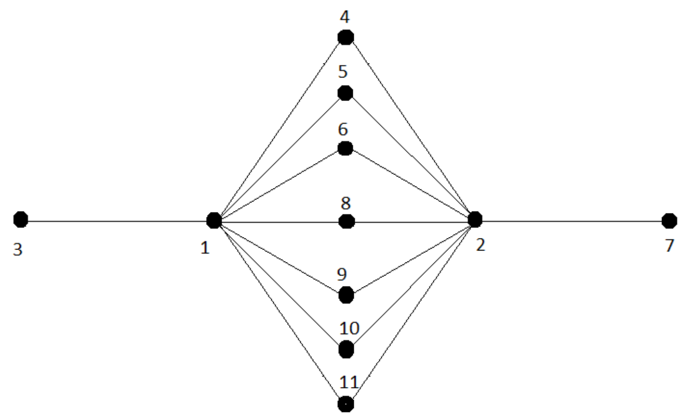


Figure 3.2: A PCC-labeling of the graph obtained by duplication of an edge in $K_{1,7}$

Theorem 3.3 The graph obtained by duplication of a vertex by an edge $K_{1,n}$ is a parity combination cordial graph.

Proof Let G be a graph obtained by duplication of a vertex v_j by an edge $e = v'_j v''_j$ in $K_{1,n}$ then the resultant graph G will have $n + 3$ vertices and $n + 3$ edges.

$$|V(G)| = n + 3; \ |E(G)| = n + 3$$

We consider the following cases.

Case (i) : Duplication of apex vertex v_0 by an edge $v'_0 v''_0$.

We define $f: V(G) \rightarrow \{1, 2, 3, ..., n + 3\}$ as follows.

$$f(v_0) = 1;$$

$$f(v'_0) = 2;$$

$$f(v''_0) = 3;$$

$$f(v_j) = j + 3; \ 1 \leq j \leq n$$

Then we get $|e_f(0) - e_f(1)| = 0$ when n is odd and $|e_f(0) - e_f(1)| = 1$ when n is even

Hence, G is a PCC-graph.

Case (ii) : Duplication of pendant vertex v_j by an edge $v'_j v''_j$.

Without loss of generality we assume that $v_j = v_1$.

We define $f: V(G) \rightarrow \{1, 2, 3, ..., n + 3\}$ as follows:

$$f(v_0) = 1;$$

$$f(v_1) = 3;$$
$$f(v'_1) = 2;$$
$$f(v''_1) = 4;$$
$$f(v_j) = j + 3;\ 2 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 0$ when n is odd and $|e_f(0) - e_f(1)| = 1$ when n is even

Hence, G is a PCC-graph.

Illustration. A parity combination cordial labeling of the graph obtained by duplication of aapex vertex v_0 by an edge $v'_0v''_0$ in $K_{1,5}$ is shown in Figure .

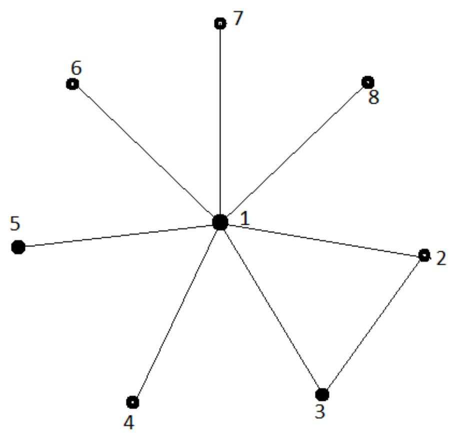


Figure 3.3: A PCC-labeling of the graph obtained by duplication of aapex vertex v_0 in $K_{1,5}$

Illustration. A parity combination cordial labeling of the graph obtained by duplication of pendant vertex v_1 by an edge $v'_1v''_7$in $K_{1,5}$ is shown in Figure .

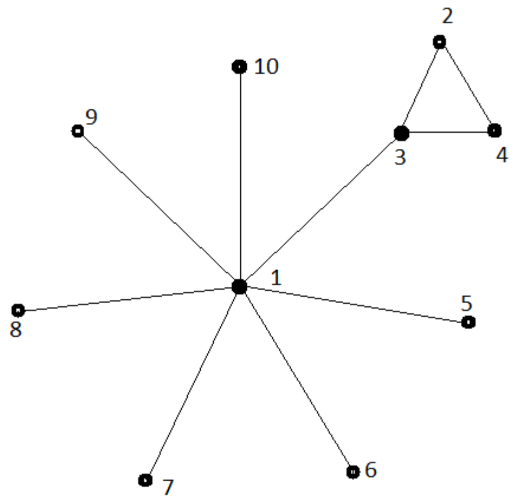


Figure 3.4: A PCC-labeling of the graph obtained by duplication of aapex vertex v_1 in $K_{1,7}$

Theorem 3.4. The graph obtained by duplication of an edge by a vertex in $K_{1,n}$ is a prime graph.

Proof Let G be a graph obtained by duplication of the edge v_0v_1 by a vertex v'_1

Now the resultant graph G will have n + 2 vertices and n + 2 edges.

We define $f: V(G) \rightarrow \{1, 2, 3,..., n + 2\}$ as follows.

$$f(v_0) = 1;$$

$$f(v_1) = 3;$$

$$f(v'_1) = 2;$$

$$f(v_j) = j + 2;\ 2 \leq j \leq n$$

Then we get $|e_f(0) - e_f(1)| = 1$ when n is odd and $|e_f(0) - e_f(1)| = 0$ when n is even

Hence, G is a PCC-graph.

Illustration: A parity combination cordial labeling of the graph obtained by duplication of an edge by a vertex in $K_{1,6}$ is shown in Figure .

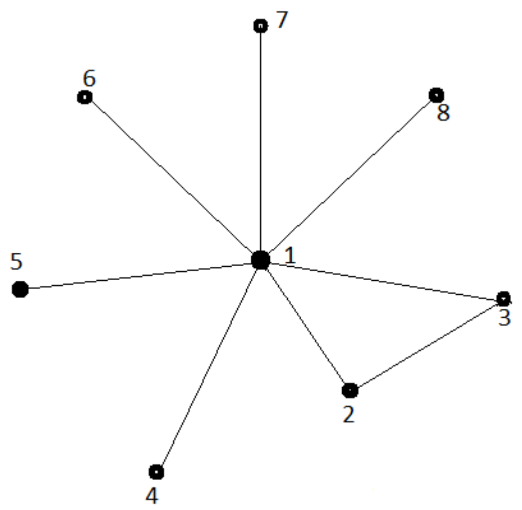


Figure 3.5: A PCC-labeling of the graph obtained by duplication of an edge by a vertex in $K_{1,6}$

Theorem 3.5: Graph obtained by duplication of each vertex by an edge in star $K_{1,n}$ is a parity combination cordial graph.

Proof: Let v_0 be the apex vertex and $v_1, v_2, v_3,..., v_n$ be the consecutive pendant vertices of $K_{1,n}$. let G be the graph obtained by duplicating each of the vertices v_j in $K_{1,n}$ by an edge $v'_j v''_j$ for $j = 0, 1, 2, 3,..., n$. Then G is a graph with $3n + 1$ vertices and $4n$ edges.

$$|V(G)| = 3n + 1; \ |E(G)| = 4n$$

Define an injective map $f: V(G) \rightarrow \{1, 2, 3,..., 3n + 1\}$as

$$f(v_0) = 1;$$

$$f(v'_0) = 2;$$

$$f(v''_0) = 3;$$

$$f(v_j) = 3j + 1 ; \ 1 \leq j \leq n$$

$$f(v'_j) = 3j + 2 ; \ 1 \leq j \leq n$$

$$f(v''_j) = 3j + 3 ; \ 1 \leq j \leq n$$

Here $e_f(0) = 2n; \ n \equiv 0(mod\ 4)$

and $e_f(0) = 2n - 1$;*otherwise*

Here $e_f(1) = 2n - 1;\ n \equiv 0(mod\ 4)$

and $e_f(0) = 2n$;*otherwise*

Then we get $\left|e_f(0) - e_f(1)\right| = 1$.

As the labeling defined above satisfies the conditions of parity combination cordial labeling and the graph under consideration is parity combination cordial graph in both cases.
Hence, G is a parity combination cordial graph.

Theorem 3.6: Graph obtained by duplication of each edge by a vertex in star $K_{1,n}$ is a parity combination cordial graph.

Proof: Let v_0 be the apex vertex and $v_1, v_2, v_3,..., v_n$ be the consecutive pendant vertices of $K_{1,n}$. let G be the graph obtained by duplication of each of the edges v_0v_j in $K_{1,n}$ by a vertex v'_j. Then G is a graph with $2n + 1$ vertices and $3n$ edges.

$$|V(G)| = 2n + 1;\ |E(G)| = 3n$$

Define an injective map $f: V(G) \rightarrow \{1, 2, 3,..., 2n + 1\}$ as

$$f(v_0) = 3;$$

$$f(v_1) = 1;$$

$$f(v_2) = 2;$$

$$f(v_j) = 2j\ ;\ \ 2 \le j \le n$$

$$f(v_{2j-1}) = 2j + 1\ ;\ \ 2 \le j \le n$$

Then we get $\left|e_f(0) - e_f(1)\right| = 0$ if n is odd and $\left|e_f(0) - e_f(1)\right| = 1$ if n is even.

As the labeling defined above satisfies the conditions of parity combination cordial labeling and the graph under consideration is parity combination cordial graph in both cases.
Hence, G is a parity combination cordial graph.

Conclusion

Here we investigate parity combination cordial labelling for some graph obtained by duplication of graph elements on $K_{1,n}$.and $K_{2,n}$.

References

[1]. D.M.Burton, elementary number theory, Tata McGraw Hill Edition, New Delhi, (2010).

[2]. J.A.Gallian, a dynamic survey of graph labelling,the elementary journal of combinatorics (2018), #DS6

[3]. I. Cahit, Cordial Graph: A Weaker Version of Graceful and Harmonious Graph, Ars Combin.,23, (1987), 201-207

[4]. F. Harary, Graph Theory, Narosa Publishing House, New Delhi (2001).

[5]. S. M. Hegde and SudhakarShetty, Combinatorial Labeling of Graphs, Applied Mathematics E-Notes, 6(2006), 251-258.

[6]. A. Rosa, On Certain Valuations of The Vertices of A Graph, Theory of Graphs(Internat. Symposium, Rome, July 1966), Gordon and Breach, N. Y. and Dunod Paris, (1967), 349-355.

[7]. M.A.Seoud and A.E.I.AbdelMaqsoud, On Cordial And Balanced Labeling of Graphs, J. Egyptain Math. Soc., 7(1999), 127-135.

[8]. S. C. Shee and Y. S. Ho, The Cordiality of One-point Union of n-copies of A Graph, Discrete Math., 117(1993), 225-243.

[9]. R. Ponraj, S.Sathish Narayanan and A. M. S. Ramasamy, Parity Combination Cordial Labeling of Graphs, Jordan Journal of Mathematics and statistics (JJMS), 8(4)(2015), 293-308.

[10]. R. Ponraj, Rajpal Singh and S. Sathish Narayanan, On Parity Combination Cordial Graphs, Palestine Journal of Mathematics, 6(1)(2017), 211-218.

[11]. S. K. Vaidya and U. M. Prajapati, Prime Labeling In The Context of Duplication of Graph Elements, International Journal of Mathematics and Soft Computing,3(1)(2013), 13-20.

[12] D. G. Adalja , G. V. Ghodasara, Sum Divisor Cordial Labeling in the Context of Duplication of Graph Elements,Mathematics Today.35 (2019) 61-83

[13] U M Prajapati , B N Suthar, Prime labeling in the context of duplication of graph elements in K2,n, International Journal of Mathematics and Soft Computing. 7(1)(2017), 117 - 125.