

The Eccentricity Version Of Some Square Reduced Indices Of Nanostar Dendrimer D[N].

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Abstract: In this present paper, we introduce Eccentricity based new indices namely, Square Reduced Sum Index, Square Reduced Product Index, Square Reduced Arithmetic Index, Square Reduced Geometric Index and Square Reduced Harmonic Index of Nanostar Dendrimer.

Keywords: Nanostar Dendrimer, Eccentricity, Square Reduced, Sum, product, Arithmetic, Geometric, Harmonic.

1.Introduction:

There are a lot of chemical compounds, either organic or inorganic, which possess a level of commercial, industrial, pharmaceutical chemistry and laboratory importance. A relationship exists between chemical compounds and their molecular structures. Graph theory is a very powerful area of mathematics that has wide range of applications in many areas of science such as chemistry, biology, computer science, electrical, electronics and other fields.

Recently Furtula et al. in [4] proposed the reduced second Zagreb index, defined as

$$RM_2(G) = \sum_{uv \in E(G)} (d_G(u)-1) (d_G(v)-1)$$

The Harmonic index $H(G)$ of a graph G was first appeared in [3] and it is defined as

$$H(G) = \sum_{uv \in E(G)} \left[\frac{2}{d_u + d_v} \right]$$

Let $ec(v)$ be the eccentricity of vertex v which defined as the largest distance between v and any other vertex u of G .

we define Eccentricity based Square Reduced sum, Square Reduced product, Square Reduced Arithmetic, Square Reduced Geometric and Square Reduced Harmonic indices of a graph G . Here we introduced new indices on Eccentricity namely

Square Reduced Sum, Square Reduced Product, Square Reduced Arithmetic, Square Reduced Geometric and Square Reduced Harmonic indices as,

$$SRS(G) = \sum_{uv \in E(G)} [(e(u) - 1)^2 + (e(v) - 1)^2]$$

$$SRP(G) = \sum_{uv \in E(G)} [(e(u) - 1)^2 ((e(v) - 1)^2)]$$

$$SRA(G) = \sum_{uv \in E(G)} \left[\frac{[(e(u) - 1)^2 + (e(v) - 1)^2]}{2} \right]$$

$$SRH(G) = \sum_{uv \in E(G)} \left[\frac{2}{[(e(u) - 1)^2 + (e(v) - 1)^2]} \right]$$

$$SRG(G) = \sum_{uv \in E(G)} \sqrt{[(e(u) - 1)^2 (e(v) - 1)^2]}$$

2. Eccentricity Based Topological indices of Nanostar Dendrimer D[n].

In this paper, we focus on the special infinite class of nanostar dendrimers D[n] (here n is the step of growth) which is described as follows:

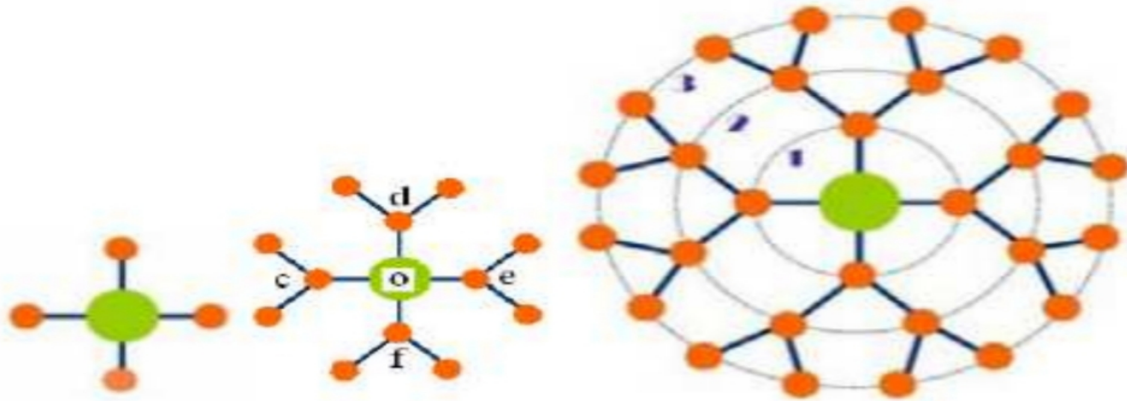


Figure 1. The structures of D[1], D[2] and D[3].

By the analysis of its molecular structure, we see that the edge set of D[n] can be divided into n parts:

$$E_1: ec(u) = n \text{ and } ec(v) = n+1;$$

$$E_2: ec(u) = n+1 \text{ and } ec(v) = n+2;$$

.....

$$E_i: ec(u) = n+i + 1 \text{ and } ec(v) = n+i;$$

.....

E_n : $ec(u) = 2n - 1$ and $ec(v) = 2n$.

Furthermore, we have $|E_1| = 4, |E_2| = 8, \dots, |E_i| = 2^{i+1}, \dots$ and $|E_n| = 2^{n+1}$. Thus, in view of the definition of eccentric related indices, we get

Theorem 1: The Square Reduced sum index of $D[n]$ is given by

$$SRS(G) = 4 \sum_{i=1}^n 2^{i-1} [(n+i-2)^2 + (n+i-1)^2]$$

Proof: we derive the Square Reduced sum index of the nanostar dendrimer $D[n]$ as follows:

$$SRS(G) = \sum_{uv \in E(G)} [(e(u) - 1)^2 + (e(v) - 1)^2]$$

$$\begin{aligned} SRS(G) &= (4 \times 2^0) [(n-1)^2 + n^2] + \\ &\quad (4 \times 2^1) [n^2 + (n+1)^2] + \\ &\quad (4 \times 2^2) [n^2 + (n+1)^2] + \dots + \\ &\quad (4 \times 2^{n-1}) [(2n-2)^2 + (2n-1)^2] \end{aligned}$$

$$SRS(G) = 4 \sum_{i=1}^n 2^{i-1} [(n+i-2)^2 + (n+i-1)^2]$$

Theorem 2: The Square Reduced product index of $D[n]$ is given by

$$SRP(G) = 4 \sum_{i=1}^n 2^{i-1} [(n+i-2)^2 (n+i-1)^2]$$

Proof: we compute the Square Reduced product index of the nanostar dendrimer $D[n]$ as follows:

$$\begin{aligned} SRP(G) &= \sum_{uv \in E(G)} [(e(u) - 1)^2 ((e(v) - 1)^2)] \\ &= (4 \times 2^0) [(n-1)^2 n^2] + \\ &\quad (4 \times 2^1) [n^2 (n+1)^2] + \\ &\quad (4 \times 2^2) [n^2 (n+1)^2] + \dots + \\ &\quad (4 \times 2^{n-1}) [(2n-2)^2 (2n-1)^2] \end{aligned}$$

$$SRP(G) = 4 \sum_{i=1}^n 2^{i-1} [(n+i-2)^2 (n+i-1)^2]$$

Theorem 3: The Square Reduced Arithmetic index of $D[n]$ is given by

$$SRA(G) = 2 \sum_{i=1}^n 2^{i-1} [(n+i-2)^2 (n+i-1)^2]$$

Proof: we find the Square Reduced Arithmetic index of the nanostar dendrimer $D[n]$ as follows:

$$\begin{aligned} \text{SRA}(G) &= \sum_{uv \in E(G)} \left[\frac{[(e(u)-1)^2 + (e(v)-1)^2]}{2} \right] \\ &= \left[\frac{1}{2} \right] \{ (4 \times 2^0) [(n-1)^2 + n^2] + \\ &\quad (4 \times 2^1) [n^2 + (n+1)^2] + \\ &\quad (4 \times 2^2) [n^2 + (n+1)^2] + \dots + \\ &\quad (4 \times 2^{n-1}) [(2n-2)^2 + (2n-1)^2] \} \\ \text{SRA}(G) &= 2 \sum_{i=1}^n 2^{i-1} [(n+i-2)^2 (n+i-1)^2] \end{aligned}$$

Theorem 4: The Square Reduced Harmonic index of $D[n]$ is given by

$$\text{SRH}(G) = 8 \sum_{i=1}^n 2^{i-1} \left[\frac{1}{[(n+i-2)^2 (n+i-1)^2]} \right]$$

Proof: we calculate the Square Reduced Harmonic index of the nanostar dendrimer $D[n]$ as follows:

$$\begin{aligned} \text{SRH}(G) &= \sum_{uv \in E(G)} \left[\frac{2}{[(e(u)-1)^2 + (e(v)-1)^2]} \right] \\ &= 2 \{ (4 \times 2^0) \frac{1}{[(n-1)^2 + n^2]} + (4 \times 2^1) \frac{1}{[n^2 + (n+1)^2]} + \\ &\quad (4 \times 2^2) \frac{1}{[n^2 + (n+1)^2]} + \dots + (4 \times 2^{n-1}) \frac{1}{[(2n-2)^2 + (2n-1)^2]} \} \\ \text{SRH}(G) &= 8 \sum_{i=1}^n 2^{i-1} \left[\frac{1}{[(n+i-2)^2 (n+i-1)^2]} \right] \end{aligned}$$

Theorem 5: The Square Reduced Geometric index of $D[n]$ is given by

$$\text{SRG}(G) = 4 \sum_{i=1}^n 2^{i-1} [(n+i-2)(n+i-1)]$$

Proof: we compute the Square Reduced Geometric index of the nanostar dendrimer $D[n]$ as follows:

$$\begin{aligned} \text{SRG}(G) &= \sum_{uv \in E(G)} \sqrt{[(e(u)-1)^2 (e(v)-1)^2]} \\ &= (4 \times 2^0) \sqrt{(n-1)^2 n^2} + \\ &\quad (4 \times 2^1) \sqrt{n^2 (n+1)^2} + \end{aligned}$$

$$(4 \times 2^2) \sqrt{n^2 (n + 1)^2} + \dots +$$

$$(4 \times 2^{n-1}) \sqrt{(2n - 2)^2 (2n - 1)^2}$$

$$SRG(G) = 4 \sum_{i=1}^n 2^{i-1} [(n + i - 2)(n + i - 1)]$$

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