Eccentricity Based On Topological Indices Of Boron Nanotubes

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Abstract: In this paper, we compute the eccentricity based on topological indices of born triangular and boron-α nanotubes.

Keywords: Boron Nanotubes, Triangular boron nanotubes, Boron-α nanotubes, Topological indices.

1. Introduction

A topological index is a real number associated with chemical constitution. It correlates the chemical structure with various physical and chemical properties and biological activity. All graphs in this paper are simple, finite and undirected. A graph G is a finite nonempty set V(G) together with a prescribed set E(G) of unordered pair of distinct elements of V.

The cardinality of V(G) and E(G) are represented by $|V(G)|$ and $|E(G)|$, respectively. Let, $d_G(v)$ be the degree of a vertex v of G and $N_G(v)$ be the neighborhood of a vertex v of G. The distance between the vertices u and v of a connected graph G is represented by $d_G(u, v)$. It is defined as the number of edges in a shortest path connects the vertices u and v. The eccentricity $e_G(v)$ of a vertex v in G is the largest distance between v and any other vertices u of G.

[3]Wiener index and its various applications are discussed in Randic index, $R_{-1/2}(G)$, is introduced by Milan Randic in 1975 defined as $R_{-1/2}(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u d_v}}$.

For general detains about $R_{-1/2}(G)$ and its generalized Randic index, $R_\alpha(G) = \sum_{uv\in E(G)} \frac{1}{(d_u d_v)^{2}}$

And the inverse Randic index is defined as $RR\alpha(G) = \sum_{uv\in E(G)} (d_u d_v)^{\alpha}$.

Clearly $R_{-1/2}(G)$ is a special case of $R_{-1/2}(G)$ when α = -1/2.

The Reciprocal Randic index is defined as $RR(G) = \sum_{uv\in E(G)} \sqrt{d_u d_v}$.

The Reduced reciprocal Randic index is defined as $RRR(G) = \sum_{uv\in E(G)} \sqrt{(d_u - 1)(d_v - 1)}$.

This index has many applications in diverse areas. Many paper and books such as are written on this topological index as well. Gutman and Trinajstic introduced two indices defined as

$M_1(G) = \sum_{uv\in E(G)} [d_u + d_v]$ *and* $M_2(G) = \sum_{uv\in E(G)} (d_u \times d_v)$.

The Reduced second Zagreb index is defined as $RM_2(G) = \sum_{uv\in E(G)}(d_u - 1)(d_v - 1)$.

The Forgotten index is defined as $F(G) = \sum_{uv\in E(G)}((d_u)^2 + (d_v)^2)$.

The first and second modified Zagreb index is defined as

$^{m}B_1(G) = \sum_{uv\in E(G)} \frac{1}{d_u+d_v}$ and $^{m}B_2(G) = \sum_{uv\in E(G)} \frac{1}{d_u \times d_v}$.

Inverse sum index $I(G) = \sum_{uv\in E(G)} \frac{d_u d_v}{d_u+d_v}$ and augmented Zagreb index $A(G) = \sum_{uv\in E(G)} \{\frac{d_u d_v}{d_u+d_v-2}\}$.

[2] Randic connectivity index called the geometric-arithmetic index, which is presented by Vukicevic

and Furtula is defined as $GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{(d_u+d_v)}$ and also defined multiplicative indices of a graph as follows:

2. Some eccentric indices of Boron Triangular and Boron-α Nanotubes:

In last 20 years, various types of boron – containing nanomaterials such as boron nanoclusters, boron nanowires, boron nanotubes. Boron nanobelts, boron nanoribbons, boron nanosheets, and boron fullerences have been experimentally synthesized and identified.

In this section, we introduced the molecular graphs of eccentricity of boron triangular nanotubes by *EBTN[m, n]* respectively, where *m* is the number of rows and *n* is the number of columns in a *EBTN[m, n]*as shown in order *3mn/2* and size *3n(3m - 2)/2.*

Molecular graph	**Order**	**Size**
EBTN[m, n]	*3mn/2*	*3n(3m - 2)/2*
EBAN(X)[m, n]	*n(4m + 1)/3*	*n(7m - 2)/2*
EBAN(Y)[m, n]	*4mn/3*	*n(7m - 4)/2*

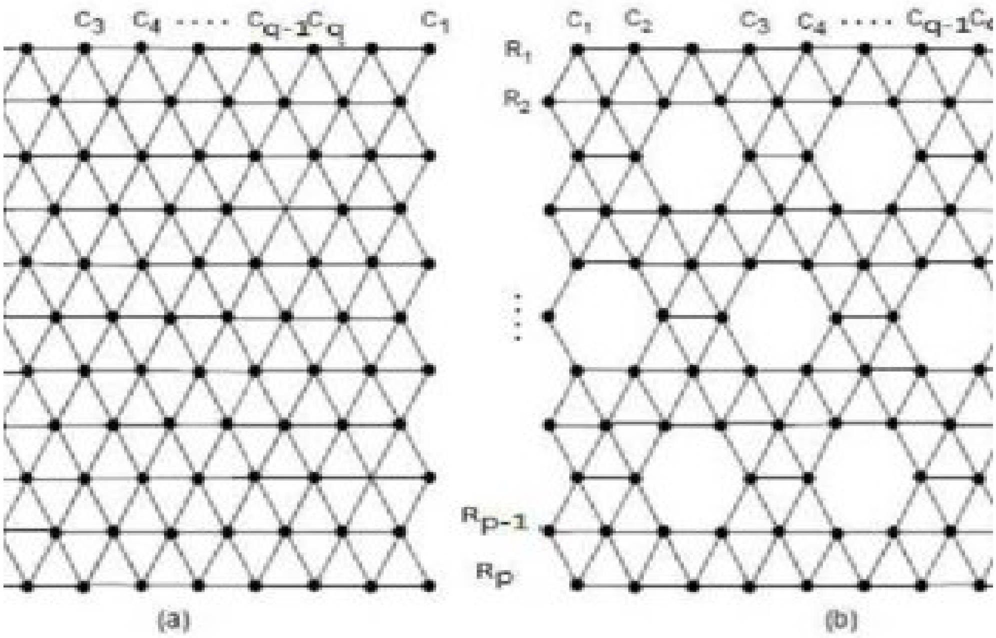

Fig . (a) 2D-sheet of boron triangular nanotube $BT[m,n]$,

(b) 2D-sheet of boron-α nanotube $BA[m,n]$.

We introduced the Milan Randic eccentric boron triangular nanotubes index, defined as $R_{-1/2}$ (EBTN[m, n]) $= \sum_{ue)} \frac{1}{\sqrt{e_{(G)}(u) + e_{(G)}(v)}}$.

For general detains about $R_{-1/2}(G)$ and its generalized Randic eccentric boron triangular nanotubes index, defined as R_{α} (EBTN[m, n]) $= \sum_{ue} \frac{1}{(e_{(G)}(u) + e_{(G)}(v))^{2}}$

We introduced the inverse Randic eccentric boron triangular nanotubes index, defined as $RR\alpha$ (EBTN[m, n]) $= \sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^{\alpha}$.

We introduced the reciprocal Randic eccentric boron triangular nanotubes index, defined as RR(EBTN[m, n]) $= \sum_{uv\in E(G)} \sqrt{e_{(G)}(u) \times e_{(G)}(v)}$.

We introduced the Reduced reciprocal Randic eccentric boron triangular nanotubes index is defined as

$$\text{RRR(EBTN[m, n])} = \sum_{uv\in E(G)} \sqrt{(e_{(G)}(u) - 1)(e_{(G)}(v) - 1)}$$

Also, we introduced two indices, defined as

M_1(EBTN[m, n])= $\sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^{\alpha}$ *and*

$M_2(G) = \sum_{ue}(e_{(G)}(u) \times e_{(G)}(v))^{\alpha}$.

We introduced the reduced second Zagreb eccentric boron triangular nanotubes index, defined as

$$\text{RM}_2(\text{EBTN}[\text{m},\text{n}]) = \sum_{uv\in E(G)}(e_{(G)}(\text{u}) - 1)(e_{(G)}(v) - 1).$$

We introduced the Forgotten eccentric boron triangular nanotubes index, defined as $\text{F}(\text{EBTN}[\text{m},\text{n}]) = \sum_{uv\in E(G)}(e_{(G)}(u))^2 + (e_{(G)}(v))^2)$.

We introduced the first & second modified Zagreb eccentric boron triangular nanotubes index, defined as

$$^{m}\text{B}_1(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\frac{1}{e_{(G)}(\text{u})+ e_{(G)}(v)}$$

and $^{m}\text{B}_2(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\frac{1}{e_{(G)}(\text{u})\times e_{(G)}(v)}$

We introduced the harmonic eccentric boron triangular nanotubes index defined as $\text{H}(\text{EBTN}[\text{m},\text{n}]) = \sum_{uv\in EG)}\frac{2}{e_{(G)}(\text{u})+ e_{(G)}(v)}$

We introduced inverse sum eccentric boron triangular nanotubes index, defined as

$$\text{I}(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\frac{e_{(G)}(\text{u})\times e_{(G)}(v)}{e_{(G)}(\text{u})+ e_{(G)}(v)}$$

We introduced the augmented zagreb eccentric boron triangular nanotubes index, defined as $\text{A}(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\{\frac{e_{(G)}(\text{u})\times e_{(G)}(v)}{e_{(G)}(\text{u})+ e_{(G)}(v)-2}\}$.

We introduced the Randic connectivity eccentric index called the geometric-arithmetic eccentric boron triangular nanotubes index, defined as

$$\text{GA}(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\frac{2\sqrt{(e_{(G)}(\text{u})\times e_{(G)}(v))}}{(e_{(G)}(\text{u})+e_{(G)}(v))}$$

We introduced the eccentricity based connectivity eccentric boron triangular nanotubes index, defined as

$$\chi\text{E}(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\frac{1}{\sqrt{e_G(u)\, e_{L(G)}(e)}}.$$

We introduced the sum connectivity eccentric boron triangular nanotubes index, defined as

$$XE(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\frac{1}{\sqrt{e_G(u) + e_{L(G)}(e)}}.$$

We introduced the sum line connectivity eccentric boron triangular nanotubes index, defined as

$$SLCEII(\text{EBTN}[\text{m},\text{n}]) = \sum_{ue}\sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}}.$$

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in L(G), $e_{L(G)}$ (e)
E_1	2(m-1)	(m-1, m-1)	m
E_2	4(m-1)	(m-1, m-2)	m-1
E_3	2(m-1)	(m-2, m-2)	m-1
E_4	4(m-1)	(m-2, m-3)	m-2
E_5	2(m-1)	(m-3, m-3)	m-2
E_6	4(m-1)	(m-3, m-4)	m-3
E_7	2(m-1)	(m-4, m-4)	m-3
E_8	4(m-1)	(m-4, m-5)	m-4
E_9	2(m-1)	(m-5, m-5)	m-4
E_{10}	4(m-1)	(m-5, m-6)	m-5
.	.	.	.
.	.	.	.
E_{m-5}	2(m-1)	((m+5)/2, (m+3)/2))	(m+5)/2
E_{m-4}	4(m-1)	((m+3)/2, (m+3)/2))	(m+5)/2
E_{m-3}	2(m-1)	((m+3)/2, (m+1)/2))	(m+3)/2
E_{m-2}	4(m-1)	((m+1)/2, (m+1)/2))	(m+3)/2
E_{m-1}	2(m-1)	((m+1)/2, (m+1)/2))	(m+1)/2
E_m	(m-1)	((m+1)/2, (m+1)/2))	(m+1)/2

We calculate the eccentricity based on topological indices of boron-α nanotubes.

Categorize the boron-α nanotubes into two classes with respect to m.

We denote these classes as *EBAN(X)[m, n]* and *EBAN(Y)[m, n]* for m ≡ 2 mod 3 and m ≡ 0 mod 3, respectively.

Consider the eccentricity of boron-α nanotube EBAN(X) [m, n] nanotube.

Let $V_{m,\ n}$ be the vertex set and $E_{m,\ n}$ be the edge set in EBAN(X) [m, n], then

$$|V_{m,\ n}| = \frac{n(4m+1)}{3} and |E_{m,\ n}| = \frac{n(7m-2)}{2}.$$

Also the number of edges with eccentricities of end vertices of G and L(G) are given as follows:

Edge set	No. of edges e = uv	Eccentricity of end vertices (e(u), e(v))	Eccentricity of e in L(G), $e_{L(G)}$ (e)
E_1	18	(m-1, m-1)	m
E_2	36	(m-1, m-2)	m-1
E_3	18	(m-2, m-2)	m-1
E_4	24	(m-2, m-3)	m-2
E_5	6	(m-3, m-3)	m-2
E_6	24	(m-3, m-4)	m-3
E_7	18	(m-4, m-4)	m-3
.	.	.	.
.	.	.	.
E_{m-5}	18(m-2)	((m+4)/2, (m+4)/2))	(m+8)/2
E_{m-4}	24(m-2)	((m+4)/2, (m+2)/2))	(m+4)/2
E_{m-3}	6	((m+2)/2, (m+2)/2))	(m+4)/2
E_{m-2}	24(m-1)	((m+2)/2, m/2)	(m+2)/2
E_{m-1}	18(m-1)	(m/2, m/2)	(m+2)/2
E_m	18m	(m/2, m/2)	m/2

Theorem 2.1: Let the graph G = EBTN[m, n] be a eccentricity of boron triangular nanotubes respectively, then (i) $EBTNB_1$[m, n] , (ii) $EBTNB_2$[m, n] , (iii) $EBTNHB_1$[m, n] and (iv) $EBTNHB_2$[m, n] .

Proof: Consider the eccentricity of boron triangular nanotube EBTN[m, n] nanotube.

From fig.4.2.4 EBTN [m, n] nanotube

Let $V_{m,\ n}$ be the vertex set and $E_{m,\ n}$ be the edge set in EBTN [m, n], then $|V_{m,\ n}| = 3mn/2 and |E_2| = 3n(3m - 2)/2.$

Also the number of edges with eccentricities of end vertices of G and L(G) are given

(i) $EBTNB_1[m, n] = \sum_{ue}[e_G(u) + e_{L(G)}(e)]$

$$= \sum_{e=uv\in E(G)}\left[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)\right]$$

$$= \sum_{uv\in E_1(G)}\left[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)\right]+\ldots..$$

$$+ \sum_{uv\in E_m(G)}\left[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)\right]$$

$$= 2(m-1)\ [\ \sum_{i=1}^{n-1}(4m + (2 - 4i)\ ]$$

$$+ 4(m-1)\ [\ \sum_{i=1}^{n-1}(4m - (1 + 4i)\ ] + 2m\ (m-1)$$

$$(ii)\ \mathrm{EBTNB}_2\,[m,n] = \sum_{ue}\left[e_G(u) \times e_{L(G)}(e)\right]$$

$$= \sum_{uv\in E_1(G)}\left[(e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e))\right] +\ldots\ldots$$

$$..+\sum_{uv\in E_m(G)}[(e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e))]$$

$$= 2(m-1)\ [2\sum_{i=1}^{n-1}(m - i)(m - 1 + i)\ ]$$

$$+\ 4(m-1)\ [\ \sum_{i=1}^{n-1}(m - i)^2 + (m - (1 + i)(m - i)\ \ ]\ + \frac{1}{2}\ (m-1)$$

$(m^2$ □ $-1)$

$$(iii)\ \mathrm{EBTNHB}_1\,[m,n] = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]^2$$

$$= \sum_{uv\in E_1(G)}[e_G(u) + e_{L(G)}(e)]^2 + [e_G(v) + e_{L(G)}(e)]^2]+\ldots.$$

$$..+\sum_{uv\in E_m(G)}\left[[e_G(u) + e_{L(G)}(e)]^2 + [e_G(v) + e_{L(G)}(e)]^2\right]$$

$$= 2(m-1)\sum_{i=1}^{n-1}[2(2m - (2i - 1))^2\ ]$$

$$+ 4(m-1)\ [\ \sum_{i=1}^{n-1}[2(m - i)]^2 + [(2m - (2i + 1)]^2 + (m-1)\ (2m^2\ \square)$$

$$(iv)\quad \mathrm{EBTNHB}_2\,[m,n] = \sum_{ue}\left[e_G(u) \times e_{L(G)}(e)\right]^2$$

$$= \sum_{e=uv\in E_1(G)}[[e_G(u)\times e_{L(G)}(e)]^2 + [e_G(v)\times e_{L(G)}(e)]^2]+\ldots$$

$$..+\sum_{e=uv\in E_m(G)}[[e_G(u)\times e_{L(G)}(e)]^2 + [e_G(v)\times e_{L(G)}(e)]^2]$$

$$= 2(m-1)\sum_{i=1}^{n-1}2[((m - i)(m - 1 + i))^2]\ ]$$

$$+ 4(m-1)\sum_{i=1}^{n-1}[(m - i)^4 + (m - (1 + i)(m - i)]^2$$

$$+ (m-1)\ [2(\frac{m^2 - 1}{4})^2\ ]$$

Theorem 2.2: Let the graph G = EBTN [m, n] be a eccentric indices of boron triangular nanotubes respectively, then (i) $R_{-1/2}$ (EBTN[m, n]), (ii) R_α (EBTN[m, n]), (iii) $RR\alpha$ (EBTN[m, n]),

(iv) M_1(EBTN[m, n]), (v) M_2(EBTN[m, n]), (vi)RR(EBTN[m, n]) (vii) RRR(EBTN[m, n]),

(viii) RM_2(EBTN[m,n]) and (ix) F(EBTN[m,n]).

Proof: Consider the eccentricity of boron triangular nanotubes G = EBTN [m, n].

(i) $R_{-1/2}(\mathrm{EBTN[m,n]}) = \sum_{ue} \frac{1}{\sqrt{e_{(G)}(u) + e_{(G)}(v)}}$

$$= \sum_{uv \in E_1(G)} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)+e_G(v)+e_{L(G)}(e)}} + \ldots..$$

$$+ \sum_{uv \in E_m(G)} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)+e_G(v)+e_{L(G)}(e)}}$$

$$= \frac{1}{\sqrt{2(m-1)[\sum_{i=1}^{n-1}(4m+(2-4i) + 4(m-1)[\sum_{i=1}^{n-1}(4m-(1+4i)] + 2m(m-1)}}$$

(ii) $R_\alpha(\mathrm{EBTN[m,n]}) = \sum_{ue} \frac{1}{(e_{(G)}(u) + e_{(G)}(v))^2}$

$$= \sum_{uv \in E_1(G)} \frac{1}{(e_G(u)+e_{L(G)}(e)+e_G(v)+e_{L(G)}(e))^2} + \ldots.. +$$

$$\sum_{uv \in E_m(G)} \frac{1}{(e_G(u)+e_{L(G)}(e)+e_G(v)+e_{L(G)}(e))^2}$$

$$= \frac{1}{[2(m-1)[\sum_{i=1}^{n-1}(4m+(2-4i) + 4(m-1)[\sum_{i=1}^{n-1}(4m-(1+4i)] + 2m(m-1)]^2}$$

(iii) $RR\alpha(\mathrm{EBTN[m,n]}) = \sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^\alpha$

$$= \sum_{uv \in E_1(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^\alpha + \ldots\ldots +$$

$$\sum_{uv \in E_m(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^\alpha$$

$$= [2(m-1)[\sum_{i=1}^{n-1}(4m + (2-4i)]$$

$$+ 4(m-1)[\sum_{i=1}^{n-1}(4m - (1+4i)] + 2m(m-1)]^\alpha$$

(iv) $M_1(\mathrm{EBTN[m,n]}) = \sum_{ue}(e_{(G)}(u) + e_{(G)}(v))^\alpha$

$$= \sum_{uv \in E_1(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^\alpha + \ldots\ldots +$$

$$\sum_{uv \in E_m(G)}(e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e))^\alpha$$

$$= [2(m-1)[\sum_{i=1}^{n-1}(4m + (2-4i)]$$

$$+ 4(m-1)[\sum_{i=1}^{n-1}(4m - (1+4i)] + 2m(m-1)]^\alpha$$

(v) $M_2(\mathrm{EBTN[m,n]}) = \sum_{ue}(e_{(G)}(u) \times e_{(G)}(v))^\alpha$

$$= \sum_{uv \in E_1(G)}(((e_G(u) \times e_{L(G)}(e)) + (e_G(v) \times e_{L(G)}(e)))^\alpha + \ldots\ldots +$$

$$\sum_{uv \in E_m(G)}(((e_G(u) \times e_{L(G)}(e)) + (e_G(v) \times e_{L(G)}(e)))^\alpha$$

$$= [2(m-1)[2\sum_{i=1}^{n-1}(m-i)(m-1+i)]$$

$$+ 4(m-1)[\sum_{i=1}^{n-1}(m-i)^2 + (m-(1+i)(m-i)]$$

$$+ \frac{1}{2}(m-1)(m^2 \,?\, -1)]^\alpha$$

(vi) $RR(EBTN[m,n]) = \sum_{uv\in E(G)} \sqrt{e_{(G)}(u)\times e_{(G)}(v)}$

$= \sum_{uv\in E_1(G)} \sqrt{(e_G(u) + e_{L(G)}(e)) \times (e_G(v) + e_{L(G)}(e))}$

$+ \ldots\ldots + \sum_{uv\in E_m(G)} \sqrt{(e_G(u) + e_{L(G)}(e)) \times (e_G(v) + e_{L(G)}(e))}$

$\sqrt{2(m-1)[2\sum_{i=1}^{n-1}(m-i)(m-1+i)] + 4(m-1)[\sum_{i=1}^{n-1}(m-i)^2 + (m-(1+i)(m-i)]\frac{1}{2}(m-1)(m^2-1)}$

(vii) $RRR(EBTN[m,n]) = \sum_{uv\in E(G)} \sqrt{(e_{(G)}(u)-1)(e_{(G)}(v)-1)}$

$= \sum_{uv\in E_1(G)} \sqrt{(e_G(u)-1) + (e_{L(G)}(e)-1)) \times (e_G(v)-1) + (e_{L(G)}(e)-1))}$
$+ \ldots\ldots +$
$\sum_{uv\in E_m(G)} \sqrt{((e_G(u)-1) + (e_{L(G)}(e)-1)) \times ((e_G(v)-1) + (e_{L(G)}(e)-1))}$

$= \sqrt{2(m-1)\sum_{i=1}^{n-1}[(2m-(2i+1))]^2 + 4(m-1)\sum_{i=1}^{n-1}(2m-(2i+2))((2m-(2i+3)) + (m-1)(m-2)^2}$

$(viii)$ $RM_2(EBTN[m,n]) = \sum_{uv\in E(G)}(e_{(G)}(u)-1)(e_{(G)}(v)-1)$

$= \sum_{uv\in E_1(G)}(e_G(u)-1) + (e_{L(G)}(e)-1)) \times (e_G(v)-1) + (e_{L(G)}(e)-1))$

$+ \ldots + \sum_{uv\in E_m(G)}(e_G(u)-1) + (e_{L(G)}(e)-1)) \times (e_G(v)-1) + (e_{L(G)}(e)-1))$

$= 2(m-1)\sum_{i=1}^{n-1}[(2m-(2i+1))]^2$

$+ 4(m-1)\sum_{i=1}^{n-1}(2m-(2i+2))((2m-(2i+3)) + (m-1)(m-2)^2$

(ix) $F(EBTN[m,n]) = \sum_{uv\in E(G)}(e_{(G)}(u))^2 + (e_{(G)}(v))^2)$

$= \sum_{uv\in E_1(G)}\left(e_{(G)}(u) + e_{L(G)}(e)\right)^2 + (e_{(G)}(v) + e_{L(G)}(e))^2) + \ldots\ldots +$

$\sum_{uv\in E_m(G)}\left(e_{(G)}(u) + e_{L(G)}(e)\right)^2 + (e_{(G)}(v) + e_{L(G)}(e))^2).$

$= 2(m-1)\sum_{i=1}^{n-1}[2(2m-(2i-1))^2]$

$+ 4(m-1)[\sum_{i=1}^{n-1}[2(m-i)]^2 + [(2m-(2i+1)]^2 + (m-1)(2m^2$ [illegible] $)$

Theorem 2.3: Let the graph G = EBTN [m, n] be a eccentricity of boron triangular nanotubes respectively, then (i) $^{m}B_1(EBTN[m,n])$, (ii) $^{m}B_2(EBTN[m,n])$ and (iv) $H_b(EBTN[m,n])$.

Proof: Consider the boron triangular nanotube $G = EBTN[m,n]$.

(i) $^{m}B_1(EBTN[m,n]) = \sum_{ue} \frac{1}{e_{(G)}(u) + e_{(G)}(v)}$

$= \sum_{uv\in E_1(G)}[\frac{1}{e_G(u) + e_{L(G)}(e)} + \frac{1}{e_G(v) + e_{L(G)}(e)}] + \ldots\ldots +$

$\sum_{uv\in E_m(G)}[\frac{1}{e_G(u) + e_{L(G)}(e)} + \frac{1}{e_G(v) + e_{L(G)}(e)}]$

$= 2(m-1)\sum_{i=1}^{n-1}[\frac{4m-(4i-2)}{((m-i)+(m-1-i))^2}$

$$+ 4(m-1) \sum_{i=1}^{n-1} \frac{4m-(4i+1)}{(2m-2i)(2m-(2i+1))} + (m-1)\frac{2}{m}$$

(ii) $$^{m}B_2(EBTN[m,n]) = \sum_{uv\in EG)} \frac{1}{e_G(u) \times e_{L(G)}(e)}$$

$$= \sum_{uv\in E_1(G)} [\frac{1}{e_G(u) \times e_{L(G)}(e)} + \frac{1}{e_G(v) \times e_{L(G)}(e)}] + \ldots +$$

$$\sum_{uv\in E_m(G)} [\frac{1}{e_G(u) \times e_{L(G)}(e)} + \frac{1}{e_G(v) \times e_{L(G)}(e)}]$$

$$= 2(m-1) \sum_{i=1}^{n-1} [\frac{2(m-i)(m-i+1)}{((m-i)(m-i+1))^2}$$

$$+ 4(m-1) \sum_{i=1}^{n-1} \frac{(m-(1+i))(m-i)+(m-i)^2}{(m-i)^3 (m-(1+i))} + (m-1)(\frac{8}{m^2-1})$$

(iii) $$H_b(EBTN[m,n]) = \sum_{uv\in EG)} \frac{2}{e_G(u)+ e_{L(G)}(e)}$$

$$= \sum_{uv\in E_1(G)} [\frac{2}{e_G(u)+ e_{L(G)}(e)} + \frac{2}{e_G(v)+ e_{L(G)}(e)}] + \ldots\ldots +$$

$$\sum_{uv\in E_m(G)} [\frac{2}{e_G(u)+ e_{L(G)}(e)} + \frac{2}{e_G(v)+ e_{L(G)}(e)}]$$

$$= 2(m-1) \sum_{i=1}^{n-1} [\frac{8m-(8i-4)}{((m-i)+(m-1-i))^2}$$

$$+ 4(m-1) \sum_{i=1}^{n-1} \frac{(4m-(4i+1)}{(m-i)(2m-(2i+1))} + (m-1)(\frac{4}{m})$$

Theorem 2.4: Let the graph G = EBTN [m, n] be a eccentricity of boron triangular nanotubes respectively, then (i) I(EBTN[m, n]) $(ii) A$(EBTN[m, n]) and (iii) GA(EBTN[m, n]).

Proof: Consider the eccentricity of boron triangular nanotube G = EBTN[m, n].

(i) $$I(EBTN[m,n]) = \sum_{ue} \frac{e_{(G)}(u)\times e_{(G)}(v)}{e_{(G)}(u)+ e_{(G)}(v)}$$

$$= \sum_{uv\in E_1(G)} [\frac{(e_{(G)}(u)\times e_{L(G)}(e))+(e_{(G)}(v)\times e_{L(G)}(e))}{(e_{(G)}(u)+e_{L(G)}(e))+(e_{(G)}(v)+ e_{L(G)}(e))}] + \ldots +$$

$$\sum_{uv\in E_m(G)} \left[\frac{(e_{(G)}(u)\times e_{L(G)}(e))+(e_{(G)}(v)\times e_{L(G)}(e))}{(e_{(G)}(u)+e_{L(G)}(e))+(e_{(G)}(v)+ e_{L(G)}(e))}\right]$$

$$=[\frac{2(m-1)\,[2\sum_{i=1}^{n-1}(m-i)(m-1+i)\,]+ 4(m-1)\,[\sum_{i=1}^{n-1}(m-i)^2 +(m-(1+i)(m-i)\,]+\frac{1}{2}\,(m-1)\,(m^2-1)}{2(m-1)\,[\sum_{i=1}^{n-1}(4m+(2-4i)\,]+ 4(m-1)\,[\sum_{i=1}^{n-1}(4m-(1+4i)\,] + 2m\,(m-1)}]$$

(ii) $$A(EBTN[m,n]) = \sum_{ue} \{\frac{e_{(G)}(u)\times e_{(G)}(v)}{e_{(G)}(u)+ e_{(G)}(v)-2}\}$$

$$= \sum_{uv\in E_1(G)} [\frac{(e_{(G)}(u)\times e_{L(G)}(e))+(e_{(G)}(v)\times e_{L(G)}(e))}{(e_{(G)}(u)+e_{L(G)}(e))-2)+(e_{(G)}(v)+ e_{L(G)}(e))-2)}] + \ldots +$$

$$\sum_{uv\in E_m(G)} [\frac{(e_{(G)}(u)\times e_{L(G)}(e))+(e_{(G)}(v)\times e_{L(G)}(e))}{(e_{(G)}(u)+e_{L(G)}(e))-2)+(e_{(G)}(v)+ e_{L(G)}(e))-2)}]$$

$$=\left\{\frac{2(m-1)\,[2\sum_{i=1}^{n-1}(m-i)(m-1+i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(m-i)^2+(m-(1+i)(m-i)\,]+\frac{1}{2}\,(m-1)\,(m^2-1)}{2(m-1)\,[\sum_{i=1}^{n-1}(4m+(2-4i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(4m-(1+4i)\,]+2m\,(m-1)}\right\}$$

(iii) $$GA(EBTN[m,n])=\sum_{ue}\frac{2\sqrt{(e_{(G)}(u)\times e_{(G)}(v))}}{(e_{(G)}(u)+e_{(G)}(v))}$$

$$=\sum_{uv\in E_1(G)}\frac{2\sqrt{(e_{(G)}(u)\times e_{(G)}(e))+(e_{(G)}(v)\times e_{(G)}(e)}}{((e_{(G)}(u)+e_{(G)}(e))+((e_{(G)}(v)+e_{L(G)}(e))}+\dots+$$

$$\sum_{uv\in E_m(G)}\frac{2\sqrt{(e_{(G)}(u)\times e_{(G)}(e))+(e_{(G)}(v)\times e_{(G)}(e)}}{((e_{(G)}(u)+e_{(G)}(e))+((e_{(G)}(v)+e_{L(G)}(e))}$$

$$=\frac{2\sqrt{2(m-1)\,[2\sum_{i=1}^{n-1}(m-i)(m-1+i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(m-i)^2+(m-(1+i)(m-i)\,]+\frac{1}{2}\,(m-1)\,(m^2-1)}}{2(m-1)\,[\sum_{i=1}^{n-1}(4m+(2-4i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(4m-(1+4i)\,]+2m\,(m-1)}$$

Theorem 2.5: Let the graph G = EBTN[m, n] be a eccentricity of boron triangular nanotubes respectively, then $(i)\ \chi E(EBTN[m,n])$, $(ii)\ XE(EBTN[m,n])$ *and (iii) SLCEII*(EBTN[m, n]).

Proof: Consider the eccentricity of boron triangular nanotube G = EBTN[m, n].

$$(i)\ \chi E(EBTN[m,n])=\sum_{ue}\frac{1}{\sqrt{e_G(u)e_{L(G)}(e)}}$$

$$=\sum_{uv\in E_1(G)}\frac{1}{\sqrt{(e_G(u)e_{L(G)}(e))+(e_G(u)e_{L(G)}(e))}}+\dots$$

$$\dots+\sum_{uv\in E_m(G)}\frac{1}{\sqrt{(e_G(u)e_{L(G)}(e))+(e_G(u)e_{L(G)}(e))}}$$

$$=\frac{1}{\sqrt{2(m-1)\,[2\sum_{i=1}^{n-1}(m-i)(m-1+i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(m-i)^2+(m-(1+i)(m-i)\,]+\frac{1}{2}\,(m-1)\,(m^2-1)}}$$

$$(ii)\ XE(EBTN[m,n])=\sum_{ue}\frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}}$$

$$=\sum_{uv\in E_1(G)}\frac{1}{\sqrt{(e_G(u)+e_{L(G)}(e))+(e_G(u)+e_{L(G)}(e))}}+\dots$$

$$..+\sum_{uv\in E_m(G)}\frac{1}{\sqrt{(e_G(u)+e_{L(G)}(e))+(e_G(u)+e_{L(G)}(e))}}$$

$$=\frac{1}{\sqrt{2(m-1)\,[\sum_{i=1}^{n-1}(4m+(2-4i)\,]+4(m-1)\,[\sum_{i=1}^{n-1}(4m-(1+4i)\,]+2m\,(m-1)}}$$

(iii) $SLCEII$ (EBTN[m, n]) = $\sum_{ue}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$.

$$= \sum_{uv\in E_1(G)}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}} + \ldots + \sum_{uv\in E_m(G)}\sqrt{\frac{e_{L(G)}(e)}{e_G(u)+e_G(v)}}$$

$$= \sqrt{2(m-1)\sum_{i=1}^{n-1}[\frac{m-i+1)}{2m-2i} + 4(m-1)\sum_{i=1}^{n-1}\frac{m-i}{2m-(2i+1)} + \frac{m+1}{2}}$$

3. Some eccentric indices of boron-α nanotubes

In the following theorem, we compute the some indices for *EBAN(X)[m, n]*.

Theorem 3.1: Consider the graph G = EBAN(X) [m, n] be a eccentricity of boron-α nanotubes respectively, then

(i) $EB_1AN(X)$ [m, n] $= 18\sum_{i=1}^{n-1}\left[4m - 2\times 3^{i-1}\right] + 36\sum_{i=1}^{n-1}[(4m - 12i +$
7]

$+ 24\sum_{i=1}^{n-1}[4m - 4i - 5] + 6\sum_{i=1}^{n-1}[4m - 12i - 2]$
$+ 18(2m+2) + 18(2m)$

(ii) $EB_2AN(X)$ [m, n] $= 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)]$
$+ \quad 36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+$
2)]

$+ 24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1)]$
$+ 6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)]$

$+ 18\left(\frac{m(m+2)}{2}\right) + 18\left(\frac{m^2}{2}\right)$

(iii) $HEB_1AN(X)$ [m, n] $= 18\sum_{i=1}^{n-1} 2\left[(2m-2i+1)^2\right]$

$+ 36\sum_{i=1}^{n-1}[(2m-6i+4)^2 + (2m-6i+3)^2]$

$+ 24\sum_{i=1}^{n-1}[(2m-2i-2)^2 + (2m-2i-3)^2]$

$+ 6\sum_{i=1}^{n-1} 2\left[(2m-6i+1)^2\right]$

$+ 18\,(2(m+1)^2) + 18\,(2m^2 ⍰)$

(iv) $HEB_2AN(X)$ [m, n] $= 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)]^2$
$+ \quad 36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+$
$2))^2]$

$+ 24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1))^2]$
$+ 6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)]^2$

$+ 18\left(\frac{(m(m+2))^2}{8}\right) + 18\left(\frac{m^4}{8}\right)$

Proof

Consider the eccentricity of boron-α nanotube EBAN(X) [m, n] nanotube.

Let V_k be the vertex set of EBAN(X) [m, n] and E_k be the edge set in EBAN(X) [m, n], then $|V_{m,\ n}| = \frac{n(4m+1)}{3}$ $and |E_{m,\ n}| = \frac{n(7m-2)}{2}$ for the structure of EBAN(X) [m, n].

Also the number of edges with eccentricities of end vertices of G and L(G). We give these values in the above Table.

$$(i) EB_1AN(X)\ [m,n] = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]$$

$$= \sum_{e=uv\in E_1(G)}\left[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)\right] + \ldots + \sum_{e=uv\in E_m(G)}\left[e_G(u) + e_{L(G)}(e) + e_G(v) + e_{L(G)}(e)\right]$$

$$= 18\sum_{i=1}^{n-1}\left[4m - 2\times 3^{\,i-1}\right] + 36\sum_{i=1}^{n-1}[(4m - 12i + 7] + 24\sum_{i=1}^{n-1}[4m - 4i - 5] + 6\sum_{i=1}^{n-1}[4m - 12i - 2] + 18(2m+2) + 18(2m)$$

$$(ii)\quad EB_2AN(X)\ [m,n] = \sum_{ue}\left[e_G(u)\times e_{L(G)}(e)\right]$$

$$= \sum_{e=uv\in E_1(G)}\left[(e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e))\right] + \ldots + \sum_{e=uv\in E_m(G)}\left[(e_G(u)\times e_{L(G)}(e)) + (e_G(v)\times e_{L(G)}(e))\right]$$

$$= 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)] + 36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+2)] + 24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1)] + 6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)] + 18\left(\frac{m(m+2)}{2}\right) + 18\left(\frac{m^2}{2}\right)$$

$$(iii)\quad HEB_1AN(X)\ [m,n] = \sum_{ue}\left[e_G(u) + e_{L(G)}(e)\right]^2$$

$$= \sum_{e=uv\in E_1(G)}\left[[e_G(u) + e_{L(G)}(e)]^2 + [e_G(v) + e_{L(G)}(e)]^2\right] + \ldots + \sum_{e=uv\in E_m(G)}\left[[e_G(u) + e_{L(G)}(e)]^2 + [e_G(v) + e_{L(G)}(e)]^2\right]$$

$$= 18\sum_{i=1}^{n-1} 2\left[(2m-2i+1)^2\right] + 36\sum_{i=1}^{n-1}[(2m-6i+4)^2 + (2m-6i+3)^2] + 24\sum_{i=1}^{n-1}[(2m-2i-2)^2 + (2m-2i-3)^2]$$

$$+ 6\sum_{i=1}^{n-1} 2\left[(2m-6i+1)^{2}\right] + 18\,(2(m+1)^{2}) + 18\,(2m^{2}\,?)$$

$$(iv)\ HEB_2AN(X)\,[m,n] = \sum_{ue}\left[e_G(u)\times e_{L(G)}(e)\right]^2$$

$$= \sum_{e=uv\in E_1(G)}\left[[e_G(u)\times e_{L(G)}(e)]^2 + [e_G(v)\times e_{L(G)}(e)]^2\right] + \ldots + \sum_{e=uv\in E_m(G)}\left[[e_G(u)\times e_{L(G)}(e)]^2 + [e_{H_k}(v)\times e_{L(H_k)}(e)]^2\right]$$

$$= 18\sum_{i=1}^{n-1} 2[(m-i)\times(m-i+1)]^2$$

$$+ 36\sum_{i=1}^{n-1}[(m-3i+2)^2 + ((m-3i+1)\times(m-3i+2))^2]$$

$$+ 24\sum_{i=1}^{n-1}[(m-i-1)^2 + ((m-i-2)\times(m-i-1))^2]$$

$$+ 6\sum_{i=1}^{n-1} 2[(m-3i)\times(m-3i+1)]^2$$

$$+ 18\left(\frac{(m(m+2))^2}{8}\right) + 18\left(\frac{m^4}{8}\right)$$

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