International Journal of Scientific Engineering and Applied Science (IJSEAS) – Volume-6, Issue-6, June 2020
ISSN: 2395-3470
www.ijseas.com

Cube root of a Number / Polynomial by Vedic Method

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Introduction

Jagadguru Shankaracharya Shri Bharati Krishna Thirtha Swamiji of Puri Mutt, had introduced a novel method of division consisting of a Common Divisor, Dividend, Quotient and Remainder for the root determination of any perfect / imperfect number and also of decimal numbers with the assistance of expansion of the type $(a+b+c+d+e+f\ldots)^n$ "n" being the order of the root and by grouping the digits of the given number, each containing n digits. It is noticed that the combination or merging of groups into one, facilitates the ease and simplicity in working. The procedure is extendable to polynomials of any order as well.

Abstract

The novel method given out by Swamiji for the determination of the cube roots is well exemplified here in several types.

A) 3 digited number 971
B) Larger number 179856214207
C) Combination of two groups for the above number which indicates the ease in working and the results are compared.
D) Number having a decimal
E) Number having a decimal at different position than in D
F) Using combination of groups (merging) into single group to indicate the ease with which the reduction can be avoided / simplified is clearly shown in decimal numbers
G) A Polynomial having perfect root
H) A polynomial having imperfect root
I) Root determination of a polynomial in ascending / descending order.

The method is applicable to determine any root provided, one prepares from the expansion table of the type $(a+b+c+\ldots.)^n$ the deductions as the combinations of expansion terms. The paper contains the results upto a specified decimal along with the corresponding tables for the cube root.

Cube root of a Number

A) Number having only 3 digits

The given number is 971. The digits are to be grouped from right to left in units of 3 each. Thus the number having 3 digits has only "one group". Hence the cube root consists of one integer followed by decimals if the number is imperfect. The maximum cube of the first group on the left isde is to be considered as the value "a" followed by digit by digit method thereon.

The cube root consists of one integer part and the rest are the decimals, or in the form of a.bcdefg…. Let us consider the cubic expansion in two elements maintaining homogeneity and symmetry i.e., $(a+b)^3=a^3+3a^2b+3ab^2+b^3$.

The expansion terms upto 6 decimals, along with the maximum integer "a" are worked out. $(a+b+c+d+e+f+g)^3=a^3+3a^2b+3ab^2+b^3+3a^2d+6abc+3a^2e+3ac^2+3b^2c+6abd+3a^2f+b^2d+3bc^2+6abe+6acd+3a^2g+3ad^2+3b^2e+6abf+6ace+6bcd+c^3$ One has to consider for the cube root, cubic expansion, in general, inclusive of the required decimals, and the positions of each expansion term as well. The required decimals, here are considered as "6".

The maximum number of the cube root that can be accommodated in the first group 971 is 9. If the maximum value of its cube i.e. 729 is deducted from the given number, the result will contain the sum of the remaining expansion terms. Thus, one is left with 242, for working out digit by digit to obtain the decimal parts, in the cube root. The procedure, suggested by Swamiji is (Straight Division method using Dividends, Quotients and Remainders), with the Common Divisor being the first derivative of the a^3 in the general expansion.

Let the cube root be a.bcdefg, where "a" stands for the maximum number (integer) of the cube root contained in the first group of the given number 971, (the only group). This is imperfect cube with b,c,d… being the 1st, 2nd, 3rd …… decimal points upto the required choice, "", i.e. 6th decimal.

Now, if one considers the given number as the expansion of $(a+b+c+d+e+f+g)^3$ keeping the homogeneity and symmetry, the expansion terms contributed upto 6th decimal point, are as given below (Table-1)

Table – 1

The various terms of the expansion belonging to the various decimal positions.

1st decimal	$3a^2b$	b is evaluated	b=9
2nd decimal	$3a^2c, 3ab^2$	c is evaluated	c=0
3rd decimal	$3a^2d, b^3, 6abc$	d is evaluated	d=2
4th decimal	$3a^2e, 3ac^2, 3b^2c, 6abd$	e is evaluated	e=3
5th decimal	$3a^2f, 3b^2d, 3bc^2, 6abe, 6acd$	f is evaluated	f=8
6th decimal	$3a^2g, 3ad^2, 3b^2e, 6abf, 6ace, 6bcd, c^3$	g is evaluated	g=3

Table-2

Decimal Positions	**1**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**4**	**4**
	$3a^2b$	$3ab^2$	$3a^2c$	b^3	6abc	$3a^2d$	$3ac^2$	$3b^2c$	$3a^2e$	6abd

5	**5**	**5**	**5**	**5**	**6**	**6**	**6**	**6**	**6**	**6**
$3a^2f$	$3b^2d$	$3bc^2$	6abe	6acd	$3a^2g$	$3ad^2$	$3b^2e$	6abf	6ace	6bcd

1) The maximum integer value of the cube root contained in the 1st group (971) is 9. Hence, the value of "a" is 9. The cube root of the given number is 9 followed by decimals. (The first derivative of $a^3=3a^2=243$ is the common Divisor (CD). Applying Swamiji's principle of Divison,
2) From the given number, the value of a^3 is to be deducted, which gives the new dividend as 242-0. This is the starting point for obtaining the decimals. The new dividend is to be divided by the Common Divisor 243. The quotient is the value of "b", the first decimal, b=9, and the remainder 233, is to be carried into the next dividend and thus it is 233-0

3) As the answer is aimed upto 6th decimal, we have to consider the deductions of the terms of the expansion successively and in the order as shown in the tables – 1 & 2 to obtain the corresponding (next) dividends along with remainders, and also quotients as decimals by using the Common Divisor. Similar procedure is to be continued until the final decimal value is obtained. The dividend is now 2330. The sum of the terms, in the expansion, contributing to the positions of different decimals as given in the tables 1,2 & 3 are to be deducted from the respective dividends and then the result is divided by the Common divisor for obtaining the corresponding quotient (decimal).

IJSEAS

International Journal of Scientific Engineering and Applied Science (IJSEAS) – Volume-6, Issue-6, June 2020
ISSN: 2395-3470
www.ijseas.com

Table-3

Maximum Value of a=9	971	0	0	0	0	0	0	0	0
CD= $3a^2=243$		242	233	143	215	206	116	80	206
Common Divisor (CD) is the first derivative of $a^3=729$						449	359	323	
							602	566	
			$3ab^2$	b^3+6abc	$3ac^2+3b^2c+6abd$	$3b^2d+3bc^2+6abe+6acd$	$3ad^2+3b^2e+6abf+6ace+6bcd+c^3$	$3b^2f+3bd^2+3c^2d+6abg+6acf+6bce+6ade$	
			2187	729	972	1944	4725	3510	
	a	b	c	d	e	f	g	h	
	9	9	0	2	4 3	10 9 8	5 4 3	8	

4) From this Dividend, in order to get the value of “c”, the 2nd decimal, one has to deduct the value of $3ab^2$ and obtain the new dividend ($3ab^2=2187$ to be deducted from 2330) and by dividing the result by CD, one gets the remainder as 143, the quotient “c” as 0. Hence c=0 and the new dividend is 1430.

5) From this, one has to deduct (b^3+6abc) to get the value of “d” (1430-729=701). When this result is divided by the CD (243), the value of “d”, the third decimal is found as 2 with the remainder 215, the new dividend being 2150.

6) From this dividend, one has to get the value of “e”, the fourth decimal. Hence one has to deduct from 2150, the value of $3ac^2+3b^2c+6abd=972$, and by dividing the result by the CD, the value of “e” is 4 and the new dividend is 2060. One has to proceed to get the value “f”, the fifth decimal.

7) Now, the deduction expression $3bc^2+6acd+3b^2d+6abe=2430$ itself is greater than the dividend 2060 and hence one has to reduce the previously evaluated decimal value “e” by “1” , so that the new value of “e” is now 3 (Table – 3) .

8) The procedure after the reduction of the immediately previous decimal value “e” (4) by 1 is as follows : One has to add the common divisor 243 to the previous remainder 206, so that, the new value, 449 is now to be considered as the dividend for the evaluation of “f”. Thus 4490 is the new dividend. One has to deduct $3bc^2+6acd+3b^2d+6abe+6acs=1944$ after incorporation of the reduced value of “e” (3) from the dividend 4490. The result is divided by the CD to get the value for “f”as 9. Proceeding further in the similar way, it is noticed that the value of f is to be still reduced to 8 and the new dividend is 1160. In order to find out the value of “g”, $3ad^2+3b^2e+6abf+6ace+6bcd$ has to be deducted from its dividend to get the correct value of “g”. Thus, until one gets the deduction value, less than the dividend, wherever necessary, one has to apply a series of reductions in the previously obtained decimal value by “1” in succession, and only after that, one gets the exact value for “g”. This is shown in the Table-3, when the reduction of ”e” value is from 4 to 3 and of “f” value is from 10 to 8 by steps of one and the corrected value is to be finally as f=8. Similarly, it is noticed that the value for “g” also is to be reduced from 5 upto 3, by steps of one each time.
Now, the cube root of 971 is

a . b c d e f g
9 . 9 0 2 3 8 3 …… (6 decimals)

When one has to consider 6 decimals, one has to expand the general expansion considering all the required decimals together with the integer part of the 1st group, "a". as $(a+b+c+d+e+f+g)^3$ Inorder to get the complete combination of terms in the expansion.

The author has prepared an algorithm for the general expansion of $(a+b+c+d+e+f+g+\ldots)^n$ "n" being positive integer. In that table while the homogeneity is maintained and symmetry being followed, combination, of the sum of the terms of the expansion, which needs to be deducted at each stage successively and finally to arrive at the value of the decimals and the next dividends as well is also clearly indicated. (Tables – 1 - 3)

Tables were prepared by the author for the 11 decimal places, the combinations of deduction terms of the expansion to be used in the determination of roots upto n=7 and the decimals upto 11 positions). The general expansion is given in the Author's Lecture Notes with the deduction factors, (combinations of the expansion terms to be considered for obtaining different dividends and consequent decimal values). The results of the expansion of $(a+b+c+\ldots.)^4$ is published (Reference–3)

B) Cube root of the number having more than 3 digits (more than one group) 179856214027, with the 1st group as (179)

The given number is (179)(856)(214)(027) for which, the cube root is worked out using Swamiji's method.

1) The number could be grouped into 4 groups, starting from the right end, with each group of 3 digits. Hence the final cube root consists of 4 digited integer value. The working is carried out from the 1st group 179 and thereafter digit by digit method is applied. The maximum cube value of the 1st group (179) is designated as "a".
2) The same concept of a.bcdef… is adopted while only at the end, the first four values of the result will be the integer part of the cube root of the number having 4 groups and after which starts the decimals of one's choice. This concept is adopted and carried out for the purpose of calculations. Such an adoption has enabled one to carry out determination of integer root of any number, with ease, which will be followed by the values of the decimals.
3) The method that has been applied for the number 971 is similarly carried out. At the end, the first 4 digits of the result , are to be treated as the integer part "a" of the cube root as it is derived from 4 groups into which the given number can be grouped and the remaining are the successive decimals b,c,d, etc. The results are shown in the table – 4 taking care to see that the deductions for obtaining the dividend values for the division by the CD, be smaller than the dividend, and until then the reduction of the decimal of the previously determined value is being reduced step wise by "1" each time, in the same manner as in the case of the working of the cube root for the number 971.

   Table-3 gives the terms that should be deducted as the combinations of the expansion terms from dividends in order to get the values. (Refer also Tables – 1 & 2)

Table-4

From 179	8	5	6	2	1	4	0	2	7
$3a^2$ =CD=75	54	23	70	70	60	16	58	64	36
a^3=125									
		98	145	145	135	91	133	139	
				220	210	166	208	214	
					285	241	283	289	
						316	358	364	
								439	

a	b	c	d	e	f	g	h	i
5	7	5	6	10	5	6	7	11
The max. cube root of the 1st group (179)	6	4	5	9	4	5	6	
			4	8	3	4	5	
				7	2	3	4	
					1	2	3	

a
Maximum Integer value of the Cube Root of the given number=5644 and the decimal part is 0.7123.
At the end, one has to consider the integer part of the cube root as the number is divided into 4 groups as the first 4 digits,
5644 is followed by decimals. In order to evaluate b to h, the deductions are given in the Table – 4.

* Dividends are larger than the contributing deductions, as otherwise the process of reduction of previously determined decimal has to be reduced successively in steps of 1 until the correct figure is reached.

The values in circles are the successive reductions of the previously determined decimals. The cube root of the given number (179)(856)(214)(027) is thus
5 6 4 4 . 7 1 2 3

The first digit in the value "a": The maximum value of the cube root of the 1st group of the number, i.e. 179 is equal to 5.

As the maximum value of the 1st group is the 1st digit of the integer part, all the remaining working is carried out digit by digit

Common Divisor (CD) = $3a^2 = 75$

The following three values out of four in the integer part of the cube root is derived from three digits of the following group (856)

The second digit of the value : 6 after reduction from 7

The third digit of the cube root is : 4 after reduction from 5

The fourth digit of the cube root is : 4 after two reductions from 6

The decimals :

7 after three reductions from 10

1 after four reductions from 5

2 after four reductions from 6

3 after four reductions from 8

the cube root of 179 856 214 027 is 5644.7123 (upto 4 decimals)

C) **Combination of two groups, [(**179)(856)] **for the above number which indicates the ease of working with two groups merged into one [(179)(856)] and the results, given in Tables – 4 & 5 are compared with**

Swamiji has suggested that one can reduce the labour of reductions, in the quotients, as some times in larger numbers and decimal numbers, one may have to have many successive reductions before a correct one is achieved by considering merging of groups into one and the integer value of the root

from such merging as the maximum cube root of the number, put together. This is shown for the number 179 856 214 027
The maximum root value of the combined group [(179)(856)] is 56

The method with the 1st group as (179) leads to laborious in the sense that at every step it appears to have reduction process (Table – 4) hence considering merging of the first two groups (179) & (856) as one unit, the results are shown in Table – 5. Digit by digit work is carried out to find the quotients, from "b" to "g".

Table - 5

$CD=3a^2=9408$	[(179)(856)]	2	1	4	0	2	7
a=56		4240	4770	7381	2518	3484	4746
a3=175616							
			37632	65856	12288	11280	13048
	56	4	4	7	1	2	3
	a.	b	c	d	e	f	g

The maximum value of the 1st group is 5 and the maximum value of the cube root of merged part is obtained by trial and error as lying between 50 and 59. The maximum integer 56 is the value for "a". As the total number of groups of the number is 4, one has to consider the maximum integer value of the cube root as 5644, after that, 7,1,2 and 3 are the four decimals..
Upto 4 decimals the Cube root of given number is 5644.7123

D) To find the cube root of the number 17985621.4027 (having a decimal in it) before the digit "4"

Consider the integer part of the number to the left of the decimal 17985621 and group it so that it is equal to (17)(985)(621) – 3 groups
The number can be grouped as (17)(985)(621).4027 with merging of the 1st two groups.
The maximum value of the cube root of the merged group [(17)(985)] is now 26. For example, as it is evaluated out of two groups 17 and 985, it is clearly seen that 2 is the value of maximum cube root that can be accommodated in 17 and the maximum root value of the combined group [17985] is by trial and error method, one has to find out that the second digit to be lying between 20 and 29. It is seen that 26 is the maximum value with its cube as 17576. Hence one has to start with 26 as the value of "a" and proceed to digit by digit evaluation. It is clearly seen from the table -6 that the value of the roots have absolutely no reductions.

Table -6

[(17)(985)]	6	2	1	4	0	2	7
a=26	409	40	90	893	822	888	1814
$CD=3a^2=2028$							
$a^3=17576$							
26	2	0	0	4	3	3	8
a.	b	C	D	E	F	g	

In order to determine the integer part of the cube root of the given number in the result, one has to consider the total number of groups that can be formed finally in the given number i.e. to the left of the decimal in the given number, i.e. 3 groups and the integer part of the cube root of the number is 262 and the rest of the quotients are as decimals.

Therefore the cube root of the given number is 262.004338

E) To find the cube root of 1798562140.27 The number with the decimal positioned at a different digit, after the digit "0"

The number can be grouped as (1)(798)(562)(140).27 into 4 groups to the left of the decimal. Merging of the first two groups (1) & (798) is considered.

Similar procedure is adopted as above.

We can combine the groups (1) and (798), As originally, there are 4 groups, the resulting integer part of the cube root of the given number, will have 4 digits to be determined at the end.

When the first two groups are merged into one,[(1)(798)] the first having 1 as the maximum cube value, while the second has to be worked out by trial and error as lying between 1 and 9 i.e 11 and 19. It is seen that 12 represents the maximum value of the integer part of the cube root of the given number. Hence a=12, a^3=1728 and Common Divisor (CD) = $3a^2$=432
With this initiation the digit by digit method is applied and the results are given in Table – 7.

Table-7

[(1)(798)]	5	6	2	1	4	0	2	7
a=12	70	273	108	217	353	327	387	323
CD=$3a^2$=432								
a^3=1728								
12	1	6	1	1	6	4	1	1
a	b	c	d	e	f	g	h	i

In order to determine the integer part of the cube root of the given number, one has to consider the total number of groups that can be formed on the left side of the decimal in the given number, i.e. as (1)(798)(562)(140)– 4 groups. Therefore the cube root of the given number is 1216.116411, by grouping the first two i.e. (1)(798).

Hence, the cube root value of the given number is 1216.116411

In case of numbers having decimals also, it is noticed that the merging of the first two groups facilitated a direct determination of the result, without any reduction, as in the case of number without decimal. (Tables – 5, 6 & 7)

Cube Root of Polynomials

In this section, the working of cube roots of perfect / imperfect polynomials and in descending / ascending order as well are exemplified.

For a polynomial of any order (of the perfect nature), the cube root is same either in the descending or ascending order but, the cube root is different when it is considered in descending or ascending order in case of imperfect polynomials. This is applicable for any order of polynomial.

Using Swamiji's method of finding out the roots of any numbers perfect / imperfect one can determine for polynomials as well of any order the roots, provided the general expansion of $(a+b+c+d+\dots)^n$, n being the order of the root is considered.

By Swamiji's method, the cube root of the perfect polynomial. Swamiji's digit by digit method can be applied for the cube root determination of a polynomial as given below :

This is exemplified in the determination of cube root of the given polynomial

$\mathbf{8x^6 + 12x^5 + 42x^4 + 37x^3 + 63x^2 + 27x + 27}$

Descending Powers of x

a=$2x^2$, CD=$3a^2$=$12x^4$

$12x^4$	$8x^6 + 12x^5 + 42x^4 + 37x^3 + 63x^2 + 27x + 27$
	$2x^2 + x + 3 + \frac{0}{x} + \frac{0}{x^2} + \frac{0}{x^3} + \frac{0}{x^4}$
	a b c

$\therefore$ $2x^2 + x + 3$ is one cube root and the other two roots are obtained by multiplying with cube roots of unity, $\frac{-1 \pm \sqrt{3}i}{2}$

$27 + 27x + 63x^2 + 37x^3 + 42x^4 + 12x^5 + 8x^6$

Ascending Powers of x

a=3, CD=$3a^2$=27

27	$27 + 27x + 63x^2 + 37x^3 + 42x^4 + 12x^5 + 8x^6$
	$3 + x + 2x^2 + 0x^3 + 0x^4 + 0x^5 + 0x^6$
	a b c

This method is applicable to any order, for example :

$$\sqrt[3]{x^9 + 6x^8 + 3x^7 - 25x^6 + 3x^5 + 50x^4 - 66x^3 + 33x^2 - 9x + 1}$$

CD=$3a^2$ =$3x^6$	x^9	+	$6x^8$	+	$3x^7$	–	$25x^6$	+	$3x^5$	+	$48x^4$	–	$60x^3$	+	$33x^2$	–	$9x$	+	1
	x^3	+	$2x^2$	–	$3x$	+	1	+	$\frac{0}{x}$	+	$\frac{0}{x^2}$		$\frac{0}{x^3}$		$\frac{0}{x^4}$		$\frac{0}{x^5}$		$\frac{0}{x^6}$
	a		b		c		d		e		F		g		h		i		j

$\therefore$ $x^3 + 2x^2 - 3x + 1$ is one cube root and the other two roots are obtained by multiplying with cube roots of unity, $\frac{-1 \pm \sqrt{3}i}{2}$

The cube root is same either in descending or ascending order when the polynomial has perfect roots.

In Ascending order and Descending order, the cube root is different in the case of imperfect Polynomial as shown below :

Cube root of $x^3 + 6x^2 + 15x + 27$

Descending Powers of x

$3x^2$	$x^3 + 6x^2 + 15x + 27$
	$x + 2 + \frac{1}{x} + \frac{7}{3x^2} + \ldots.$

Ascending Powers of x

27	$27 + 15x + 6x^2 + x^3$

$$3 + \frac{5x}{9} + \frac{29x^2}{243} - \frac{752x^3}{6561} + \ldots$$

Acknowledgements

The author is deeply indebted to Prof. GSN Raju, the Vice Chancellor of Centurion University of Technology and Management AP, Visakhapatnam and also equally acknowledges Prof. D.N.Rao, the Vice President of the Institution for his interest in this line of Investigation. The author extends her special thanks to Prof. Mukti Kanta Mishra, the President of the Institution for giving the position of Emeritus Professor in the Institution.

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