Measure of Mutated Slope Rotatability for Response Surface Designs of Second Order Model Using a Pair of Similar Incomplete Block Designs

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Abstract

To assess the degree of mutated slope rotatability for a given response surface designs of second order model and variances of the estimated responses using a pair of similar incomplete block designs (like a pair of symmetrical unequal block arrangements with two unequal block sizes) is developed in this paper.

Keywords: Response surface designs, Rotatability, mutated slope rotatability, measure

1. Introduction

An important criterion in response surface designs was initiated by Box and Hunter (1957). Another important idea of construction of rotatable designs using incomplete block designs was developed by Das and Narasimham (1962). The concept of slope rotatability for second order response surface model was studied by Hader and Park (1978). Victorbabu and Narasimham (1991a, 91b) further developed construction of slope rotatable designs of second order model using balanced incomplete block designs (BIBD) and pair of incomplete block designs respectively. Measure of slope rotatability for second order model was studied by Park and Kim (1992). Mutated (modified) response surface design of second order model was developed by Das et al (1999).

Similarly several authors like Victorbabu (2005, 2006a, 2006b, 2006c, 2007) Victorbabu and Surekha (2012a, 12b, 12c, 2016), Chiranjeevi and Victorbabu (2020), Victorbabu and Jyostna (2021a, 2021b) and Jyostna and Victorbabu (2020a, 2020b) contributed for the development of mutated slope rotatability, measure of slope rotatability and measure of mutated slope rotatability respectively.

In this paper, to assess the degree of mutated slope rotatability for a given response surface designs of second order model and variances of the estimated responses using a pair of similar incomplete block designs (like a pair of symmetrical unequal block arrangements (SUBA) with two unequal block sizes with two unequal block sizes) is developed.

2. Conditions for slope rotatable design of second order model

Suppose we want to fit a response surface design of second order model $D=((P_{iu}))$ to fit the surface is,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i p_{iu} + \sum_{i=1}^{v} b_{ii} p_{iu}^2 + \sum_{i=1}^{v}\sum_{j=1}^{v} b_{ij} p_{iu} p_{ju} + e_u \qquad (1)$$

The conditions to get slope rotatability for second order model are given below [cf. Box and Hunter (1957), Hader and Park (1978) and Victorbabu and Narasimham (1991a)].

1. $\sum p_{iu}=0,\ \sum p_{iu}p_{ju}=0,\ \sum p_{iu}p_{ju}^2=0,\ \sum p_{iu}p_{ju}p_{ku}=0,\ \sum p_{iu}^3=0,\ \sum p_{iu}p_{ju}^3=0,$
   $\sum p_{iu}p_{ju}p_{ku}^2=0,\ \sum p_{iu}p_{ju}p_{ku}p_{lu}=0;$ for $i \neq j \neq k \neq l$

2. (i) $\sum p_{iu}^2 = \text{constant} = N\gamma_2;$
   (ii) $\sum p_{iu}^4 = \text{constant} = cN\gamma_4;$ for all i

3. $\sum p_{iu}^2 p_{ju}^2 = \text{constant} = N\gamma_4;$ for $i \neq j$ (2)

4. $$\frac{\gamma_4}{\gamma_2^2} > \frac{v}{(c+v-1)} \qquad (3)$$

5. $$[v(5-c)-(c-3)^2]\,\gamma_4 + [v(c-5)+4]\,\gamma_2^2 = 0 \qquad (4)$$

where c, γ_2 and γ_4 are constants.

The variances and covariances of the estimated parameters are

$$V(\hat{b}_0) = \frac{\gamma_4(c+v-1)\sigma^2}{N\left[\gamma_4(c+v-1)-v\gamma_2^2\right]},$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\gamma_2},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\gamma_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\gamma_4}\left[\frac{\gamma_4(c+v-2)-(v-1)\gamma_2^2}{\gamma_4(c+v-1)-v\gamma_2^2}\right],$$

$$\mathrm{Cov}(\hat{b}_0,\hat{b}_{ii}) = \frac{-\gamma_2\sigma^2}{N[\gamma_4(c+v-1)-v\gamma_2^2]},$$

$$\mathrm{Cov}(\hat{b}_{ii},\hat{b}_{jj}) = \frac{(\gamma_2^2-\gamma_4)\sigma^2}{(c-1)N\gamma_4[\gamma_4(c+v-1)-v\gamma_2^2]} \qquad (5)$$

and other covariances vanish.

3. Conditions for mutated slope rotatable designs of second order model

Equations 1, 2 and 3 of (2), (3), (4), (5) and $(\sum p_{iu}^2)^2 = N\sum p_{iu}^2 p_{ju}^2$ provide the necessary and sufficient conditions for mutated slope rotatable designs of second order model (cf. Hader and Park (1978), Das et al. 1999, Victorbabu, 2005, 06a).

Here $(\sum p_{iu}^2)^2 = N\sum p_{iu}^2 p_{ju}^2$ leads to $\gamma_2^2 = \gamma_4$. (6)

Substituting $\gamma_2^2 = \gamma_4$ in (4) and on simplification we get c=1 or c=5. From the non-singularity condition we get c=5. The variances and covariances for mutated slope rotatable designs of second order model are given below

$$V(\hat{b}_0) = \frac{(v+4)\sigma^2}{4N},$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\gamma_4}},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\gamma_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{4N\gamma_4}$$

$\mathrm{Cov}(\hat{b}_0,\hat{b}_{ii}) = \frac{-\sigma^2}{4N\sqrt{\gamma_4}}$ and other covariances are zero. (7)

$$V\left(\frac{\partial\hat{Y}}{\partial x_i}\right) = \left[\frac{\sqrt{\gamma_4}+d^2}{N\gamma_4}\right]\sigma^2.$$ (cf. Victorbabu 2005, 06a) (8)

4. Conditions of measure of slope rotatability for response surface designs of second order model

Following Hader and Park (1978), Victorbabu and Narasimham (1991a), Park and Kim (1992), equations (2), (3), (4) and (5) provide the necessary and sufficient conditions for a measure of slope rotatability for response surface designs of second order model. Further variances and covariances are given below,

$V(b_i)$, $V(b_{ii})$ are the same for all i,

$V(b_{ij})$ are the same for all i, j; where $i \neq j$,

$Cov(b_i,b_{ii}) = Cov(b_i,b_{ij}) = Cov(b_{ii},b_{ij}) = Cov(b_{ij},b_{il})$ for all $i \neq j \neq l$.

The measure of slope rotatability ($Q_v(D)$) for response surface design of second order model can be achieved by the following equation (cf. Park and Kim, 1992, page 398).

$$Q_V(D)=\frac{1}{\sigma^4}\left[4V(b_{ii})-V(b_{ij})\right]^2$$

Note: $Q_v(D)$ is zero, if and only if a design D is slope-rotatable. $Q_v(D)$ becomes more as D deviates from a slope-rotatable design.

5. Measure of slope rotatability for second order response surface designs model using a pair of similar incomplete block designs

The result of measure of slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs (like a pair of SUBA with two unequal block sizes) is given below (cf. Chiranjeevi and Victorbabu, 2020).

Result: The design points,

$$\left[1\text{-}(v,d_1,r_1,q_{11},q_{12},d_{11},d_{12},\lambda_1)\right]2^{t(q_1)}U\left[a\text{-}(v,d_2,r_2,q_{21},q_{22},d_{21},d_{22},\lambda_2)\right]2^{t(q_2)}U(n_0)$$

will give a v-dimensional measure of slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs $N = d_1 2^{t(q_1)} + d_2 2^{t(q_2)} + n_0$ where

$D_1=(v,d_1,r_1,q_{11},q_{12},d_{11},d_{12},\lambda_1)$, $q_1=\sup(q_{11},q_{12})$, $q_{11}+q_{12}=q_1$, $r_1 \lesseqgtr c\lambda_1$ and

$D_2=(v,d_2,r_2,q_{21},q_{22},d_{21},d_{22},\lambda_2)$, $q_2=\sup(q_{21},q_{22})$, $d_{21}+d_{22}=d_2$, $r_2 \underset{<}{\overset{>}{-}} c\lambda_2$ are two SUBA with two unequal block sizes. Further,

$$Q_V(D) = \left[\frac{\sum p_{iu}^2}{N}\right]^4 \left[4e\text{-}V(b_{ij})\right]^2$$

If $Q_v(D)$ is zero, if and only if, a design 'D' is slope-rotatable. $Q_v(D)$ becomes more as 'D' deviates from a slope rotatable design (cf. Park and Kim (1992), Chiranjeevi and Victorbabu (2020)).

6. Mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs

Following the method of construction of mutated slope rotatability for response surface designs of second order model using a pair of BIBD of Victorbabu (2006b), a new method of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs (like SUBA with two unequal block sizes) is given in the following result.

If $D_1=(v,d_1,r_1,q_{11},q_{12},d_{11},d_{12},\lambda_1)$, and $D_2=(v,d_2,r_2,q_{21},q_{22},d_{21},d_{22},\lambda_2)$, are pair of similar incomplete block designs. Let $q_1=\sup(q_{11},q_{12})$, $q_{11}+q_{12}=q_1$, $r_1 \underset{>}{\overset{<}{-}} c\lambda_1$ of D_1 and $q_2=\sup(q_{21},q_{22})$, $d_{21}+d_{22}=d_2$, $r_2 \underset{<}{\overset{>}{-}} c\lambda_2$ of D_2 designs respectively. Let $2^{t(q_1)}$ and $2^{t(q_2)}$ resolution V fractional replicates of 2^{q_1} and 2^{q_2} factorials with levels ± 1, such that no interaction with less than five factors is confounded. Let $[1\text{-}(v,d_1,r_1,q_{11},q_{12},d_{11},d_{12},\lambda_1)]$ denote the design points obtained from the transpose of incidence matrix of a pair of similar incomplete block designs with two unequal block sizes, $[1\text{-}(v,d_1,r_1,q_{11},q_{12},d_{11},d_{12},\lambda_1)]2^{t(q_1)}$ are the $d_1 2^{t(q_1)}$ design points obtained from design D_1 by "multiplication" (cf. Raghavarao 1971). Repeat these design points y_1 times. Let $[a\text{-}(v,d_2,r_2,q_{21},q_{22},d_{21},d_{22},\lambda_2)]2^{t(q_2)}$ are the $d_2 2^{t(q_2)}$ design points obtained from design D_2 by multiplication, repeat these design by y_2 times and n_0 central points. The new method suggested in the following theorem.

Theorem:

The design points,

$y_1[1\text{-}(v,d_1,r_1,q_{11},q_{12},d_{11},d_{12},\lambda_1)]2^{t(q_1)} \cup y_2\,[a\text{-}(v,d_2,r_2,q_{21},q_{22},d_{21},d_{22},\lambda_2)]2^{t(q_2)} \cup (n_0)$ will give a v-dimensional mutated slope rotatability for response surface design of second order model.

Proof:

From 2(i), (ii) of (2) and (3) equations, the simple symmetric conditions are

$$\sum p_{iu}^2 = y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2 = N\gamma_2 \quad (9)$$

$$\sum p_{iu}^4 = y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2 = 5N\gamma_4 \quad (10)$$

$$\sum p_{iu}^2 p_{ju}^2 = y_1 \lambda_1 2^{t(q_1)} + y_2 \lambda_2 2^{t(q_2)} a^4 = N\gamma_4 \quad (11)$$

From (9) and (11), we get

$$N = \frac{(y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2)^2}{y_1 \lambda_1 2^{t(q_1)} + y_2 \lambda_2 2^{t(q_2)} a^4}$$ design points if,

$$a^4 = \frac{y_1(5\lambda_1 - r_1)}{y_2(r_2 - 5\lambda_2)} 2^{t(q_1) - t(q_2)},$$

$$n_0 = \frac{(y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2)^2}{y_1 \lambda_1 2^{t(q_1)} + y_2 \lambda_2 2^{t(q_2)} a^4} - y_1 b_1 2^{t(q_1)} - y_2 b_2 2^{t(q_2)}$$ and n_0 turns out to be an integer.

Example: We illustrate the new method for $v=12$ factors with the help of a mutated pair of similar incomplete block designs (like SUBA with two unequal block sizes) with parameters

$D_1 = (v=12,\ d_1=13,\ r_1=4,\ q_{11}=3,\ q_{12}=4,\ d_{11}=4,\ d_{12}=9,\ \lambda_1=1)$ and

$D_2 = (v=12,\ d_2=26,\ r_2=6,\ q_{21}=2,\ q_{22}=3,\ d_{21}=6,\ d_{22}=20,\ \lambda_2=1)$.

The design points,

$$y_1[1\text{-}(v=12,d_1=13,r_1=4,q_{11}=3,q_{12}=4,d_{11}=4,d_{12}=9,\lambda_1=1)]2^4 \cup$$
$$y_2\,[a\text{-}(v=12,d_2=26,r_2=6,q_{21}=2,q_{22}=3,d_{21}=6,d_{22}=20,\lambda_2=1)]2^3 \cup (n_0)$$

will provide a mutated slope rotatability for response surface design of second order model in N=784 design points. From (9), (10) and (11), we get

$$\sum p_{iu}^2 = y_1 64 + y_2 48a^2 = N\gamma_2 \quad (12)$$

$$\sum p_{iu}^4 = y_1 64 + y_2 48a^4 = 5N\gamma_4 \quad (13)$$

$$\sum p_{iu}^2 p_{ju}^2 = y_1 16 + y_2 8a^4 = N\gamma_4 \qquad (14)$$

$\sum p_{iu}^4 = 5\sum p_{iu}^2 p_{ju}^2$, we have

$$y_1 64 + y_2 48a^4 = 5(y_1 16 + y_2 8a^4)$$

$$y_2 8a^4 = y_1 16$$

$$a^4 = \frac{16y_1}{8y_2}$$

$$a^4 = 4 \text{ for } (y_1 = 2,\ y_2 = 1)$$

$$a^2 = 2$$

using the mutated (modified) condition (6) using (12) and (14) and on simplification we get N=784, and $n_0 = 160$.

7. Measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs

The proposed measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs (like a pair of SUBA with two unequal block sizes) is suggested here.

The design points,

$$y_1[1\text{-}(v,d_1,r_1,q_{11},q_{12},d_{11},d_{12},\lambda_1)]2^{t(q_1)} \cup y_2[a\text{-}(v,d_2,r_2,q_{21},q_{22},d_{21},d_{22},\lambda_2)]2^{t(q_2)} \cup (n_0)$$

generated from a pair of similar incomplete block designs in $N = \dfrac{(y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2)^2}{y_1\lambda_1 2^{t(q_1)} + y_2\lambda_2 2^{t(q_2)} a^4}$ design points, will give a v-dimensional measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs with

$$a^4 = \frac{y_1(5\lambda_1 - r_1)}{y_2(r_2 - 5\lambda_2)} 2^{t(q_1) - t(q_2)},$$

$$\gamma_2 = \frac{y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2}{N} \text{ and } \gamma_4 = \frac{y_1\lambda_1 2^{t(q_1)} + y_2\lambda_2 2^{t(q_2)} a^4}{N}$$

(Alternatively N may be obtained directly as $N = y_1 d_1 2^{t(q_1)} + y_2 d_2 2^{t(q_2)} + n_0$ design points). For the above design points generated from a pair of similar incomplete block designs using (6) and on simplification, we get,

$$n_0 = \frac{(y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2)^2}{y_1 \lambda_1 2^{t(q_1)} + y_2 \lambda_2 2^{t(q_2)} a^4} - y_1 b_1 2^{t(q_1)} - y_2 b_2 2^{t(q_2)}$$

$$Q_V(D) = \left[\frac{\sum p_{iu}^2}{N}\right]^4 \left[4e - V(b_{ij})\right]^2$$

where

$$e = V(\hat{b}_{ii}) = \frac{y_1 d_1 2^{t(q_1)} + y_2 d_2 2^{t(q_2)} + n_0}{4[y_1^2 r_1^2 2^{2t(q_1)} + y_1 y_2 2 r_1 r_2 2^{t(q_1)+t(q_2)} a^2 + y_2^2 r_2^2 2^{2t(q_2)} a^4]} \qquad (\text{since } \gamma_2^2 = \gamma_4)$$

Measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs is

$$Q_v(D) = \left[\frac{y_1 r_1 2^{t(q_1)} + y_2 r_2 2^{t(q_2)} a^2}{N}\right]^4 \left[4e - \frac{1}{y_1 \lambda_1 2^{t(q_1)} + y_2 \lambda_2 2^{t(q_2)} a^4}\right]^2$$

The following Table gives the values of measure of mutated slope rotatability for response surface designs of second order model using various parameters of pair of similar incomplete block designs for different level of 'a'. Variances of the estimated responses are also obtained.

Example: We illustrate the new measure of mutated slope rotatability for response surface of second order model for $v=12$ factors with the help of a pair of similar incomplete block designs (like a pair of SUBA with two unequal block sizes) with parameters $D_1 = (v=12, d_1=13, r_1=4, q_{11}=3, q_{12}=4, d_{11}=4, d_{12}=9, \lambda_1=1)$ and

$D_2 = (v=12, d_2=26, r_2=6, q_{21}=2, q_{22}=3, d_{21}=6, d_{22}=20, \lambda_2=1)$.

The design points,

$$y_1[1-(v=12, d_1=13, r_1=4, q_{11}=3, q_{12}=4, d_{11}=4, d_{12}=9, \lambda_1=1)]2^4 \cup$$
$$y_2[a-(v=12, d_2=26, r_2=6, q_{21}=2, q_{22}=3, d_{21}=6, d_{22}=20, \lambda_2=1)]2^3 \cup (n_0)$$

will give a measure of mutated rotatability for response surface design of second order model in N=784 design points. From (9), (10) and (11), we have

$$\sum p_{iu}^2 = y_1 64 + y_2 48a^2 = N\gamma_2 \quad (15)$$

$$\sum p_{iu}^4 = y_1 64 + y_2 48a^4 = cN\gamma_4 \quad (16)$$

$$\sum p_{iu}^2 p_{ju}^2 = y_1 16 + y_2 8a^4 = N\gamma_4 \quad (17)$$

From equations (16) and (17) with slope rotatability value $c=5$, $y_1=2$ and $y_2=1$, we get $a^4 = 4 \Rightarrow a^2 = 2 \Rightarrow a=1.414213$. From equations (15) and (17) using the mutated condition with $(\gamma_2^2=\gamma_4)$ along with $a^2=2$, $y_1=2$ and $y_2=1$, we get $N=784$, $n_0=160$. At $a=1.4142$, we get $e = 0.00391$ then $Q_v(D)$ is zero. Then the design is mutated slope rotatable. Variance of the estimated response for measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs is

$$V\left(\frac{\partial \hat{y}}{\partial p_i}\right) = 4.4643\times10^{-3}\sigma^2 + 1.5625\times10^{-2} d^2\sigma^2 .$$

Suppose if we take $a = 2.5$ instead of taking $a = 1.4142$ for 12- factors we get $e = 0.0011$ then $Q_v(D) = 1.6844\times10^{-7}$. Here $Q_v(D)$ becomes larger it deviates from mutated slope rotatability. Variance of the estimated response for measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs is

$$V\left(\frac{\partial \hat{y}}{\partial p_i}\right) = 1.924\times10^{-3}\sigma^2 + 2.903\times10^{-3} d^2\sigma^2 .$$

Table gives the values of measure of mutated slope rotatability $(Q_v(D))$ for response surface designs of second order model using a pair of similar incomplete block designs, at different values of 'a' for 12-factors. It can be verified that $Q_v(D)$ is zero, if and only if a design 'D' is mutated slope rotatable of second order model. $Q_v(D)$ becomes larger as 'D' deviates from a mutated slope rotatable of second order model. Variances of the estimated responses for measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs for different values of 'a' are also included in the table.

Conclusion: In this paper, mutated and measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs with two unequal block sizes (like a pair of SUBA with two unequal block sizes) has been proposed which enables us to assess the degree of mutated slope rotatability for a given response surface design. This measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs, $Q_v(D)$ has the value zero, if and only if, the design 'D' is reformed slope rotatable design, and becomes larger as 'D' deviates from a mutated slope rotatable design. Variances of the estimated response for measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs (like a pair of SUBA with two unequal block sizes) are also obtained.

Table: Measure of mutated slope rotatability for response surface designs of second order model using a pair of similar incomplete block designs (like SUBA with two unequal block sizes).

$(12,13,4,3,4,4,9,1)(12,26,6,2,3,6,20,1)$, N=784, a=1.4142, $n_0=160, y_1=2, y_2=1$		
a	$Q_v(D)$	$V\left(\partial\hat{y}/\partial p_i\right)$
1.0	2.4394×10^{-10}	$5.6469\times10^{-3}\ \sigma^2+2.5\times10^{-2}\ d^2\sigma^2$
*1.4142	0	$4.4643\times10^{-3}\ \sigma^2+1.5625\times10^{-2}\ d^2\sigma^2$
1.5	6.5904×10^{-10}	$4.194\times10^{-3}\ \sigma^2+1.379\times10^{-2}\ d^2\sigma^2$
2.0	5.4886×10^{-8}	$2.8235\times10^{-3}\ \sigma^2+6.25\times10^{-3}\ d^2\sigma^2$
2.5	1.6844×10^{-7}	$1.924\times10^{-3}\ \sigma^2+2.903\times10^{-3}\ d^2\sigma^2$
3.0	2.7585×10^{-7}	$3.6958\times10^{-3}\ \sigma^2+1.4706\times10^{-3}\ d^2\sigma^2$
3.5	3.5855×10^{-7}	$1.0173\times10^{-3}\ \sigma^2+8.114\times10^{-4}\ d^2\sigma^2$
4	4.1934×10^{-7}	$7.8309\times10^{-4}\ \sigma^2+4.8077\times10^{-4}\ d^2\sigma^2$
4.5	4.6406×10^{-7}	$6.2053\times10^{-4}\ \sigma^2+3.0189\times10^{-4}\ d^2\sigma^2$
5.0	4.9747×10^{-7}	$5.0347\times10^{-4}\ \sigma^2+1.9873\times10^{-4}\ d^2\sigma^2$

***=** exact mutated (modified) slope rotatability value using a pair of similar incomplete block designs.

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