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Second Quantization of Electromagnetic Field in Terms of Divisors

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Abstract

In previous works, Maxwell's equations for electromagnetic field have been written in different forms. In particular, Maxwell's equations have been written in covariant form through divisors K_μ and m_μ. Furthemore, the Lagrange formalism for electromagnetic field in terms of divisors has been elaborated.

In this work, in development of the above ideas, we elaborated the second quantization of electromagnetic field in terms of divisors K_μ and m_μ.

***Keywords:** Second quantization, electromagnetic field, divisors.*

Introduction

In previous works, Maxwell's equations for electromagnetic field have been written in different forms. In particular, Maxwell's equations have been written in covariant form through divisors K_μ and m_μ. The formulation of Maxwell's equations through divisors has been followed by the elaboration of the Lagrange formalism for electromagnetic field in terms of divisors K_μ and m_μ. Here, expressions for dynamical variables (energy, momentum, charge and spin) conserved in time, have been written through divisors K_μ and m_μ.

In this work, we shall develop the second quantization of electromagnetic field in terms of divisors K_μ and m_μ. The wave function m_μ will be expanded in Fourier series. The coefficients of expansion will then be replaced by the corresponding creation and annihilation operators. Finally, dynamical variables (energy, momentum, charge and spin) conserved in time will be expressed through the number of particles.

Research Method

In previous works, Maxwell's equations have been written in different forms. In particular, Maxwell's equations have been written through divisors K_μ and m_μ. The Lagrange formalism for electromagnetic field in terms of divisors K_μ and m_μ has been elaborated. In this work, using the traditional procedure for second quantization, we shall elaborate the second quantization of electromagnetic field in terms of divisors K_μ and m_μ.

Second Quantization of Electromagnetic Field in Terms of Divisors

Maxwell's equations for electromagnetic field in vacuum

$$\begin{cases} \mathrm{rot}\,\vec{E} + \frac{\partial \vec{H}}{\partial t} = 0 \\ \mathrm{rot}\,\vec{H} - \frac{\partial \vec{E}}{\partial t} = 0 \\ \mathrm{div}\vec{E} = 0 \\ \mathrm{div}\vec{H} = 0 \end{cases} . \tag{1}$$

can be written in terms of isotropic complex vectors as follows

$$\begin{cases} D_0\vec{F} = i\vec{D} \times \vec{F} \\ \vec{D}\vec{F} = 0 \end{cases} . \tag{2}$$

Where

$$\vec{F} = \vec{E} + i\vec{H},$$

$$D_0 = \frac{i}{2}\frac{\partial}{\partial t}, \ \vec{D} = -\frac{i}{2}\vec{\nabla}. \tag{3}$$

However, in general the solution of Maxwell's equations does not satisfy non-linear isotropic condition $\vec{F}^2 = 0$, but for electromagnetic field in vacuum this condition is satisfied.

Let us introduce bivector field

$$F_{\mu\nu} = K_\mu \wedge m_\nu, \tag{4}$$

satisfying isotropic condition

$$F_{\mu\nu}\tilde{F}_{\mu\nu} = 0. \tag{5}$$

Here K_μ is a real four vector; m_μ is a complex four vector. In Eq(4), the sign "∧", means antisymmetrization over indices μ and ν.

The four vectors K_μ and m_μ are called divisors and are expressed through $\vec{E}$ and $\vec{H}$ as follows

$$K_\mu = \left(|\vec{E}|, \frac{\vec{E}\times\vec{H}}{|\vec{E}|}\right), \qquad m_\mu = \left(0, \frac{\vec{F}}{|\vec{E}|}\right). \tag{6}$$

Maxwell's equations, Eq (1) written through divisors take the form

$$D^\nu F_{\mu\nu} = 0. \tag{7}$$

In previous work, it has been proved that Maxwell's equations, Eq (2) can be obtained by variation principle from the Lagrange function

$$L = \frac{i}{2}\left\{\left[D_0\vec{F} - i\vec{D}\times\vec{F}\right]\vec{F}^* - \left[D_0\vec{F}^* + i\vec{D}\times\vec{F}^*\right]\vec{F}\right\} / \left(\frac{\vec{F}\vec{F}^*}{2}\right)^{1/2} \tag{8}$$

It is easy to prove that, the Lagrange function, Eq(8) written through divisors takes the form

$$L = \left(D^\nu F_{\mu\nu}\right)m^{\mu*} - \left(D^\nu F^*_{\mu\nu}\right)m^\mu. \tag{9}$$

Using Noether's theorem, we can derive from the Lagrange function, Eq (9) expressions for fundamental dynamical variables conserved in time.

Energy is determined by the formula

$$E = \int T^{00} d^3x, \tag{10}$$

where

$$T^{00} = \frac{i}{4}\left(m^{\nu*}\frac{\partial m_\nu}{\partial t} - m^\nu\frac{\partial m^*_\nu}{\partial t}\right). \tag{11}$$

Considering the solution for free particle in the form of plane waves

$$m_\nu = \left[0, \frac{(\vec{E}^0 + i\vec{H}^0)}{|\vec{E}|} e^{-2ikt+2i\vec{k}\vec{r}}\right], \tag{12}$$

we find

$$T^{00} = k|\vec{E}|. \tag{13}$$

Similarly for momentum, we have

$$P^j = \int T^{0j} d^3x, \tag{14}$$

where

$$T^{0j} = \frac{i}{4}\left[m^\nu\vec{\nabla}_j m^*_\nu - m^{\nu*}\vec{\nabla}_j m_\nu\right]. \tag{15}$$

Replacing Eq(12) into Eq(15), we obtain

$$T^{0j} = k_j|\vec{E}|. \tag{16}$$

For charge, we find

$$Q = \int j^0 d^3x, \tag{17}$$

where

$$j^0 = m^\nu m_\nu^* + m_\nu^* m^\nu. \tag{18}$$

Replacing Eq(12) into Eq(18), we get

$$j^0 = |\vec{E}|. \tag{19}$$

In the same way, the density of the spin pseudo vector is determined by the formula

$$s_k = \frac{1}{2}\varepsilon_{klm} s_{lm}^0, \tag{20}$$

where

$$s_{lm}^0 = \left[-\frac{\partial L}{\partial m_{i,0}} m_j A_{i,lm}^j - \frac{\partial L}{\partial m_{i,0}^*} m_j^* A_{i,lm}^j\right]. \tag{21}$$

Here

$$A_{i,lm}^j = g_{il}\delta_m^j - g_{im}\delta_l^i. \tag{22}$$

Then, we obtain

$$\vec{s} = \frac{\vec{E}\times\vec{H}}{|\vec{E}|}. \tag{23}$$

Let us expand the wave function $m_\mu(x)$ in Fourier series

$$m_\mu(x) = \sum_{k,s} a_s(\vec{k}) m_{\mu s}(\vec{k}) e^{-2ikt+2i\vec{k}\vec{r}}, \tag{24}$$

$$m_\mu^*(x) = \sum_{k,s} a_s^*(\vec{k}) m_{\mu s}^*(\vec{k}) e^{2ikt-2i\vec{k}\vec{r}}. \tag{25}$$

Replacing Eqs(24)- (25) in Eqs(10), (14), (17) and Eq(23) and considering the normalization condition

$$\int \left[\frac{m_{ks}^{\nu *} m_{k's'}^{\nu}}{2[m_{ks}^{\nu *} m_{k's'}^{\nu}/2]^{1/2}}\right] d^3x = \delta_{kk'}\delta_{ss'}, \tag{26}$$

we obtain

$$E = \sum_{k,s} k[a_s^*(\vec{k}) a_s(\vec{k})], \tag{27}$$

$$P_j = \sum_{k,s} k_j [a_s^*(\vec{k}) a_s(\vec{k})], \tag{28}$$
$$Q = \sum_{k,s} [a_s^*(\vec{k}) a_s(\vec{k})], \tag{29}$$
$$S_j = \sum_{k,s} æ_j [a_s^*(\vec{k}) a_s(\vec{k})]. \tag{30}$$

Here $\vec{æ}$ is a unit vector in the direction of polarization vector $\vec{S}$.

Changing $a_s(\vec{k})$ by operator $\hat{a}_s(\vec{k})$, we find the following operators for observable physical quantities

$$\hat{E} = \sum_{k,s} k [\hat{a}_s^+(\vec{k}) \hat{a}_s(\vec{k})], \tag{31}$$
$$\hat{P}_j = \sum_{k,s} k_j [\hat{a}_s^+(\vec{k}) \hat{a}_s(\vec{k})], \tag{32}$$
$$\hat{Q} = \sum_{k,s} [\hat{a}_s^+(\vec{k}) \hat{a}_s(\vec{k})], \tag{33}$$
$$\hat{S}_j = \sum_{k,s} æ_j [\hat{a}_s^+(\vec{k}) \hat{a}_s(\vec{k})]. \tag{34}$$

To ensure the positive determination of energy, we must require the following commutation relations

$$[\hat{a}_s^+(\vec{k}), \hat{a}_{s'}(\vec{k}')]_- = \delta_{kk'} \delta_{ss'}. \tag{35}$$

Using Eqs(31)-(34), we find expressions for eigenvalues of the above operators

$$E = \sum_{k,s} k [N_{ks}], \tag{36}$$
$$P_j = \sum_{k,s} k_j [N_{ks}], \tag{37}$$
$$Q = \sum_{k,s} [N_{ks}], \tag{38}$$
$$S_j = \sum_{k,s} æ_j [N_{ks}]. \tag{39}$$

Here N_{ks} is the number of particles.

Discussion and Conclusion

In previous works, Maxwell's equations for electromagnetic field have been written through divisors K_μ and m_μ and the Lagrange formalism for electromagnetic field in terms of divisors K_μ and m_μ has been elaborated. In this work, we developed the second quantization of electromagnetic field in terms of divisors K_μ and m_μ. Here, dynamical variables (energy, momentum, charge and spin) conserved in time have been expressed through the number of particles.

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