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Effect of Dust Particles on a Rotating Couple-stress Ferromagnetic Fluid Heated From Below

Sudhir Kumar, Rimple Pundir, Pulkit Kumar Nadian

Department of Mathematics, S.D. (P.G.) College, Muzaffarnagar,

Uttar Pradesh- 251001, India

Abstract

The purpose of this paper is to study the effects of dust particles, couple-stress, rotation and magnetization on thermal stability of a layer of couple-stress ferromagnetic fluid. Using linearized theory and normal mode analysis, dispersion relation has been obtained. In case of stationary convection, it is found that dust particles always have destabilizing effect on the system. Couple-stress has both stabilizing and destabilizing effect under certain conditions while in the absence of rotation, it has stabilizing effect on the system. Both rotation and magnetization have stabilizing effect on the system. The principle of exchange of stabilities is found to hold true in the absence of rotation under certain conditions.

Keywords: *Thermal instability, couple-stress fluid, rotation, dust particles, ferromagnetic fluid.*

1. Introduction

The theory of Bénard convection in viscous Newtonian fluid layer heated from below has been given by Chandrasekhar [1]. Chandra [2] ascertained that in an air layer, convection take place at much lower gradients than anticipated if the layer depth was less than 7mm, and called this motion 'Columnar instability'. However, a Bénard type cellular convection was discovered for layers deeper than 10mm. Thus, there is a contradiction between the theory and the experiment. The effect of dust particles on the onset of Bénard convection has been considered by Scanlon and Segel [3] and found that the critical Rayleigh number was reduced solely because of the heat capacity of the pure fluid. The investigations on non-Newtonian fluids are desirable with the raising importance in modern technology and industries. Stokes' [4] proposed the theory of couple-stress fluid. Applications of couple-stress fluid occur in aid of the study of the mechanism of lubrication of synovial joints, that has become the object of scientific research. Human joint is a dynamically loading bearing with articular cartilage as the bearing and synovial fluid as the lubricant. The joints of shoulder, hip, knee and ankle are the loaded-bearing synovial joints of the human body and have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid. The synovial fluid has been modeled as a couple-stress fluid in human joints by Walicki and Walicka [5]. The effect of suspended particles on couple-stress fluid heated and soluted from below in a porous medium has been investigated by Sunil et al. [6] and found that suspended particles have destabilizing effect on the system. Sharma and Sharma [7] have discussed the effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field. Sharma and Aggarwal [8] have studied the effect of compressibility and suspended particles on thermal convection in Walters' B' elastico-viscous fluid in hydromagnetics. The combined effect of magnetic field and rotation on couple-stress fluid heated from below in the presence of suspended particles has been discussed by Aggarwal and Makhija [9]. Sharma and Rana [10] have studied the thermosolutal instability of Walters (model B') visco-elastic rotating fluid permeated with suspended particles and and variable gravity field in porous medium. The effect of compressibility, rotation and magnetic field on thermal instability of Walters' fluid permeated with suspended particles in porous medium has been investigated by Aggarwal nad Verma [11]. Kumar [12] have discussed the Rayleigh-Taylor instability of Rivlin-Ericksen visco-elastic plasma in presence of a variable gravity field and suspended particles in porous medium.

An authoritative introduction to the research on magnetic liquids has been given by Rosenweig [13] in his monograph. This monograph follow-up several applications of heat transfer through ferromagnetic fluids. Finlayson [14] have considered convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform magnetic field. Sunil et al. [15] have studied the effect of dust particles on thermal convection in ferromagnetic fluid saturating a porous medium. Also Sunil et al. [16] have discussed the effect of dust particles on a rotating ferromagnetic fluid heated from below saturating a porous medium. Effect of dust particles on ferrofluid heated and soluted from below has been investigated by Sunil et al. [17]. Also, effect of rotation on a ferromagnetic fluid heated and soluted from below in the presence of dust particles has been studied by Sunil et al. [18].

In the present paper, we have studied the effect of dust particles on thermal instability of rotating couple-stress ferromagnetic fluid.

2. Formulation of the problem

Here, we consider a static state in which an incompressible couple-stress ferromagnetic fluid layer of thickness d, confined between two infinite horizontal planes situated at $z=0$ and $z=d$ is arranged, which is acted upon by a uniform rotation $\boldsymbol{\Omega}(0,0,\Omega)$, vertical magnetic field $(0,0,H)$ and gravity $\boldsymbol{g}(0,0,-g)$. The particles are assumed to be non-conducting. The couple-stress ferromagnetic fluid layer is heated from below leading to an adverse temperature gradient $\beta=\frac{T_0-T_1}{d}$, where T_0 and T_1 are constant temperatures of the lower and upper boundaries with $T_0>T_1$.

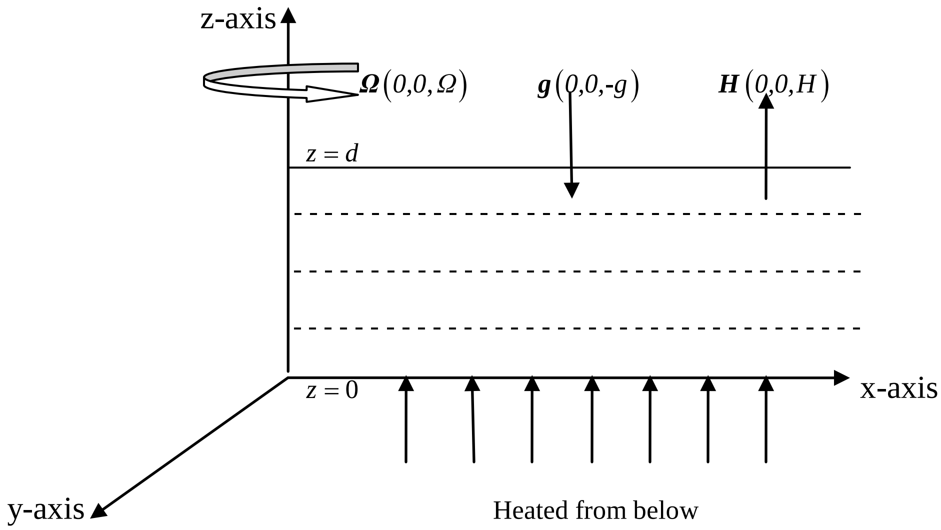

Fig.1. Geometrical configuration

Since, ferromagnetic fluids react speedily to a magnetic torque, so we assume the following condition to be hold,

$$\boldsymbol{M} \times \boldsymbol{H} = 0. \tag{1}$$

Now, taking the fluid electrically non-conducting and that displacement current is negligible, Maxwell's equations become,

$$\nabla.\boldsymbol{B} = 0, \nabla \times \boldsymbol{H} = 0. \tag{2}$$

Also, in Chu formulation of electrodynamics, magnetic field intensity, magnetization and magnetic induction relates as under,

$$\boldsymbol{B} = \mu_0 (\boldsymbol{H} + \boldsymbol{M}). \tag{3}$$

Here, $\boldsymbol{M}$ denotes magnetization, $\boldsymbol{H}$ denotes magnetic field intensity and $\boldsymbol{B}$ denotes magnetic induction.

We assume that the magnetization is aligned with the magnetic field but depends on the magnitude of the magnetic field and temperature so that,

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M(\boldsymbol{H}, \boldsymbol{T}). \tag{4}$$

Let $p, \rho, T, \alpha, \mu', \nu, \kappa_T$ and $\boldsymbol{q}(u, v, w)$ denote respectively pressure, density, temperature, thermal coefficient of expansion, couple-stress viscosity, kinematic viscosity, thermal diffusivity and velocity of fluid. Also let $\boldsymbol{q_d}(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of particles respectively. $K = 6\pi\mu\eta$ (where η is the particle radius) is a constant and $\bar{x} = (x, y, z)$.Then the equations of motion and continuity of couple-stress ferromagnetic rotating dusty fluid in hydromagnetics are,

$$\frac{\partial \boldsymbol{q}}{\partial t} + (\boldsymbol{q}.\nabla)\boldsymbol{q} = -\frac{1}{\rho_0}\nabla p + \boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right) + \frac{1}{\rho_0}\boldsymbol{M}.\nabla\boldsymbol{H} + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\boldsymbol{q} + \frac{KN}{\rho_0}(\boldsymbol{q}_d - \boldsymbol{q}) + 2(\boldsymbol{q}\times\boldsymbol{\Omega}), \tag{5}$$

$$\nabla.\boldsymbol{q} = 0, \tag{6}$$

The density equation of state is,

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right] , \tag{7}$$

Where, the suffix zero refers to value at reference level $z = 0$. α be the coefficient of thermal expansion and $\nabla\boldsymbol{H}$ is the magnetic field gradient. Also,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, H = |\boldsymbol{H}|, B = |\boldsymbol{B}|, M = |\boldsymbol{M}|.$$

The presence of particles add an extra term of force which is proportional to the velocity difference between particles and fluid and appears in eq. of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to the force exerted by the particles on the fluid, so there must be an extra force term (which is equal in magnitude but opposite in sign) in equation of motion for the particles. The buoyancy force on the particles is neglected. Interparticle reactions are not considered for we assume that the distance between particles is quite large as compared with their diameter. The equations of motion and continuity for the particles, under the above approximation are,

$$mN\left[\frac{\partial \boldsymbol{q_d}}{\partial t} + (\boldsymbol{q_d}.\nabla)\boldsymbol{q_d}\right] = KN(\boldsymbol{q} - \boldsymbol{q_d}) \tag{8}$$

and

$$\frac{\partial N}{\partial t} + \nabla(N.\boldsymbol{q_d}) = 0 \tag{9}$$

where mN is the mass of particles per unit volume.

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Let c_v, c_{pt} be the heat capacity of the fluid at constant volume and heat capacity of the particles respectively. Let us assume that the particles and fluids are in thermal equilibrium, so the equation of heat conduction gives,

$$\rho_0 c_v \left(\frac{\partial}{\partial t} + \boldsymbol{q}.\boldsymbol{\nabla} \right) T + mNc_{pt} \left(\frac{\partial}{\partial t} + \boldsymbol{q_d}.\boldsymbol{\nabla} \right) T = q \nabla^2 T$$

Or

$$\frac{\partial T}{\partial t} + (\boldsymbol{q}.\boldsymbol{\nabla}) T + \frac{mNc_{pt}}{\rho_0 c_v} \left(\frac{\partial}{\partial t} + \boldsymbol{q_d}.\boldsymbol{\nabla} \right) T = \kappa_T \nabla^2 T \tag{10}$$

where, kinematic viscosity ν, couple-stress viscosity μ', thermal diffusivity κ_T and coefficient of thermal expansion α are assumed to be constant.

Generally, the equation of state will specify $\boldsymbol{M}$ in two thermodynamic variables only ($\boldsymbol{H}$ and $\boldsymbol{T}$) is necessary to complete the system. Here, we consider magnetization to be independent of magnetic field intensity so that $M = M(t)$ only. As a first approximation, we consider that,

$$M = M_0 \left[1 - \gamma (T - T_0) \right] , \tag{11}$$

where M_0 is the magnetization at $T = T_0$ with T_0 being the reference temperature and

$$\gamma = \frac{1}{M_0} \left(\frac{\partial M}{\partial T} \right)_H .$$

3. Basic state and perturbation equations

In the undisturbed state, the fluid is at rest. The basic state of which we wish to examine the stability is characterized by,

$$\boldsymbol{q} = (0,0,0), \boldsymbol{q_d} = (0,0,0), p = p(z), \rho = \rho(z) = \rho_0 [1 + \alpha \beta z], T = T(z) = T_0 - \beta z, \boldsymbol{\Omega} = (0,0,\Omega), M = M_0 [1 + \gamma \beta z],$$
$$\boldsymbol{M} = \boldsymbol{M}(z) \text{ and } N = N_0 . \tag{12}$$

Here, β may be either positive or negative.

The character of equilibrium is examined by assuming that the system is slightly perturbed so that every physical quantity is supposed to be the sum of a mean and fluctuating component, later designated as prime quantities and suppose to be very small in comparison to their equilibrium state values. Here, we assume that the small disturbances are the functions of space and time variables. Hence, the perturbed flow may be represented as,

$$\boldsymbol{q} = (0,0,0) + (u,v,w), \boldsymbol{q_d} = (0,0,0) + (l,r,s), T = T(z) + \theta, \rho = \rho(z) + \delta\rho, \; p = p(z) + \delta p, \; M = M(z) + \delta M, \quad (13)$$

where $\boldsymbol{q}(u,v,w), \boldsymbol{q_d}(l,r,s), \theta, \delta\rho, \delta p, \delta M$ are respectively the perturbations in fluid velocity $\boldsymbol{q} = (0,0,0)$,dust particles velocity $\boldsymbol{q_d}(0,0,0)$, temperature $\boldsymbol{T}$, density ρ , pressure p and magnetization $\boldsymbol{M}$. Now using Eq. (13) in governing Eqs. (5) to (11) and linearizing them, we get,

$$\frac{\partial \boldsymbol{q}}{\partial t} = -\frac{\nabla \delta p}{\rho_0} + \frac{\delta\rho}{\rho_0}\boldsymbol{g} + \frac{\delta \boldsymbol{M}}{\rho_0}.\nabla \boldsymbol{H} + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 \boldsymbol{q} + \frac{KN_0}{\rho_0}(\boldsymbol{q_d} - \boldsymbol{q}) + 2(\boldsymbol{q} \times \boldsymbol{\Omega}), \quad (14)$$

$$\nabla.\boldsymbol{q} = 0, \quad (15)$$

$$\frac{m}{K}\frac{\partial \boldsymbol{q_d}}{\partial t} + \boldsymbol{q_d} = \boldsymbol{q}, \quad (16)$$

$$(1+b)\frac{\partial \theta}{\partial t} = \beta(w+bs) + \kappa_T \nabla^2 \theta, \quad (17)$$

$$\delta\rho = -\rho_0 \alpha \theta, \quad (18)$$

$$\delta M = -\gamma M_0 \theta. \quad (19)$$

where, $\kappa_T = \dfrac{q}{\rho_0 c_v}$ and $b = \dfrac{mN_0 c_{pt}}{\rho_0 c_v}$.

In Cartesian form, Eqs. (14)-(16) with the help of Eqs. (18) and (19) can be written as,

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial x}\delta p + \left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 u - \frac{mN_0}{\rho_0}\frac{\partial u}{\partial t} + 2\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\Omega v, \quad (20)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial y}\delta p + \left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 v - \frac{mN_0}{\rho_0}\frac{\partial v}{\partial t} - 2\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\Omega u, \quad (21)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial z}\delta p - \frac{\delta\rho}{\rho_0}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)g + \frac{\delta M}{\rho_0}\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\nabla H + \left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 w - \frac{mN_0}{\rho_0}\frac{\partial w}{\partial t}, \quad (22)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (23)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)l=u, \tag{24}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)r=v, \tag{25}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)s=w, \tag{26}$$

Also, by Eq. (17), z-component is given by,

$$(1+b)\frac{\partial\theta}{\partial t}=\beta(w+bs)+\kappa_T\nabla^2\theta, \tag{27}$$

Now, operate $(\partial/\partial x)$ on Eq. (20) and $(\partial/\partial y)$ on Eq. (21) and adding them, we get,

$$\left\{\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}-\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\right]+\left(\frac{mN_0}{\rho_0}\frac{\partial}{\partial t}\right)\right\}\frac{\partial w}{\partial z}=\frac{1}{\rho_0}\delta p\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-2\Omega\zeta\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right), \tag{28}$$

Also operate $\left[\nabla^2-\left(\partial^2/\partial z^2\right)\right]$ on Eq. (22) and $(\partial/\partial z)$ on Eq. (28) and adding them, we get,

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1+\frac{mN_0}{\rho_0}\right)\frac{\partial}{\partial t}\nabla^2 w=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\alpha g-\frac{\gamma M_0\nabla H}{\rho_0}\right)\theta+\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^4 w-2\Omega\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial z}, \tag{29}$$

Again operate $-(\partial/\partial y)$ on Eq. (20) and $(\partial/\partial x)$ on Eq. (21) and adding them, we get,

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\zeta-\frac{mN_0}{\rho_0}\frac{\partial\zeta}{\partial t}+2\Omega\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial z}. \tag{30}$$

4. The dispersion relation

Analyzing the perturbations into normal modes, we suppose that the perturbation quantities are of the form,

$$(w,\theta,\zeta)=\left[W(z),\Theta(z),Z(z)\right].\exp\left(ik_x x+ik_y y+nt\right) \tag{31}$$

where, k_x and k_y are wave numbers in x and y directions respectively and $k=\left(k_x{}^2+k_y{}^2\right)^{1/2}$ is the resultant wave number of the disturbance and n is the frequency of any arbitrary disturbance (which is a complex constant in general).

Now on using relation (31), Eqs. (29),(30) and (27) in non-dimensional form become,

$$\left(D^2-a^2\right)\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W+\frac{\alpha a^2 d^2}{\nu}\left(g-\frac{\gamma M_0 \nabla H}{\rho_0\alpha}\right)\Theta+\frac{2\Omega d^3}{\nu}DZ=0, \tag{32}$$

$$\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]Z=\frac{2\Omega d}{\nu}DW, \tag{33}$$

$$\left(D^2-a^2-\sigma Bp_1\right)\Theta=-\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{\beta d^2}{\kappa_T}W. \tag{34}$$

where, we eliminate the physical quantities using the non-dimensional parameters $a=kd, \sigma=\left(nd^2/\nu\right), \tau=\left(m/K\right), p_1=\left(\nu/\kappa_T\right), F=\left(\mu'/\rho_0 d^2\nu\right), \tau_1=\left(\tau\nu/d^2\right), M=\left(mN_0/\rho_0\right), B=\left(1+b\right)$ and $D^*=dD$ and dropping (*) for convenience.

Now, eliminating Θ and Z among Eqs. (32) - (34), we obtain the stability governing equation,

$$\left(D^2-a^2\right)\left(D^2-a^2-\sigma Bp_1\right)\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]^2 W$$

$$-R_f a^2\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W+T_A\left(D^2-a^2-\sigma Bp_1\right)D^2W=0 \tag{35}$$

where $R_f=\left(\alpha\beta d^4/\nu\kappa_T\right)\left[g-\left(\gamma M_0\nabla H/\rho_0\alpha\right)\right]$ is the Rayleigh number for ferromagnetic fluids and $T_A=\left[\left(2\Omega d^2/\nu\right)\right]^2$ is the modified Taylor number.

The perturbation in the temperature is zero at the boundaries because the boundaries are maintained at constant temperature. The suitable boundary conditions are,

$$W=0, Z=0, \Theta=0 \text{ at } z=0 \text{ and } z=1 \text{ also } DZ=D^2W=D^4W=0 \text{ at } z=0 \text{ and } z=1. \tag{36}$$

From Eq. (36), it is obvious that all the even order derivatives of W vanish at the boundaries. Therefore, the proper solution of Eq. (35) characterizing the lowest mode is,

$W = W_0 sin\pi z$, where W_0 is a constant. (37)

Now using Eq. (37) in Eq. (35), we get,

$$R_1 = \frac{(1+x)}{x}\left(1+x+iBp_1\sigma_1\right)\left[i\sigma_1\left(1+\frac{M}{1+i\sigma_1\tau_1\pi^2}\right)+F_1(1+x)^2+(1+x)\right]\left(\frac{1+i\sigma_1\tau_1\pi^2}{B+i\sigma_1\tau_1\pi^2}\right)$$
$$+\frac{T_{A_1}\left(1+x+iBp_1\sigma_1\right)}{x\left[i\sigma_1\left(1+\frac{M}{1+i\sigma_1\tau_1\pi^2}\right)+F_1(1+x)^2+(1+x)\right]}\left(\frac{1+i\sigma_1\tau_1\pi^2}{B+i\sigma_1\tau_1\pi^2}\right) \tag{38}$$

where, $x=\left(a^2/\pi^2\right), R_1=\left(R_f/\pi^4\right), T_{A_1}=\left(T_A/\pi^4\right), i\sigma_1=\left(\sigma/\pi^2\right), F_1=\pi^2F$.

5. Analytical discussion

5.1 Stationary Convection

At stationary convection, when stability sets, the marginal state will be characterized by $\sigma_1 = 0$. So, put $\sigma_1 = 0$ in Eq. (38), we get,

$$R_1 = \frac{(1+x)^3}{Bx}\left[F_1(1+x)+1\right]+\frac{T_{A_1}}{Bx\left[F_1(1+x)+1\right]}. \tag{39}$$

This relation is the modified Rayleigh number R_1 as a function of the parameters F_1 and T_{A_1} and dimensionless wave number x. In order to study the effect of couple-stress and rotation, we examine the behavior of (dR_1/dB), (dR_1/dF_1) and $\left(dR_1/dT_{A_1}\right)$ analytically.

So, by eq. (39), we have,

$$\frac{dR_1}{dB} = -\frac{1}{xB^2}\left\{(1+x)^3\left[F_1(1+x)+1\right]+\frac{T_{A_1}}{\left[F_1(1+x)+1\right]}\right\} \tag{40}$$

which clearly shows that dust particles have destabilizing effect on thermal instability of rotating dusty couple-stress ferromagnetic fluid.

Also, by Eq. (39), we have

$$\frac{dR_1}{dF_1} = \frac{(1+x)}{Bx}\left\{(1+x)^3-\frac{T_{A_1}}{\left[F_1(1+x)+1\right]^2}\right\} \tag{41}$$

which shows that couple-stress has a stabilizing effect on the thermal instability of rotating dusty couple-stress ferromagnetic fluid under the condition,

$$T_{A_1} < \left[F_1\left(1+x\right)+1\right]^2 \left(1+x\right)^3$$

Also, couple-stress has a destabilizing effect on the thermal instability of rotating dusty couple-stress ferromagnetic fluid under the condition,

$$T_{A_1} > \left[F_1\left(1+x\right)+1\right]^2 \left(1+x\right)^3$$

In the absence of rotation, Eq. (41) becomes,

$$\frac{dR_1}{dF_1} = \frac{\left(1+x\right)^4}{Bx}.$$

which shows that couple-stress has a stabilizing effect on the system.

Also by Eq. (39), we have

$$\frac{dR_1}{dT_{A_1}} = \frac{1}{Bx\left[F_1\left(1+x\right)+1\right]}. \tag{42}$$

which shows that rotation has a stabilizing effect on the system.

Now, replace R_1 by $\left(R_f / \pi^4\right)$ in Eq. (39), we get,

$$R = \frac{\pi^4}{Bx}\left\{\left(1+x\right)^3\left[F_1\left(1+x\right)+1\right]+\frac{T_{A_1}}{\left[F_1\left(1+x\right)+1\right]}\right\}\left(1-\frac{\gamma M_0 \nabla H}{\rho_0 \alpha g}\right)^{-1}. \tag{43}$$

Now, to see the effect of magnetization, we examine the behavior of $\left(dR / dM_0\right)$ analytically.

From Eq. (37), we have,

$$\frac{dR}{dM_0} = \frac{\pi^4}{Bx}\left\{\left(1+x\right)^3\left[F_1\left(1+x\right)+1\right]+\frac{T_{A_1}}{\left[F_1\left(1+x\right)+1\right]}\right\}\left(1-\frac{\gamma M_0 \nabla H}{\rho_0 \alpha g}\right)^{-2}\left(\frac{\gamma \nabla H}{\rho_0 \alpha g}\right), \tag{44}$$

which shows that magnetization has a stabilizing effect on the system.

5.2 Stability of the system and Oscillatory modes

Multiplying Eq. (32) by conjugate of W i.e. W^* and integrate over the range of z and making use of Eqs. (33) and (34) together with the boundary conditions (36), we get,

$$\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)I_1+I_2+FI_3+d^2\left[\sigma^*\left(1+\frac{M}{1+\tau_1\sigma^*}\right)I_4+I_5+FI_6\right]-\frac{\alpha a^2\kappa_T}{\beta\nu}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left[I_7+p_1B\sigma^* I_8\right]=0. \quad (45)$$

where,

$$I_1=\int\left(|DW|^2+a^2|W|^2\right)dz,\; I_2=\int\left(|D^2W|^2+2a^2|DW|^2+a^4|W|^2\right)dz,$$

$$I_3=\int\left(|D^3W|^2+3a^2|D^2W|^2+3a^4|DW|^2+a^6|W|^2\right)dz,\; I_4=\int\left(|Z|^2\right)dz,$$

$$I_5=\int\left(|DZ|^2+a^2|Z|^2\right)dz,\; I_6=\int\left(|D^2Z|^2+2a^2|DZ|^2+a^4|Z|^2\right)dz,$$

$$I_7=\int\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\; I_8=\int\left(|\Theta|^2\right)dz.$$

All these integrals from I_1-I_8 are positive definite.

Putting $\sigma=i\sigma_i$ (where σ_i is real) in Eq. (45) and equating the imaginary part, we get

$$\sigma_i\left\{\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)I_1-d^2I_4\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)+\frac{\alpha a^2\kappa_T}{\beta\nu}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)\left[\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}I_7+\left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right)Bp_1I_8\right]\right\}=0 \quad (46)$$

In the absence of rotation, Eq. (46) becomes,

$$\sigma_i\left\{\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)I_1+\frac{\alpha a^2\kappa_T}{\beta\nu}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)\left[\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}I_7+\left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right)Bp_1I_8\right]\right\}=0 \quad (47)$$

If $g\ge(\gamma M_0\nabla H/\rho_0\alpha)$, then the terms in the bracket are positive definite, which implies that $\sigma_i=0$ .Therefore, oscillatory modes are not allowed and principle of exchange of stabilities is satisfied if $g\ge(\gamma M_0\nabla H/\rho_0\alpha)$. Hence, presence of rotation introduce the oscillatory modes in the system.

6. Numerical Computations

The dispersion relation (39) is analyzed numerically also. The numerical value of thermal Rayleigh number R_1 is determined for various values of dust particles B, couple-stress F_1, rotation T_{A_1} and magnetization M_0. Also, graphs have been plotted between R_1 and B, R_1 and F_1, R_1 and T_{A_1}, R_1 and M_0 as shown in Figs. (2) - (8).

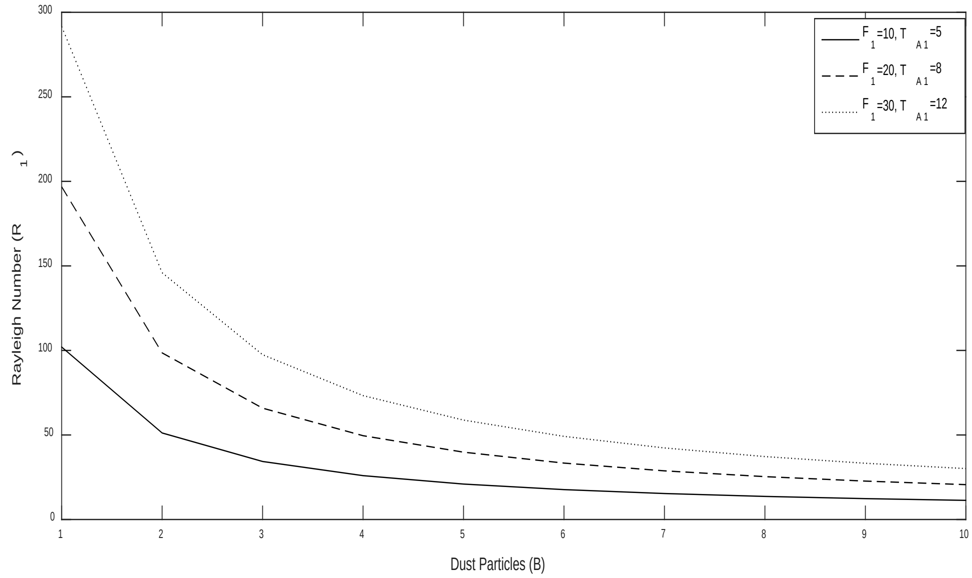

Fig: 2 -Variation of R_1 with B for fixed F_1 =10, 20, 30 and T_{A_1} =5, 8, 12.

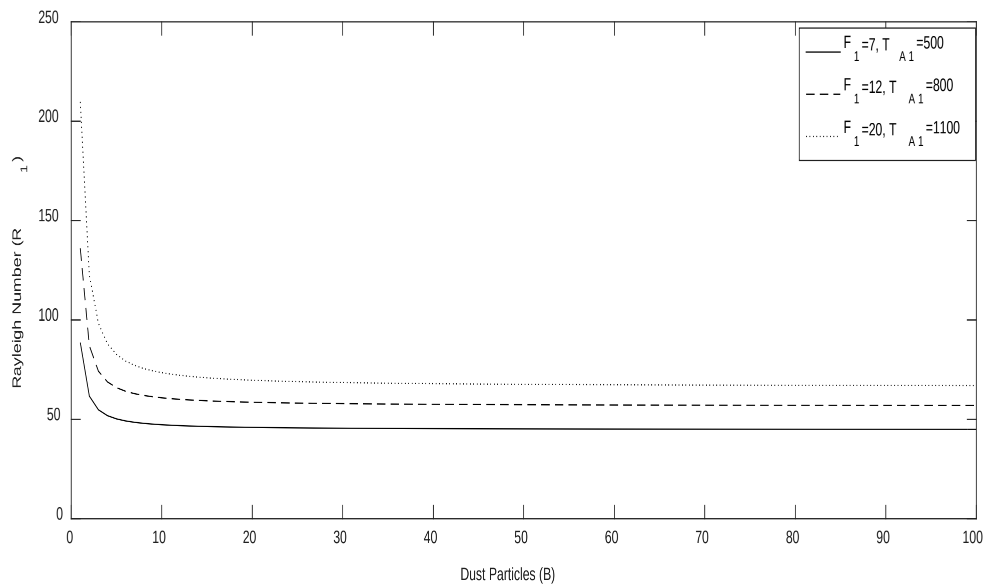

Fig: 3 -Variation of R_1 with B for fixed F_1 =7, 12, 20 and T_{A_1} =500, 800, 1100

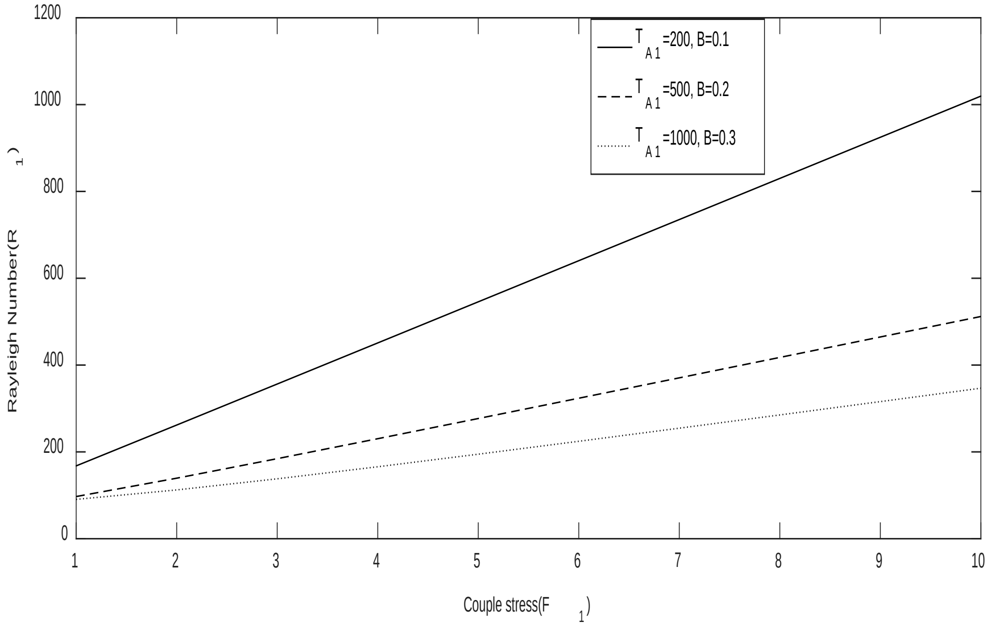

Fig: 4 -Variation of R_1 with F_1 for fixed B =0.1, 0.2, 0.3 and T_{A_1} =200, 500, 1000.

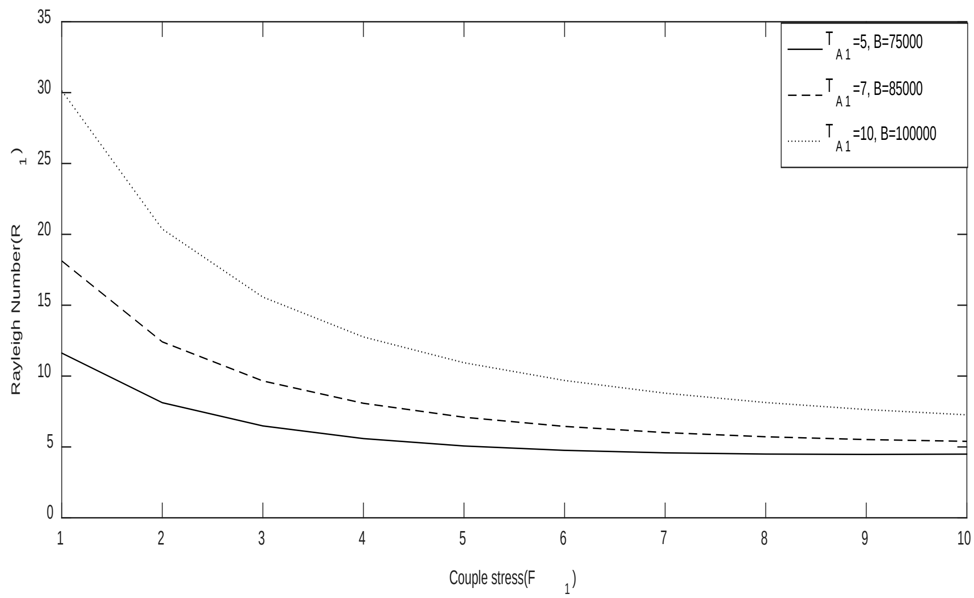

Fig: 5 -Variation of R_1 with F_1 for fixed B =75000, 85000, 100000 and T_{A_1} =5, 7, 10.

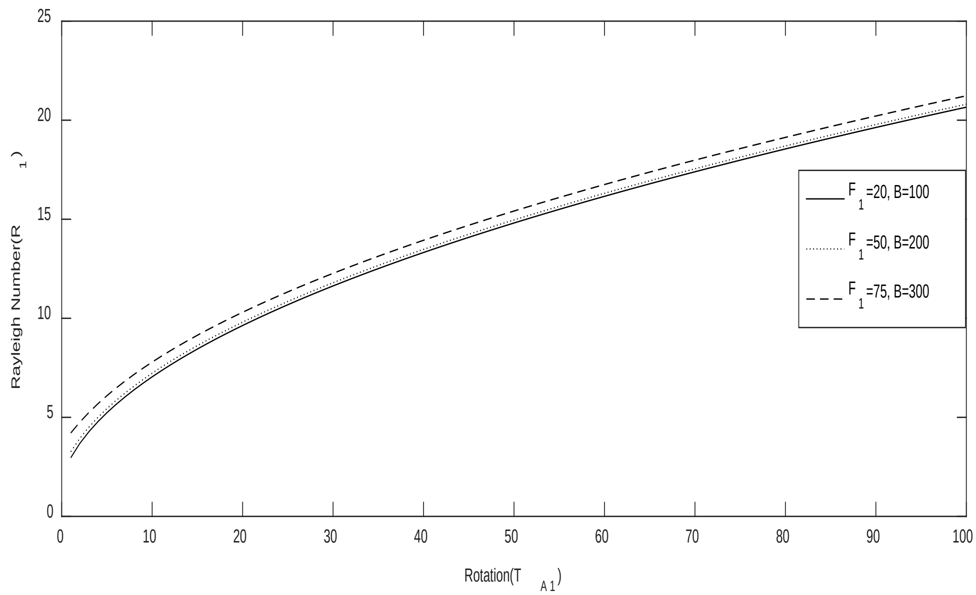

Fig: 6 -Variation of R_1 with T_{A_1} for fixed B =100, 200, 300 and F_1 =20, 50, 75.

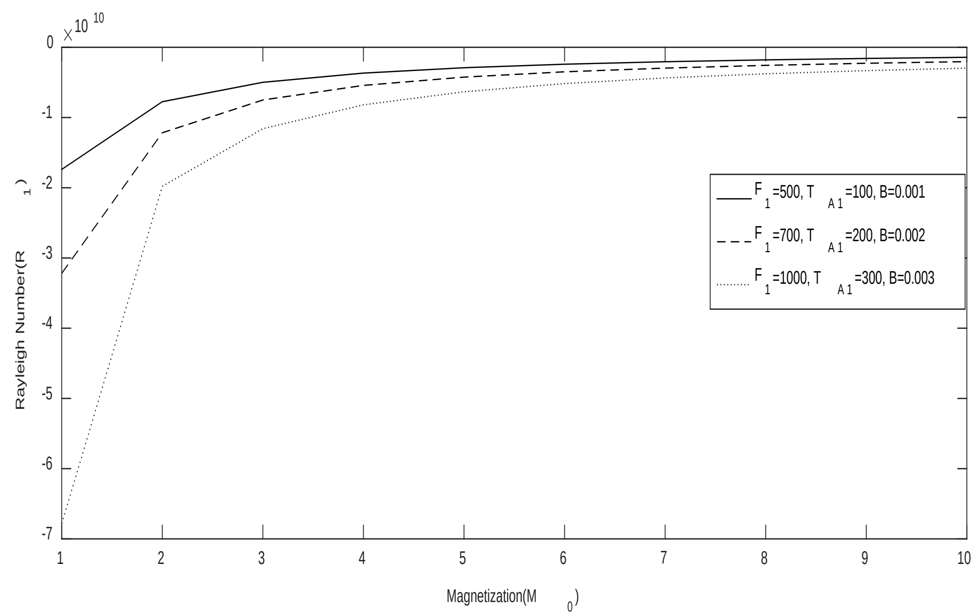

Fig: 7-Variation of R_1 with M_0 for fixed ρ_0 =10, α =10, γ =0.5, $\nabla H = 10$, F_1 =500, 700, 1000; T_{A_1} =100 ,200, 300 and B =0.001, 0.002, 0.003.

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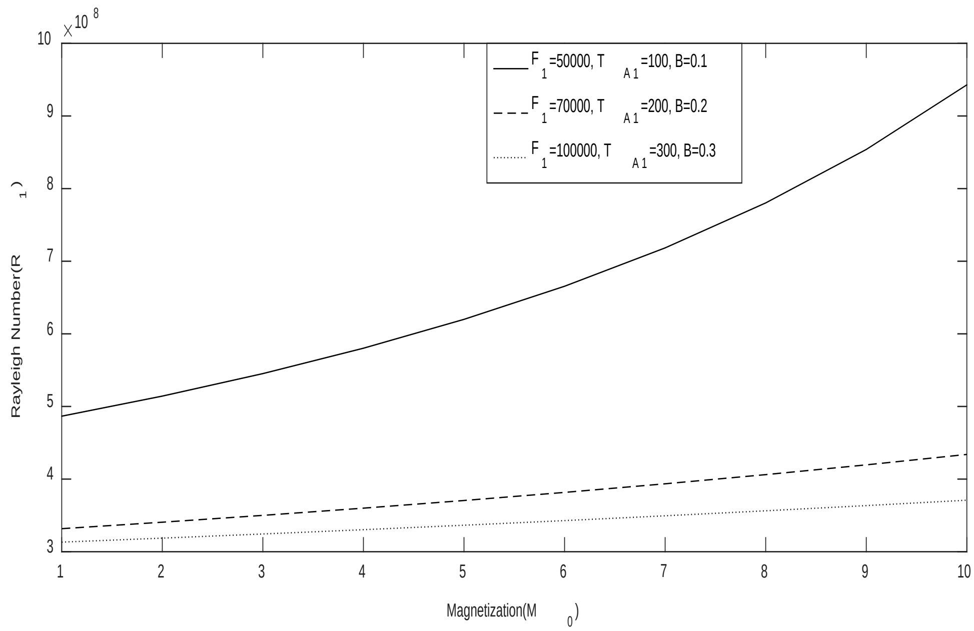

Fig: 8-Variation of R_1 with M_0 for fixed ρ_0 =10, α =10, γ =0.5, $\nabla H = 10$; F_1 =50000, 70000, 100000; T_{A_1} =100 ,200, 300 and B =0.1, 0.2, 0.3.

In Fig. 2, critical Rayleigh number R_1 is plotted against dust particles parameter B for $F_1 = 10, 20, 30$ and $T_{A_1} = 5,8,12$, which shows that critical Rayleigh number R_1 decreases with increase in dust particles parameter B. So, dust particles have a destabilizing effect on the system.

In Fig. 3, critical Rayleigh number R_1 is plotted against dust particles parameter B for $F_1 = 7,12,20$ and $T_{A_1} = 500,800,1100$, which shows that critical Rayleigh number R_1 decreases with increase in dust particles parameter B. So, dust particles have a destabilizing effect on the system.

In Fig. 4, critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for $B = 0.1, 0.2, 0.3$ and $T_{A_1} = 200,500,1000$, which shows that critical Rayleigh number R_1 increases with increase in couple-stress parameter F_1. So, couple-stress has a stabilizing effect on the system.

In Fig. 5, critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for $B = 75000,85000,100000$ and $T_{A_1} = 5,7,10$, which shows that critical Rayleigh number R_1 decreases with increase in couple-stress parameter F_1. So, couple-stress has a destabilizing effect on the system.

In Fig. 6, critical Rayleigh number R_1 is plotted against rotation parameter T_{A_1} for $B = 100, 200, 300$ and $F_1 = 20, 50, 75$, which shows that critical Rayleigh number R_1 increases with increase in rotation parameter T_{A_1}. So, rotation has a stabilizing effect on the system.

In Fig. 7, critical Rayleigh number R_1 is plotted against magnetization parameter M_0 for $B = 0.001, 0.002, 0.003$; $F_1 = 500, 700, 1000$ and $T_{A_1} = 100, 200, 300$, which shows that critical Rayleigh number R_1 increases with increase in magnetization parameter M_0. So, magnetization has a stabilizing effect on the system.

In Fig. 8, critical Rayleigh number R_1 is plotted against magnetization parameter M_0 for $B = 0.1, 0.2, 0.3$; $F_1 = 50000, 70000, 100000$ and $T_{A_1} = 100, 200, 300$, which shows that critical Rayleigh number R_1 increases with increase in magnetization parameter M_0. So, magnetization has a stabilizing effect on the system.

7. Conclusions

The main results from the analysis of this present problem are as under,

1- For stationary convection,

(i) Dust particles have a destabilizing effect on the system.

(ii) Couple-stress has a stabilizing effect on the system when $T_{A_1} < \left[F_1(1+x)+1\right]^2 (1+x)^3$. Also, couple-stress has a destabilizing effect on the system when $T_{A_1} > \left[F_1(1+x)+1\right]^2 (1+x)^3$. In the absence of rotation, couple-stress has a stabilizing effect on the system.

(iii) Rotation has a stabilizing effect on the system.

(iv) Magnetization has a stabilizing effect on the system.

2- The principle of exchange of stabilities is not valid for the present problem under consideration whereas, in the absence of rotation, principle of exchange of stabilities (PES) is valid for the present problem if $g \geq \dfrac{\gamma M_0 \nabla H}{\rho_0 \alpha}$.

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