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# The Wavefunctions in the Conversion between the Bosonic Superconducting State and the Fermionic Normal Metallic State

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## Abstract
The 4-dimensional complex spacetime world where we really live is the reason why the wavefunctions derived from the Schrödinger equation in quantum mechanics can be expressed by the complex numbers. That is, the wavefunction is not virtual and abstract function but substantial function denoting the amplitude of the substantial wave. The Heisenberg's uncertainty principle means the necessary minimum size for formation of a wave packet. The superimposition between various wavefunctions with various momentum as a consequence of the appearance of large free energy such as kinetic energy is the origin of the collapse of the wavefunction from the bosonic standing wave to the fermionic traveling wave. The increase of the entropy and the total energy of the 4-dimensional complex spacetime world is the main reason why the conversion from the fermionic traveling wave state to the bosonic standing wave state occurs significantly rapidly.
***Keywords:*** *Complex Wavefunction; Heisenberg's Uncertainty Principle; The Collapse of the Wavefunction; The Principle of the Causality.*

## 1. Introduction
The effect of vibronic interactions and electron–phonon interactions [1–7] in molecules and crystals is an important topic of discussion in modern chemistry and physics. The vibronic and electron–phonon interactions play an essential role in various research fields such as the decision of molecular structures, Jahn–Teller effects, Peierls distortions, spectroscopy, electrical conductivity, and superconductivity. We have investigated the electron–phonon interactions in various charged molecular crystals for more than 15 years [1–8]. In particular, in 2002, we predicted the occurrence of superconductivity as a consequence of vibronic interactions in the negatively charged picene, phenanthrene, and coronene [8]. Recently, it was reported that these trianionic molecular crystals exhibit superconductivity [9].

Related to the research of superconductivity as described above, in the recent research [10,11], we explained the mechanism of the Ampère's law (experimental rule discovered in 1820) and the Faraday's law (experimental rule discovered in 1831) in normal metallic and superconducting states [12], on the basis of the theory suggested in our previous researches [1–7]. Furthermore, we discussed how the left-handed helicity magnetic field can be induced when the negatively charged particles such as electrons move [13]. That is, we discussed the relationships between the electric and magnetic fields [13]. Furthermore, by comparing the electric charge with the spin magnetic moment and mass, we suggested the origin of the electric charge in a particle. Furthermore, in the previous research, we discussed the origin of the gravity, by comparing the gravity with the electric and magnetic forces. Furthermore, we showed the reason why the gravity is much smaller than the electric and magnetic forces [14]. We discussed the origin of the strong forces, by comparing the strong force with the gravitational, electric, magnetic, and electromagnetic forces. We also discussed the essential properties of the gluon and color charges, and discussed the reason why the quarks and gluons are confined in hadron [15]. Furthermore, we discussed the origin of the weak forces, and discussed the reason why the parity violation can be observed in the weak interactions [16]. We also suggested the relationships between the Cooper pairs in superconductivity and the Higgs boson in the vacuum [16,17]. Recently, we discussed the origin of the spin magnetic dipole moment, massive charge, electric monopole charge, and color charge for the particle and antiparticles at the particles and antiparticle spacetime axes, by considering that particles (antiparticles) can be formed by mixture of the wavefunction of more dominant particle (antiparticle) component and of less dominant antiparticle (particle) component [18]. We suggested the new interpretation of the spacetime axis in the special relativity [19]. We also discussed the mechanism of the particle–antiparticle pair annihilation in view of the special relativity [19].

Furthermore, we suggested the relationships between the superconducting, normal metallic, and insulating states. Related to these relationships, in particular, related to the relationships between the bosonic standing waves and the fermionic traveling waves, and between the equilibrium states and the non-equilibrium states, we also discussed the relationships between the entropy and the time [20].

In the previous research [10], we suggested the mechanism of the occurrence of the Faraday's law in terms of the particle rather than the wave. In this research, we will discuss the fundamental problems as follows. We will discuss the reason why the wavefunctions in quantum mechanics is expressed by the complex numbers with the real and imaginary numbers. We will next discuss the interpretations of the wavefunctions. Furthermore, we will also discuss the relationships between the Heisenberg's uncertainty principle and the principle of the causality. We will discuss the mechanism of the Faraday's law in terms of the wave rather than the particle. Furthermore, we will discuss the conversion between the bosonic standing wave and the fermionic traveling wave, and discuss the mechanism of the collapse of the wavefunction.

## 2. New Interpretation of the Spacetime Axis in the Special Relativity

In this article, we define the spacetime components of the particles and antiparticles as follows (Figs. 1, 2) [19].

The $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}$ and $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{a}}}$ terms denote the real space components at the real 3-dimensional real space axis for particles and antiparticles, respectively. The $\boldsymbol{r}_{\boldsymbol{t}_{\mathrm{p}}}$ and $\boldsymbol{r}_{\boldsymbol{t}_{\mathrm{a}}}$ terms denote the real space components at the real 3-dimensional time axis for particles and antiparticles, respectively.

The $\boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}}}$ and $\boldsymbol{t}_{\boldsymbol{t}_{\mathrm{a}}}$ terms denote the real time components at the real 1-dimensional time axis for particles and antiparticles, respectively. The $\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}$ and $\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{a}}}$ terms denote the real time components at the real 1-dimensional space axis for particles and antiparticles, respectively.

The $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ and $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{a}}}$ terms denote the real momentum components at the real 3-dimensional space axis for particles and antiparticles, respectively. The $\boldsymbol{p}_{\boldsymbol{t}_{\mathrm{p}}}$ and $\boldsymbol{p}_{\boldsymbol{t}_{\mathrm{a}}}$ terms denote the real momentum components at the real 3-dimensional time axis for particles and antiparticles, respectively.

The $\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}}}$ and $\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{a}}}$ terms denote the real energy components at the real 1-dimensional time axis for particles and antiparticles, respectively. The $\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}}}$ and $\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{a}}}$ terms denote the real energy components at the real 1-dimensional space axis for particles and antiparticles, respectively.

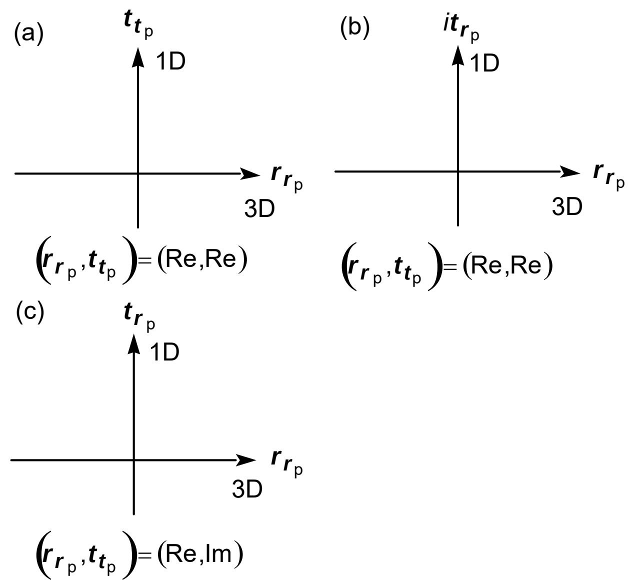



Fig. 1. Relationships between the space and time axes. (a) The 3-dimensional real space axis and the 1-dimensional real time axis. (b) The 3-dimensional real space axis and the 1-dimensional imaginary space axis. (c) The 3-dimensional real space axis and the 1-dimensional real space axis.

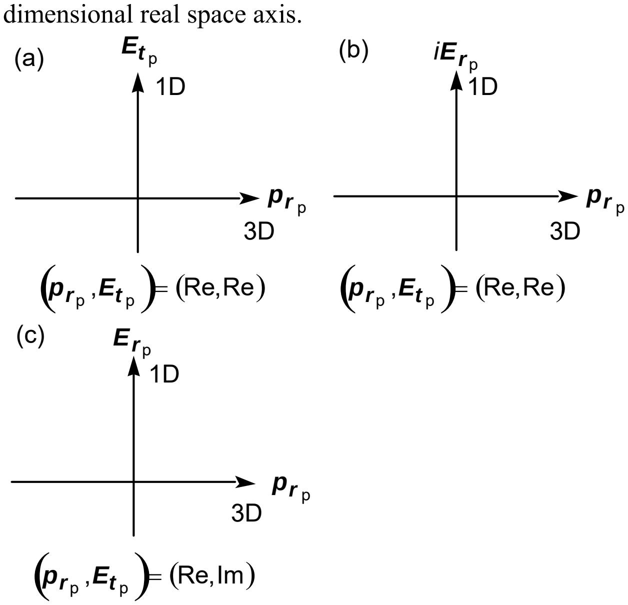



Fig. 2. Relationships between the momentum and energy axes. (a) The 3-dimensional real momentum axis and the 1-dimensional real energy axis. (b) The 3-dimensional real momentum axis and the 1-dimensional imaginary momentum axis. (c) The 3-dimensional real momentum axis and the 1-dimensional real momentum axis.

According to the special relativity and Minkowski's research, the relationships between the space ($x$, $y$, $z$) and time axes ($t$) before observation can be expressed as

$$x^2+y^2+z^2+(ict)^2=\text{const}. \quad (1)$$

where the $c$ is the speed of the light. In other words,

$$\boldsymbol{r}_{\boldsymbol{r}_\text{p}}^2+\left(c\boldsymbol{t}_{\boldsymbol{t}_\text{p}}\right)^2=\text{const}. \quad (2)$$

$$\boldsymbol{r}_{\boldsymbol{r}_\text{a}}^2+\left(c\boldsymbol{t}_{\boldsymbol{t}_\text{a}}\right)^2=\text{const}. \quad (3)$$

On the other hand, the 1-dimensional $\boldsymbol{t}_{\boldsymbol{t}_\text{p}}$ and $\boldsymbol{t}_{\boldsymbol{t}_\text{a}}$ time vectors, which are real components at the real time axis, are the imaginary components at the real 1-dimensional space axis, as expressed as (Fig. 1 (c)),

$$\boldsymbol{t}_{\boldsymbol{t}_\text{p}}=i\boldsymbol{t}_{\boldsymbol{r}_\text{p}}, \quad (4)$$

$$\boldsymbol{t}_{\boldsymbol{t}_\text{a}}=i\boldsymbol{t}_{\boldsymbol{r}_\text{a}}. \quad (5)$$

Therefore,

$$\boldsymbol{r}_{\boldsymbol{r}_\text{p}}^2+\left(c\boldsymbol{t}_{\boldsymbol{t}_\text{p}}\right)^2=\boldsymbol{r}_{\boldsymbol{r}_\text{p}}^2+\left(ic\boldsymbol{t}_{\boldsymbol{r}_\text{p}}\right)^2=\text{const}. \quad (6)$$

$$\boldsymbol{r}_{\boldsymbol{r}_\text{a}}^2+\left(c\boldsymbol{t}_{\boldsymbol{t}_\text{a}}\right)^2=\boldsymbol{r}_{\boldsymbol{r}_\text{a}}^2+\left(ic\boldsymbol{t}_{\boldsymbol{r}_\text{a}}\right)^2=\text{const}. \quad (7)$$

If we consider that we live in the real visible space axis, real time axis ($\boldsymbol{t}_{\boldsymbol{t}_\text{p}}$ and $\boldsymbol{t}_{\boldsymbol{t}_\text{a}}$) can be considered to be imaginary invisible space axis ($i\boldsymbol{t}_{\boldsymbol{r}_\text{p}}$ and $i\boldsymbol{t}_{\boldsymbol{r}_\text{a}}$) (Fig. 1 (b)). That is, we can consider that the *ct* term is related to the real time component (imaginary space component) (Fig. 1 (a)), on the other hand, the *ict* term is related to the real space component (imaginary time component) (Fig. 1 (b)).

Therefore, we can consider that the real world we live in is the complex 4-dimensional spacetime world which is formed by the real visible 3-dimensional space components (so-called, space axis) and by the imaginary invisible 1-dimensional space component (so-called, time axis), at the real 4-dimensional space axis (Fig. 1 (c)).

The 4-dimensional spacetime axis can be interpreted by various definitions as follows. The components of the 4-dimensional spacetime axis are composed of the 3-dimensional real space vectors at the 3-dimensional real space axis and of the 1-dimensional real time vectors at the 1-dimensional real time axis (Fig. 1 (a)). The components of the 4-dimensional spacetime axis are composed of the 3-dimensional real space vectors at the 3-dimensional real space axis and the 1-dimensional real time vectors at the 1-dimensional imaginary space axis (Fig. 1 (b)). The components of the 4-dimensional spacetime axis are composed of the 3-dimensional real space vectors at the 3-dimensional real space axis and of the 1-dimensional imaginary time vectors at the 1-dimensional real space axis (Fig. 1 (c)).

We can consider that the components of the 4-dimensional spacetime axis are composed of the 3-dimensional real space vectors at the 3-dimensional real space axis and of the 1-dimensional imaginary time vectors at the 1-dimensional real space axis, in which we really live (Fig. 1 (c)). On the other hand, we consider that the components of the 4-dimensional spacetime axis are composed of the 3-dimensional real space vectors at the 3-dimensional real space axis and of the 1-dimensional real time vectors at the 1-dimensional real time axis, in the non-relativistic classical mechanics, in which we have thought to live (Fig. 1 (a)).

We can see from Eq. (6) that the $\boldsymbol{r}_{\boldsymbol{r}_\text{p}}^2$ value increases with an increase in the $\boldsymbol{t}_{\boldsymbol{r}_\text{p}}^2$ value. This may be the reason why the universe is observed to expand with an increase in time $\boldsymbol{t}_{\boldsymbol{r}_\text{p}}$ from the past to the future. In other words, the universe is expanding in order for the volume of the 4-dimensional complex spacetime world, where we really live, to become constant, as can be seen from Eq. (6). Furthermore, the $\boldsymbol{t}_{\boldsymbol{r}_\text{p}}$ is closely related to the rest mass charge $q_{\text{g},0}$. Therefore, observation of the accelerated expansion of the universe may be related to the increase in the mass and time proceeding itself.

From Eqs. (6) and (7), denoting the relationships between the space and time axes, we can derive the equation, denoting the relationships between the momentum ($p_x$, $p_y$, $p_z$, $p_t$, $p_{t,0}$) and energy ($E_x$, $E_y$, $E_z$, $E_t$, $E_{t,0}$) by using the mass $q_\text{g}$ and the rest mass $q_{\text{g},0}$, as follows (Fig. 2),

$$\left(q_\text{g}v_x\right)^2+\left(q_\text{g}v_y\right)^2+\left(q_\text{g}v_z\right)^2+\left(iq_\text{g}c\right)^2=\left(iq_{\text{g},0}c\right)^2=\text{const.}<0, \quad (8)$$

$$p_x^2+p_y^2+p_z^2+p_t^2=p_{t,0}^2=\text{const.}<0, \quad (9)$$

$$p_x^2+p_y^2+p_z^2+\left(\frac{E_t}{c}\right)^2=\left(\frac{E_{t,0}}{c}\right)^2=\text{const.}<0, \quad (10)$$

where

$$p_x=q_\text{g}v_x, \quad (11)$$

$$p_y = q_g v_y, \quad (12)$$

$$p_z = q_g v_z, \quad (13)$$

$$p_t = iq_g c, \quad (14)$$

$$p_{t,0} = iq_{g,0} c, \quad (15)$$

$$E_t = cp_t, \quad (16)$$

$$E_{t,0} = cp_{t,0}. \quad (17)$$

In other words (Fig. 2 (a)),

$$\boldsymbol{p}_{\boldsymbol{r}_\mathrm{p}}^2 + \boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}}^2 = \boldsymbol{p}_{\boldsymbol{r}_\mathrm{p}}^2 + \left(\frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}}}{c}\right)^2 = \left(\frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},0}}{c}\right)^2 = \text{const.} \quad (18)$$

$$\boldsymbol{p}_{\boldsymbol{r}_\mathrm{a}}^2 + \boldsymbol{p}_{\boldsymbol{t}_\mathrm{a}}^2 = \boldsymbol{p}_{\boldsymbol{r}_\mathrm{a}}^2 + \left(\frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{a}}}{c}\right)^2 = \left(\frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{a},0}}{c}\right)^2 = \text{const.} \quad (19)$$

where

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}} = c\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}}, \quad (20)$$

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{a}} = c\boldsymbol{p}_{\boldsymbol{t}_\mathrm{a}}. \quad (21)$$

On the other hand, the 1-dimensional $\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}}$ and $\boldsymbol{p}_{\boldsymbol{t}_\mathrm{a}}$ ($\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}}$ and $\boldsymbol{E}_{\boldsymbol{t}_\mathrm{a}}$) momentum (energy) vectors, which are real components at the time axis, are the imaginary components at the real 1-dimensional space axis, as expressed as (Fig. 2 (b), (c)),

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}} = i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}, \quad (22)$$

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{a}} = i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a}}. \quad (23)$$

Therefore, the $\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}$ and $\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a}}$ can be interpreted as the energy momentum vector for the particles and antiparticles, respectively (Fig. 2 (c)). Eqs. (8)–(10) can be expressed by using vectors as (Fig. 2 (c))

$$\boldsymbol{p}_{\boldsymbol{r}_\mathrm{p}}^2 + \boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}}^2 = \boldsymbol{p}_{\boldsymbol{r}_\mathrm{p}}^2 + \left(\frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}}}{c}\right)^2$$
$$= \boldsymbol{p}_{\boldsymbol{r}_\mathrm{p}}^2 + \left(\frac{i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}}{c}\right)^2 = \left(\frac{i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p},0}}{c}\right)^2 = \text{const.} \quad (24)$$

$$\boldsymbol{p}_{\boldsymbol{r}_\mathrm{a}}^2 + \boldsymbol{p}_{\boldsymbol{t}_\mathrm{a}}^2 = \boldsymbol{p}_{\boldsymbol{r}_\mathrm{a}}^2 + \left(\frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{a}}}{c}\right)^2$$
$$= \boldsymbol{p}_{\boldsymbol{r}_\mathrm{a}}^2 + \left(\frac{i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a}}}{c}\right)^2 = \left(\frac{i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a},0}}{c}\right)^2 = \text{const.} \quad (25)$$

The momentum $p_x$, $p_y$, and $p_z$ values are related to the real components at the real space axis, $x$, $y$, and $z$, respectively. The energy $E_t$ is related to the real (imaginary) component at the real time (real space) axis (Fig. 2 (a)). The $p_t$ and $p_{t,0}$ ($E_t / c$ and $E_{t,0} / c$) terms are usually considered to be related to the energy, that is, related to the real components at the time axis (Fig. 2 (a)). On the other hand, if we consider that we live in the real visible momentum axis, which is related to the real space axis, the energy can be considered to be the imaginary invisible momentum component at the real space axis (Fig. 2 (b), (c)). Therefore, we can consider that the $p_t$ and $E_t$ terms, and the $p_{t,0}$ and $E_{t,0}$ terms, are related to the real time component (imaginary space component), on the other hand, the $ip_t$ and $iE_t$ terms, and the $ip_{t,0}$ and $iE_{t,0}$ terms, are related to the real space component (imaginary time component).

Let us next express the energy components from Eqs. (8)–(10),

$$E_x^2 + E_y^2 + E_z^2 + E_t^2 = E_{t,0}^2 = \text{const.} < 0, \quad (26)$$

$$\sqrt{-\left(E_x^2 + E_y^2 + E_z^2 + E_t^2\right)} = \sqrt{-E_{t,0}^2}, \quad (27)$$

where

$$E_x = cp_x, \quad (28)$$

$$E_y = cp_y, \quad (29)$$

$$E_z = cp_z, \quad (30)$$

$$\sqrt{-\left\{\left(c\boldsymbol{p}_{\boldsymbol{r}_\mathrm{p}}\right)^2 + \left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}\right)^2\right\}} = \sqrt{-\left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p},0}\right)^2}, \quad (31)$$

$$\sqrt{-\left\{\left(c\boldsymbol{p}_{\boldsymbol{r}_\mathrm{a}}\right)^2 + \left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a}}\right)^2\right\}} = \sqrt{-\left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a},0}\right)^2}, \quad (32)$$

$$\left(c\boldsymbol{p}_{\boldsymbol{r}_\mathrm{p}}\right)^2 + \left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}\right)^2 = \left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p},0}\right)^2, \quad (33)$$

$$\left(c\boldsymbol{p}_{\boldsymbol{r}_\mathrm{a}}\right)^2 + \left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a}}\right)^2 = \left(i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{a},0}\right)^2. \quad (34)$$

We can see from Eqs. (33) and (34) that the original point ($p_x = p_y = p_z = p_t = 0$) at the spacetime axis in energy is saddle point (massless transition state (TS)) [21] of the converting reaction between massive particle and antiparticle states in momentum-energy curves (Fig. 3).

## 3. Relationships between the Classical Dynamics and the Quantum Mechanics

### 3.1 Schrödinger Equation

In the non-relativistic classical dynamics, only the space $\boldsymbol{r}_{r_\mathrm{p},\mathrm{CL}}$ and momentum $\boldsymbol{p}_{r_\mathrm{p},\mathrm{CL}}$ at the real 3-dimensional space axis, and the time $\boldsymbol{t}_{t_\mathrm{p},\mathrm{CL}}$ and energy $\boldsymbol{E}_{t_\mathrm{p},\mathrm{CL}}$ at the real 1-dimensional time axis are used. In the non-relativistic classical dynamics, the $\boldsymbol{r}_{r_\mathrm{p},\mathrm{CL}}$ and $\boldsymbol{p}_{r_\mathrm{p},\mathrm{CL}}$ values at the real 3-dimensional space axis are not related to the $\boldsymbol{t}_{t_\mathrm{p},\mathrm{CL}}$ and $\boldsymbol{E}_{t_\mathrm{p},\mathrm{CL}}$ values at the real 1-dimensional time axis. On the other hand, in the quantum mechanics by considering the special relativity, the space $\boldsymbol{r}_{r_\mathrm{p}}$ and momentum $\boldsymbol{p}_{r_\mathrm{p}}$ can be expressed in view of the real 3-dimensional space axis, and the time $\boldsymbol{t}_{r_\mathrm{p}}$ and energy $\boldsymbol{E}_{r_\mathrm{p}}$ can be also expressed in view of the real 1-dimensional space axis. In the quantum mechanics by considering the special relativity, the $\boldsymbol{r}_{r_\mathrm{p}}$ and $\boldsymbol{p}_{r_\mathrm{p}}$ values at the real 3-dimensional space axis are closely related to the $\boldsymbol{t}_{r_\mathrm{p}}$ and $\boldsymbol{E}_{r_\mathrm{p}}$ values at the real 1-dimensional space axis (Eqs. (6) and (18)).

The relationships between the spacetime in the non-relativistic classical dynamics ($\boldsymbol{r}_{r_\mathrm{p},\mathrm{CL}}$ and $\boldsymbol{t}_{t_\mathrm{p},\mathrm{CL}}$) and in the relativistic mechanics ($\boldsymbol{r}_{r_\mathrm{p}}$ and $\boldsymbol{t}_{r_\mathrm{p}}$) can be expressed as,

$$\boldsymbol{r}_{r_\mathrm{p}} = \boldsymbol{r}_{r_\mathrm{p},\mathrm{CL}}, \qquad (35)$$

$$\boldsymbol{t}_{r_\mathrm{p}} = \boldsymbol{t}_{t_\mathrm{p},\mathrm{CL}}. \qquad (36)$$

The Schrödinger equation can be expressed by the complex numbers, in the quantum mechanics, as follows,

$$H\psi = i\frac{h}{2\pi}\frac{\partial\psi}{\partial \boldsymbol{t}_{r_\mathrm{p}}}, \qquad (37)$$

$$H\psi = \boldsymbol{E}_{t_\mathrm{p}}\,\psi, \qquad (38)$$

where

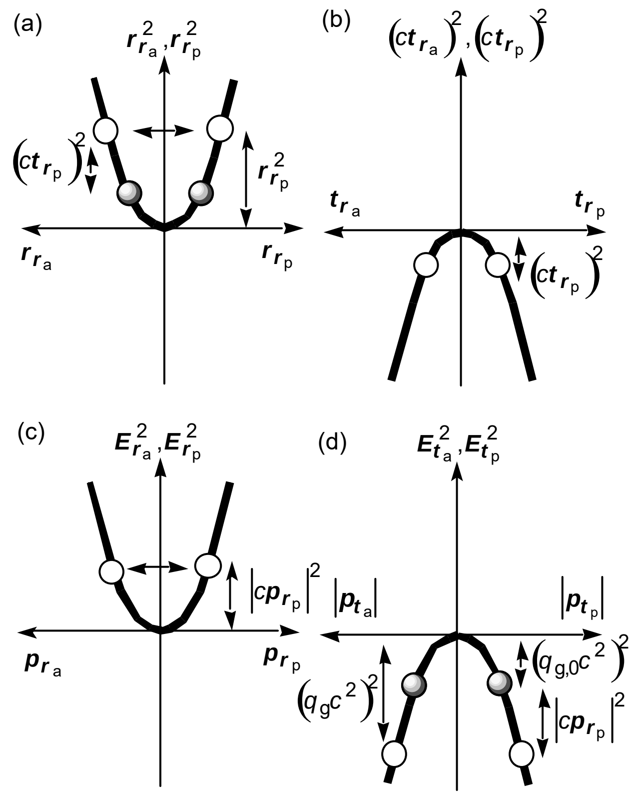



Fig. 3. (a) Scale of the space. The opened and shaded circles indicate the scales of the space and the total spacetime, respectively. (b) Scale of the time denoted by the opened circles. (c) Energy for the space axis denoted by the opened circles. (d) Energy for the time axis. The opened and shaded circles indicate the energies of the time and the total spacetime axes, respectively.

$$\begin{aligned} H &= -\frac{(h/(2\pi))^2}{2q_\mathrm{g}}\nabla^2_{r_\mathrm{p},\mathrm{CL}} + V\left(\boldsymbol{r}_{r_\mathrm{p},\mathrm{CL}}, \boldsymbol{t}_{t_\mathrm{p},\mathrm{CL}}\right) \\ &= -\frac{(h/(2\pi))^2}{2q_\mathrm{g}}\nabla^2_{r_\mathrm{p}} + V\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right) \end{aligned} \qquad (39)$$

The wavefunctions estimated from the Schrödinger equation should be complex function with real and imaginary numbers in quantum mechanics even though we have thought that we live in the world, the physical values in which should be expressed by using only real numbers in the non-relativistic classical dynamics. This can be explained by our new theory as follows. The real time (space) components are the imaginary space (time) components. The components of the 4-dimensional spacetime axis are composed of the 3-dimensional real space vectors at the 3-dimensional real space axis and of the 1-dimensional imaginary time vectors at the 1-dimensional real space axis (Fig. 1). That is, we live in the 4-dimensional complex space world with 3-

dimensional real and 1-dimensional imaginary components. Therefore, we can consider that the real world we live in is the complex 4-dimensional spacetime world which is formed by the real visible 3-dimensional space components (so-called, space axis) and by the imaginary invisible 1-dimensional space component (so-called, time axis), at the real 4-dimensional space axis (Fig. 1 (c)). This is the reason why the wavefunctions estimated from the Schrödinger equation should be complex function with the real and imaginary numbers in quantum mechanics even though we have thought that we live in the world, the physical values in which should be expressed by using only real numbers in the non-relativistic classical mechanics.

We can conclude that, even in the Schrödinger equation, which has been considered to be non-relativistic equation, the concepts of the relationships between the space and time (spacetime) (Eq. (6)) in the special relativity have been included. This is the reason why the Schrödinger equation can be expressed by the complex numbers in quantum mechanics.

### 3.2 Linear Momentum

In quantum mechanics, the operator for the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ state by considering the special relativity can be usually expressed by the imaginary number as

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow -i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}} \rightarrow -i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}} = -i\frac{h}{2\pi}\nabla_{\boldsymbol{r}_{\mathrm{p}}}. \tag{40}$$

On the other hand, the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ state in the classical dynamics by considering the special relativity can be also expressed as

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}} = q_{\mathrm{g}}\frac{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}{\partial \boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}}}} = q_{\mathrm{g}}\frac{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}{\partial\left(i\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)} = -iq_{\mathrm{g}}\frac{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}, \tag{41}$$

and thus, from Eqs. (40) and (41),

$$-iq_{\mathrm{g}}\frac{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}} = -i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}, \tag{42}$$

On the other hand, the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ state in the non-relativistic classical dynamics, in which the space $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ and time $\boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}$ can exist independently, can be expressed by using the real numbers as follows,

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}} = q_{\mathrm{g}}\frac{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}}{\partial \boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}} = q_{\mathrm{g}}\frac{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}. \tag{43}$$

Therefore from Eqs. (42) and (43), the operator for the momentum defined in the non-relativistic classical dynamics $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ value can be expressed as

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}(\mathrm{Re}) = q_{\mathrm{g}}\frac{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Re}) \rightarrow \frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Re}). \tag{44}$$

Therefore, the operator for the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ state in the non-relativistic classical dynamics can be expressed by using the real numbers, as expected. Furthermore, it should be noted that if we consider that the real time component in our real world is the imaginary component in the 4-dimensional complex space world, as described above, the $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ value can be considered to be imaginary in the 4-dimensional complex world, where we really live,

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}(\mathrm{Im}) \rightarrow -i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Im}), \tag{45}$$

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}(\mathrm{Im}) = -i\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}(\mathrm{Re}). \tag{46}$$

This is the reason why the operator for the momentum state, which should be real numbers in the world of the non-relativistic classical dynamics ($\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$) (independent real 3-dimensional space components and real 1-dimensional time components), where we have thought to live, can be expressed as the imaginary number ($\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$) in the world in quantum mechanics in the 4-dimensional complex space world, where we really live. In summary, the linear momentum is the imaginary component in the 3-dimensional real space axis and the 1-dimensional real space axis in the 4-dimensional complex spacetime world in quantum mechanics by considering the special relativity, on the other hand, is the real component in the 3-dimensional real space axis and the 1-dimensional real time axis in the non-relativistic classical dynamics.

### 3.3 Energy

In quantum mechanics, the operator for the energy $\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}}}$ state by considering the special relativity can be usually expressed by the imaginary number as,

$$\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}}} \rightarrow i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}} \rightarrow i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}, \tag{47}$$

$$\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}} = \frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}}}{c} \rightarrow \frac{i}{c}\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_\mathrm{p}}, \tag{48}$$

since $\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}} = i\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}$,

$$\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}} \rightarrow \frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}}. \tag{49}$$

On the other hand, the momentum $\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}}$ and energy $\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}}$ states in the classical dynamics by considering the special relativity can be also expressed as

$$\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}} = q_\mathrm{g}\frac{\partial\left(c\boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}\right)}{\partial \boldsymbol{t}_{\boldsymbol{t}_\mathrm{p}}} = \frac{q_\mathrm{g}c}{i}\frac{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}}{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}} = -iq_\mathrm{g}c, \tag{50}$$

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}} = c\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}} = -iq_\mathrm{g}c^2. \tag{51}$$

From Eqs. (48) and (50),

$$-iq_\mathrm{g}c = \frac{i}{c}\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}}. \tag{52}$$

Furthermore, the energy $\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}$ state in the classical dynamics by considering the special relativity can be expressed as

$$\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}} = \frac{\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}}}{i} = -q_\mathrm{g}c^2. \tag{53}$$

The momentum $\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}$ and energy $\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}$ state in the non-relativistic classical dynamics, in which the space $\boldsymbol{r}_{\boldsymbol{r}_\mathrm{p},\mathrm{CL}}$ and time $\boldsymbol{t}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}$ can exist independently, can be expressed by using the real numbers as follows,

$$\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}} = q_\mathrm{g}\frac{\partial\left(c\boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}\right)}{\partial \boldsymbol{t}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}} = q_\mathrm{g}\frac{\partial\left(c\boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}\right)}{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}} = q_\mathrm{g}c, \tag{54}$$

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}} = c\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}} = q_\mathrm{g}c^2. \tag{55}$$

Therefore, the operator for the momentum defined in the non-relativistic classical dynamics $\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}$ value can be expressed as

$$\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}(\mathrm{Re}) \rightarrow -\frac{1}{c}\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}}(\mathrm{Re})$$

$$\rightarrow -\frac{1}{c}\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}}(\mathrm{Re}), \tag{56}$$

$$\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p}}(\mathrm{Im}) = -i\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}(\mathrm{Re}), \tag{57}$$

and thus the energy defined in the non-relativistic classical dynamics $\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}$ value can be expressed as

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}(\mathrm{Re}) = c\boldsymbol{p}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}(\mathrm{Re})$$

$$\rightarrow -\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}}(\mathrm{Re}) \rightarrow -\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}}(\mathrm{Re}), \tag{58}$$

$$\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p}}(\mathrm{Im}) = -i\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}(\mathrm{Re}). \tag{59}$$

Therefore, the operator for the energy $\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}$ state in the non-relativistic classical dynamics can be expressed by using the real numbers, as expected. Furthermore, it should be noted that if we consider that the real time component in our real world is the imaginary component in the 4-dimensional complex space world, as described above, the $\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}$ value can be considered to be real in the 4-dimensional complex world, where we actually live,

$$\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}(\mathrm{Re}) \rightarrow \frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_\mathrm{p}}}(\mathrm{Re}), \tag{60}$$

$$\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}(\mathrm{Re}) = -\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}(\mathrm{Re}). \tag{61}$$

This is the reason why the operator for the energy state, which should be real numbers in the world of the non-relativistic classical dynamics ($\boldsymbol{E}_{\boldsymbol{t}_\mathrm{p},\mathrm{CL}}$) (independent real 3-dimensional space components and the real 1-dimensional time components), where we have thought to live, can also be expressed as the real number ($\boldsymbol{E}_{\boldsymbol{r}_\mathrm{p}}$) in the world in the quantum mechanics in the 4-dimensional complex spacetime world, where we really live. The energy is the real component in the 3-dimensional real space axis and the 1-dimensional real space axis in the 4-dimensional complex spacetime world in quantum mechanics, and furthermore, is also the real component in the independent 3-dimensional real space axis and the 1-dimensional real time axis in the non-relativistic classical dynamics.

3.4 Linear Momentum and Energy in the Relativistic and the Non-Relativistic Mechanics

In quantum mechanics in which the special relativity is considered, the time and space are equivalent. Furthermore, the real time (space) components are the imaginary space (time) components. That is, when we try to treat space and time equivalently, the complex numbers with the real and imaginary numbers can explicitly appear. This is the reason why the operators for the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ and energy $\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}}}$ in the conventional quantum mechanics can be expressed by imaginary numbers as (Fig. 4 (a)),

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}(\mathrm{Im}) \rightarrow -i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Im}), \qquad (62)$$

$$\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}}}(\mathrm{Im}) \rightarrow i\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Im}). \qquad (63)$$

On the other hand, in the non-relativistic classical mechanics, the time and space can be treated independently. That is, we can express any physical parameters as a function of the space and time by using only real numbers. This is the reason why we usually observe the momentum and energy as the real $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}\left(= q_{\mathrm{g}}\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}} / \partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ and $\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}\left(= q_{\mathrm{g}} c^2\right)$ values, respectively, even in the 4-dimensional complex spacetime world where we actually live, as shown below (Fig. 4 (b)),

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}} = \frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}, \qquad (64)$$

$$\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}} = -\frac{h}{2\pi}\frac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}. \qquad (65)$$

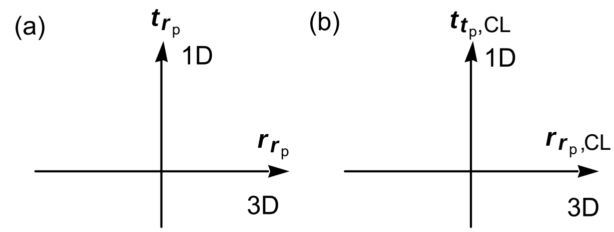
3.5 Wavefunction

Let us next look into the new interpretation of the wavefunction $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}, \boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}\right)$. The complete wavefunction has the form

$$\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}, \boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}\right) = \varphi(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}) e^{-i\boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}\boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}/(h/(2\pi))}. \qquad (66)$$

On the other hand, by considering Eqs. (61) and (66), the wavefunction $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}, \boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}\right)$ expressed by the parameters in the non-relativistic classical dynamics is the same with that $\Psi(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})^*$ expressed by the parameters in the quantum mechanics by considering the special relativity,

$$\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}, \boldsymbol{t}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}\right) = \Psi(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})^*, \qquad (67)$$




$$\begin{cases} \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}(\mathrm{Im}) \rightarrow -i\dfrac{h}{2\pi}\dfrac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Im}) \\ \boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}}}(\mathrm{Im}) \rightarrow i\dfrac{h}{2\pi}\dfrac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Im}) \end{cases} \quad \begin{cases} \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}(\mathrm{Re}) = \dfrac{h}{2\pi}\dfrac{\partial}{\partial \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Re}) \\ \boldsymbol{E}_{\boldsymbol{t}_{\mathrm{p}},\mathrm{CL}}(\mathrm{Re}) = -\dfrac{h}{2\pi}\dfrac{\partial}{\partial \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}}(\mathrm{Re}) \end{cases}$$

Fig. 4. Operators for the linear momentum and energy. (a) Quantum mechanics by considering the special relativity. (b) Non-relativistic classical dynamics.

where

$$\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right) = \varphi(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}) e^{-i\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}}}\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}/(h/(2\pi))}. \qquad (68)$$

It was suggested by Born in 1926 that $\Psi(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})^* \Psi(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})$ should be regarded as a probability density (Fig. 5 (a)). That is, according to the interpretation of Born, an electron particle moves rapidly and randomly, and the $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ is the abstract and virtual (physically, substantially meaningless) wave denoting the probability density of the existence of a particle at the position $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}$ at the time $\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}$. This is because the wavefunction $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ is expressed by the (virtual) complex numbers with the real and imaginary numbers even though all physical parameters have been believed to essentially be expressed by only real numbers in the non-relativistic classical dynamics. According to the Born's interpretation of the $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ value, we cannot precisely predict the movement (position and momentum) of a particle at each time, but can predict the probability of existence at the position $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}$ at each time $\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}$, and can predict the distribution (position) $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}$ of many particles at each time $\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}$, statistically. That is, the movement of each particle at each time can be predicted only statistically in the framework of the probability theory (Fig. 5 (a)).

On the other hand, according to our theory, it is rational to consider that the wavefunction $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ expressed by the (not virtual) complex numbers means the distribution of the physical components of a particle of the substantial concrete wave (not virtual and abstract wave)

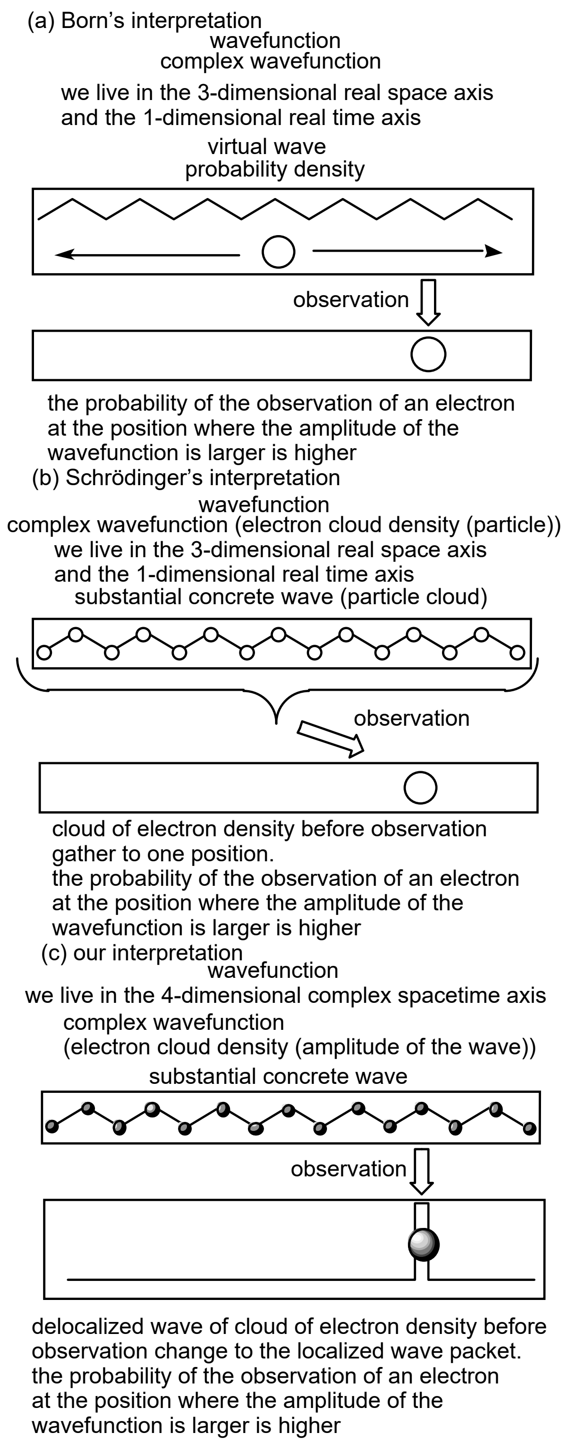



Fig. 5. Interpretation of the wavefunction. The opened and shaded circles indicate the particle and wave component, respectively. (a) Born's interpretation. (b) Schrödinger's interpretation. (c) Our interpretation.

since we really live in the 4-dimensional complex spacetime world (Fig. 5 (b), (c)). That is, the complex and imaginary numbers should not considered to be virtual but be substantial because we really live in the 4-dimensional complex spacetime world. Furthermore, it is rational to consider that the bosonic standing wave $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ before observation is the concrete substantial wave (cloud of electron) because the superimposition between various bosonic standing wave $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ forms the observable particles (or wave packets), which are considered to be concrete substantial physical quantity. The substantial physical component exists at various positions at the same time, at the 4-dimensional complex spacetime world, before observation, according to the amplitude of $\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ (Fig. 5 (b), (c)). But it should be noted that cloud of electron is considered to be formed by cloud of small particles in the Schrödinger's interpretation (Fig. 5 (b)), on the other hand, cloud of electron is considered to be substantial components (amplitude) of the substantial bosonic standing wave in our interpretation.

We should note that the particle (wave packet) is not formed by the moving of the cloud of electron particles from the various position in the bosonic standing wave to the central position $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{center}}$ of the wave packet in the 3-dimensional real space, as Schrödinger suggested (Fig. 5 (b)). On the other hand, according to our new theory, the fermionic traveling wave packet can be formed by superimposition between various bosonic standing waves with various momentum, as discussed in detail later (Figs. 5 (c), 6). Since the particle (wave packet) can be formed by the just superimposition of various waves with various $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\left(\sim \boldsymbol{k}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ values, this process can be realized during very short time (almost simultaneously). In principle, such transformation can be realized almost simultaneously (amplitude change at each position at the same time) in the 4-dimensional complex spacetime world.

### 3.6 The Uncertainty Principle

The momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ state can be expressed by the wave number $\boldsymbol{k}_{\boldsymbol{r}_{\mathrm{p}}}$ $[\mathrm{number}/\mathrm{m}]$ as follows,

$$\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}} = \frac{h}{2\pi}\boldsymbol{k}_{\boldsymbol{r}_{\mathrm{p}}}. \tag{69}$$

The uncertainty of the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ state can be expressed as

$$\left|\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\right| = \frac{h}{2\pi}\left|\Delta \boldsymbol{k}_{\boldsymbol{r}_{\mathrm{p}}}\right|. \tag{70}$$

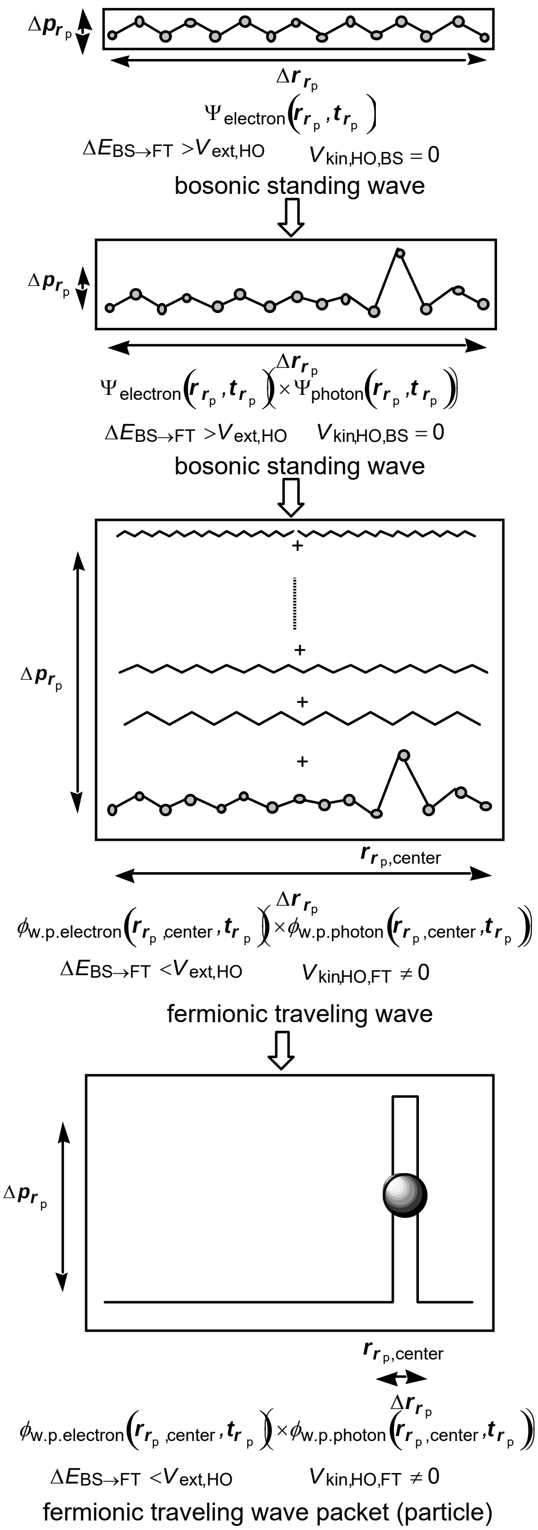



Fig. 6. Collapse of the wavefunction.

The Heisenberg's uncertainty principle between the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ state and the position $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ of a particle with respect to the direction of the $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ vector in the non-relativistic classical dynamics can be expressed as (Fig. 7 (a)),

$$\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}} \cdot \Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}} = \left|\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}\right|\left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}\right| \cos\theta = \left|\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}\right|\left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL},||}\right| \geq \frac{h}{2\pi}, \quad (71)$$

where $\theta$ denotes the internal angle between the $\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ and $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$, and the $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL},||}$ denotes the parallel component of the $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ with respect to the $\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$,

$$\left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL},\|}\right| = \left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}\right| \cos\theta. \quad (72)$$

That is, the uncertainty principle between position and linear momentum for the direction of the $\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ (i.e., $\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ and $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL},||}$) can be expressed by the scalar product between the $\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ and $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{CL}}$ with arbitrary direction.

On the other hand, the uncertainty principle between the momentum $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ state and the position $\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}$ of a particle with respect to the direction of the $\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ vector in the classical dynamics by considering the special relativity can be expressed as (Fig. 7 (a)),

$$\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}} \cdot \Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}} = \left|\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\right|\left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}\right| \cos\theta = \left|\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\right|\left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},||}\right| \geq \frac{h}{2\pi}, \quad (73)$$

where $\theta$ denotes the internal angle between the $\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$ and $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}$, and the $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},||}$ denotes the parallel component of the $\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}$ with respect to the $\Delta \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}$,

$$\left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\|}\right| = \left|\Delta \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}\right| \cos\theta. \quad (74)$$

On the other hand, according to the Bohr's quantization condition of the angular momentum, the angular momentum $\boldsymbol{l}_{\boldsymbol{r}_{\mathrm{p}}}$ can be defined as (Fig. 7 (b)),

$$\boldsymbol{l}_{\boldsymbol{r}_{\mathrm{p}}} = \boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}} \times \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}, \quad (75)$$

$$\left|\boldsymbol{l}_{\boldsymbol{r}_{\mathrm{p}}}\right| = \left|\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}} \times \boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\right| = \left|\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\right|\left|\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}\right| \sin\theta = \left|\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\right|\left|\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}},\perp}\right| = n\frac{h}{2\pi}, \quad (n = 1,2,3,\ldots), \quad (76)$$

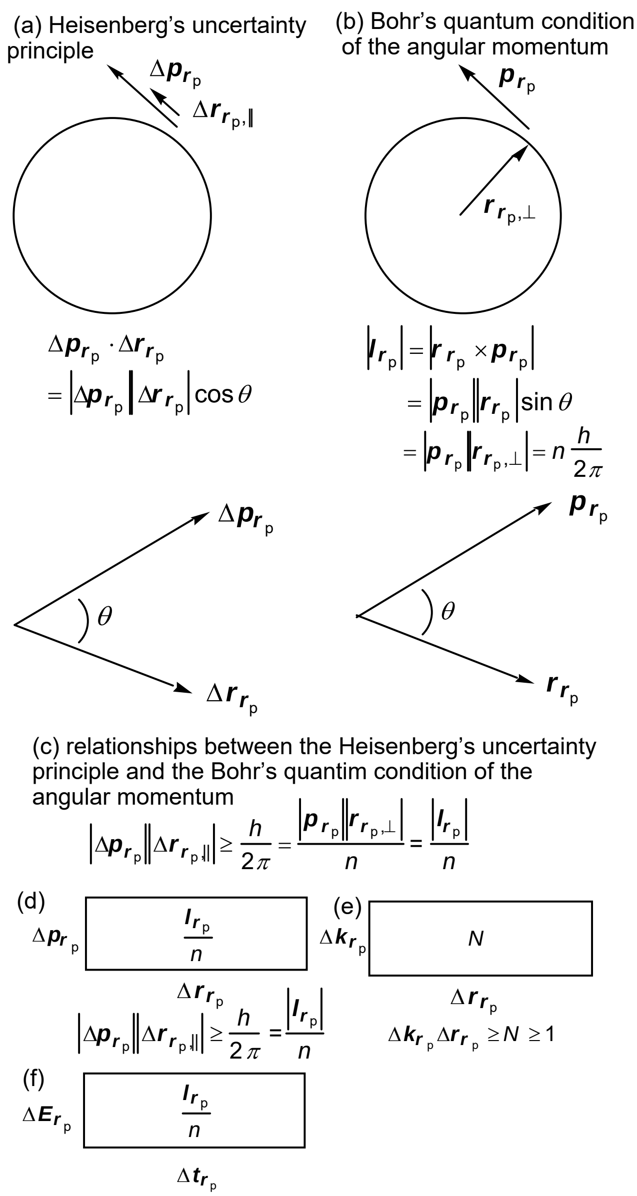



Fig. 7. Uncertainty principle and the quantum condition of the angular momentum. (a) Heisenberg's uncertainty principle. (b) Bohr's quantum condition of the angular momentum. (c) Relationships between the uncertainty principle and the quantum condition of the angular momentum. (d) Uncertainty between the position and linear momentum. (e) Uncertainty between the position and wave number. (f) Uncertainty between the time and energy.

where $\theta$ denotes the internal angle between the $p_{r_p}$ and $r_{r_p}$, and the $r_{r_p,\perp}$ denotes the perpendicular component of the $r_{r_p}$ with respect to the $p_{r_p}$,

$$|r_{r_p,\perp}| = |r_{r_p}| \sin\theta. \tag{77}$$

That is, the angular momentum $l_{r_p}$ between position and linear momentum for the direction perpendicular to the $p_{r_p}$ (i.e., $p_{r_p}$ and $r_{r_p,\perp}$) can be expressed by the vector product between the $p_{r_p}$ and $r_{r_p}$ with arbitrary direction.

We can see from Eqs. (73) and (76) that the direction of the space axis $\Delta r_{r_p,\parallel}$ considered in the uncertainty principle (scalar product) is perpendicular to that $r_{r_p,\perp}$ considered in the angular momentum (vector product). Therefore, the scalar product considered in the uncertainty principle would not be directly related to the definition of the vector product considered in the angular momentum. On the other hand, from Eqs. (73) and (76), the relationships between the uncertainty principle and the angular momentum can be expressed as (Fig. 7 (c)),

$$|\Delta p_{r_p}| |\Delta r_{r_p,\parallel}| \geq \frac{h}{2\pi} = \frac{|p_{r_p}| |r_{r_p,\perp}|}{n} = \frac{|l_{r_p}|}{n}. \tag{78}$$

Therefore, the uncertainty principle is related to the possible minimum angular momentum for formation of a wave packet. Furthermore, the Heisenberg's uncertainty principle is closely related to the Bohr's quantization condition of the angular momentum (Fig. 7 (c)).

It should be noted that since the $r_{r_p}$ and $p_{r_p}$ values are real and imaginary terms, respectively, the $\Delta p_{r_p} \cdot \Delta r_{r_p}$, $r_{r_p} \times p_{r_p}$, and $l_{r_p}$ terms are imaginary, and thus we should consider the absolute value for each term. From Eqs. (70) and (73), the uncertainty principle between the wave number $k_{r_p}$ and the position $r_{r_p}$ of a particle can be expressed as

$$|\Delta k_{r_p}| |\Delta r_{r_p,\parallel}| \geq 1. \tag{79}$$

The uncertainty principle between the energy $E_{t_p,\mathrm{CL}}$ state and the time $t_{t_p,\mathrm{CL}}$ (the direction of which is parallel to the $E_{t_p,\mathrm{CL}}$ vector (i.e., $t_{t_p,\mathrm{CL},\parallel}$)) of a particle in the non-relativistic classical dynamics can be expressed as

$$|\Delta E_{t_p,\mathrm{CL}}| |\Delta t_{t_p,\mathrm{CL},\parallel}| \geq \frac{h}{2\pi}. \tag{80}$$

Furthermore, the uncertainty principle between the energy $E_{r_p}$ and time $t_{r_p}$ (the direction of which is parallel to the $E_{r_p}$ vector (i.e., $t_{r_p,\parallel}$)) of a particle in the quantum mechanics by considering the special relativity can be expressed as

$$\left|\Delta E_{r_p}\right|\left|\Delta t_{r_p,\parallel}\right| = c\left|\Delta p_{r_p}\right|\left|\Delta t_{r_p,\parallel}\right| = \left|\Delta p_{r_p}\right|\left|c\Delta t_{r_p,\parallel}\right|$$
$$= \left|\Delta p_{r_p}\right|\left|\Delta r_{r_p,\parallel}\right| \ge \frac{h}{2\pi} = \frac{\left|p_{r_p}\right|\left|r_{r_p,\perp}\right|}{n} = \frac{\left|l_{r_p}\right|}{n}. \qquad (81)$$

The uncertainty principle originates from the fact that the $\left|\Delta k_{r_p}\right|\left|\Delta r_{r_p,\parallel}\right|$ value for the minimum one wave packet should be larger than 1, and that the $\left|\Delta p_{r_p}\right|\left|\Delta r_{r_p,\parallel}\right|$ and $\left|\Delta E_{r_p}\right|\left|\Delta t_{r_p,\parallel}\right|$ values for the minimum one wave packet should be larger than $h/(2\pi)$ (Fig. 7 (d)–(f)). That is, the uncertainty principle is not related to the break of law of the causality, but related to the formation of the necessary minimum size of one wave packet, the minimum angular momentum ($\left|p_{r_p}\right|\left|r_{r_p,\perp}\right| = \left|l_{r_p}\right|/n$) of which should be larger than $h/(2\pi)$ value (Fig. 7). Furthermore, the property of wave packet ranges between the particle ($\left|\Delta k_{r_p}\right| \to \infty$, $\left|\Delta r_{r_p,\parallel}\right| \to 0$) and wave ($\left|\Delta r_{r_p,\parallel}\right| \to \infty$, $\left|\Delta k_{r_p}\right| \to 0$) (Figs. 6, 8). The $h/(2\pi)$ value is the possible minimum angular momentum for the minimum size of one wave packet (one particle). In other words, a particle is the limit state of a wave packet with $\left|\Delta k_{r_p}\right| \to \infty$ and $\left|\Delta r_{r_p,\parallel}\right| \to 0$, and a wave is the limit state of a wave packet with $\left|\Delta r_{r_p,\parallel}\right| \to \infty$ and $\left|\Delta k_{r_p}\right| \to 0$ (Figs. 6, 8 (b), (c)). The figure and size of the wave packet depends on the environment around the wave (Figs. 6, 8).

The uncertainty principle obeys the principle of the causality under the conservation law of the angular momentum ($\left|l_{r_p}\right|/n\left(= h/(2\pi) = \left|\Delta r_{r_p,\parallel}\right|\left|\Delta p_{r_p}\right| = \left|\Delta t_{r_p,\parallel}\right|\left|\Delta E_{r_p}\right|\right)$). If there were not the uncertainty principle, and the $h/(2\pi)$ value were 0, the particles and wave packets with the finite size would not be formed, and there would be no materials in the universe.

3.7 The Stationary and Transition States and the Energy Conservation Law

The wavefunction for the stationary state with energy level $\varepsilon_{r_p}$ in the 4-dimensional complex spacetime world, where we actually live, can be expressed as

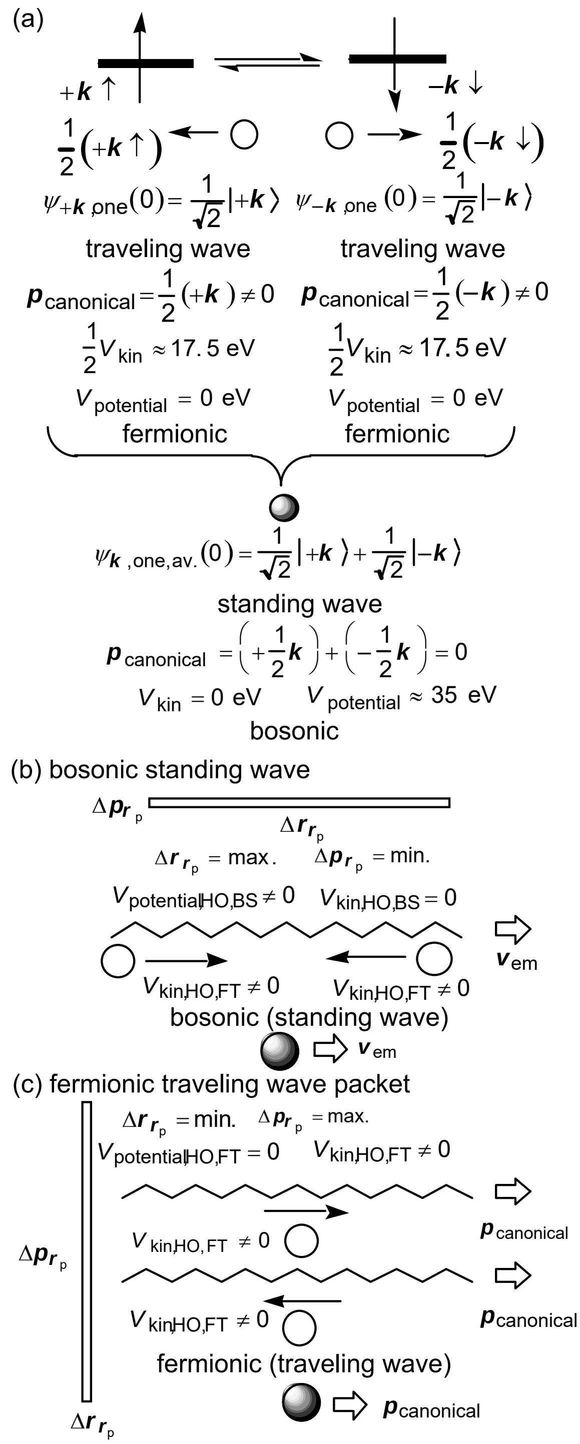



Fig. 8. (a) Relationships between the bosonic standing wave and the fermionic traveling wave. (b) Moving of the bosonic standing wave. (c) Moving of the fermionic traveling wave.

$$\Psi\left(r_{r_p}, t_{r_p}\right) = \varphi(r_{r_p})e^{-i\frac{\varepsilon_{r_p} t_{r_p}}{(h/(2\pi))}}$$

$$= \mathrm{Re}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right) + i\mathrm{Im}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right), \quad (82)$$

where

$$\mathrm{Re}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right) = \varphi(\boldsymbol{r}_{r_\mathrm{p}})\cos\left(\frac{\varepsilon_{r_\mathrm{p}}\boldsymbol{t}_{r_\mathrm{p}}}{(h/(2\pi))}\right), \quad (83)$$

$$\mathrm{Im}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right) = -\varphi(\boldsymbol{r}_{r_\mathrm{p}})\sin\left(\frac{\varepsilon_{r_\mathrm{p}}\boldsymbol{t}_{r_\mathrm{p}}}{(h/(2\pi))}\right). \quad (84)$$

The $\varphi\left(\boldsymbol{r}_{r_\mathrm{p}}\right)$ and $\exp\left(-i\varepsilon_{r_\mathrm{p}}\boldsymbol{t}_{r_\mathrm{p}}/(h/(2\pi))\right)$ terms are related to the space and time, respectively, and furthermore, are related to the particle and wave characteristics, respectively.

The wavefunction for the transition state with various energy levels $\varepsilon_{r_\mathrm{p},n}$ can be expressed as

$$\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right) = \sum_n c_n\varphi_n(\boldsymbol{r}_{r_\mathrm{p}})e^{-i\frac{\varepsilon_{r_\mathrm{p}}\boldsymbol{t}_{r_\mathrm{p}}}{(h/(2\pi))}}$$
$$= \mathrm{Re}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right) + i\mathrm{Im}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right), \quad (85)$$

where

$$\mathrm{Re}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right) = \sum_n c_n\varphi_n(\boldsymbol{r}_{r_\mathrm{p}})\cos\left(\frac{\varepsilon_{r_\mathrm{p},n}\boldsymbol{t}_{r_\mathrm{p}}}{(h/(2\pi))}\right), \quad (86)$$

$$\mathrm{Im}\left(\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right) = -\sum_n c_n\varphi_n(\boldsymbol{r}_{r_\mathrm{p}})\sin\left(\frac{\varepsilon_{r_\mathrm{p},n}\boldsymbol{t}_{r_\mathrm{p}}}{(h/(2\pi))}\right), \quad (87)$$

and furthermore, the probability density of the existence of an electron at $\boldsymbol{r}_{r_\mathrm{p}}$ and $\boldsymbol{t}_{r_\mathrm{p}}$ in the Born's interpretation (Fig. 5 (a)) and the density of substantial physical quantities of an electron in our new interpretation (Fig. 5 (c)) can be expressed as

$$\left|\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right|^2 = \Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)^* \Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)$$
$$= \sum_n\sum_l c_n^* c_l\varphi_n^*(\boldsymbol{r}_{r_\mathrm{p}})\varphi_l(\boldsymbol{r}_{r_\mathrm{p}})e^{-i\omega_{n,l}\boldsymbol{t}_{r_\mathrm{p}}}$$
$$= \mathrm{Re}\left(\left|\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right|^2\right) + i\mathrm{Im}\left(\left|\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right|^2\right), \quad (88)$$

where

$$\omega_{n,l} = \frac{\varepsilon_{r_\mathrm{p},n} - \varepsilon_{r_\mathrm{p},l}}{(h/(2\pi))}, \quad (89)$$

$$\mathrm{Re}\left(\left|\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right|^2\right)$$
$$= \sum_n\sum_l c_n^* c_l\varphi_n^*(\boldsymbol{r}_{r_\mathrm{p}})\varphi_l(\boldsymbol{r}_{r_\mathrm{p}})\cos\omega_{n,l}\boldsymbol{t}_{r_\mathrm{p}}, \quad (90)$$

$$\mathrm{Im}\left(\left|\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right|^2\right)$$
$$= -\sum_n\sum_l c_n^* c_l\varphi_n^*(\boldsymbol{r}_{r_\mathrm{p}})\varphi_l(\boldsymbol{r}_{r_\mathrm{p}})\sin\omega_{n,l}\boldsymbol{t}_{r_\mathrm{p}}. \quad (91)$$

Energy conservation law cannot necessarily be applied. The total energy ( $\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}$ ) can be expressed by the summation of the energies originating from the real components ( $\mathrm{Re}\left(\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\right)$ ) and the imaginary components ( $\mathrm{Im}\left(\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\right)$). The only $\mathrm{Re}\left(\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\right)$ value is observable energy and the $\mathrm{Im}\left(\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\right)$ value is not observable in our real 4-dimensional complex spacetime world (the 3-dimensional real space axis and the real 1-dimensional space axis).

The $\left|\varphi\left(\boldsymbol{r}_{r_\mathrm{p}}\right)\right|$ and $\exp\left(-i\omega_{n,l}\boldsymbol{t}_{r_\mathrm{p}}\right)$ terms are related to the properties of particles ( $\Delta\boldsymbol{r}_{r_\mathrm{p}}$ ) and waves ( $\Delta\boldsymbol{k}_{r_\mathrm{p}}$ ), respectively,

$$\left|\varphi\left(\boldsymbol{r}_{r_\mathrm{p}}\right)\right| \sim \Delta\boldsymbol{r}_{r_\mathrm{p}}, \quad (92)$$

$$e^{-i\omega_{n,l}\boldsymbol{t}_{r_\mathrm{p}}} \sim \Delta\boldsymbol{p}_{r_\mathrm{p}}\left(\sim \Delta\boldsymbol{k}_{r_\mathrm{p}}\right) \quad (93)$$

The total energy $\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}$ for the 4-dimensional complex spacetime world can be expressed as follows,

$$\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}} = \mathrm{Re}\left(\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\right) + \mathrm{Im}\left(\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\right) \quad (94)$$

where

$$\mathrm{Re}\left(\boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\right) = \boldsymbol{E}_{r_\mathrm{p},\mathrm{total}} \times \mathrm{Re}\left(\left|\Psi\left(\boldsymbol{r}_{r_\mathrm{p}}, \boldsymbol{t}_{r_\mathrm{p}}\right)\right|^2\right)$$
$$= \boldsymbol{E}_{r_\mathrm{p},\mathrm{total}}\sum_n\sum_l c_n^* c_l\varphi_n^*(\boldsymbol{r}_{r_\mathrm{p}})\varphi_l(\boldsymbol{r}_{r_\mathrm{p}})\cos\omega_{n,l}\boldsymbol{t}_{r_\mathrm{p}},$$

(95)

$$\mathrm{Im}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right) = \boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}} \times \mathrm{Im}\left(\left|\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)\right|^2\right)$$
$$= -\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}} \sum_n \sum_l c_n{}^* c_l \varphi_n{}^*(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}) \varphi_l(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}) \sin \omega_{n,l} \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}. \quad (96)$$

In the stationary state ($n = l$), the $\omega_{n,l}$ value becomes 0,

$$\omega_{n,n} = \frac{\varepsilon_{\boldsymbol{r}_{\mathrm{p}},n} - \varepsilon_{\boldsymbol{r}_{\mathrm{p}},n}}{(h/(2\pi))} = 0. \quad (97)$$

Therefore,

$$\mathrm{Im}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right) = 0, \quad (98)$$

$$\left|\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)\right|^2 = \mathrm{Re}\left(\left|\Psi\left(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}, \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)\right|^2\right) = \mathrm{const}. \quad (99)$$

$$\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}} = \mathrm{Re}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right) = \mathrm{const}. \quad (100)$$

The stationary state is stable and can exist for a long time ($\Delta \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow \infty$). Therefore, according to the uncertainty principle, the energy can be conserved at this stable state ($\Delta \boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow 0$). On the other hand, the $\mathrm{Re}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right)$ value is observable. In order to be in the stationary state ($\Delta \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow \infty$), the angular velocity $\omega_{n,l}$ in $\exp\left(i\omega_{n,l}\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$ should be 0. Therefore, the observable energy $\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{observe}}$ value becomes the $\mathrm{Re}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right)$ value, which is constant ($\Delta \boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow 0$). Therefore, the energy conservation law can be applied at the stationary state for a long time (Fig. 9 (a)),

$$\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{observe}} = \boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}} = \mathrm{Re}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right) = \mathrm{const}. \quad (101)$$

On the other hand, the transition state is very unstable and can exist only for very short time ($\Delta \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow 0$). Therefore, according to the uncertainty principle, the energy cannot be conserved at this unstable state ($\Delta \boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow \infty$) (Fig. 9 (b)). In the transition state ($\Delta \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow 0$), the angular velocity $\omega_{n,l}$ in $\exp\left(i\omega_{n,l}\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}\right)$

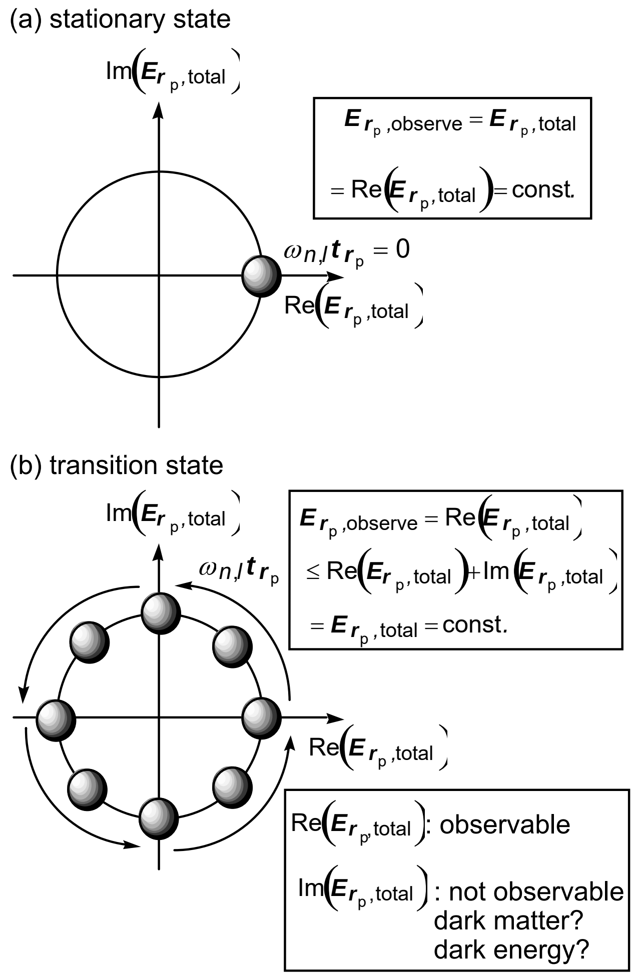



Fig. 9. Complex plane for energy. (a) Stationary state. (b) Transition state.

do not have to be 0. Therefore, the observable energy $\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{observe}}$ value becomes as

$$\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{observe}}$$
$$= \mathrm{Re}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right)$$
$$= \boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}} \sum_n \sum_l c_n{}^* c_l \varphi_n{}^*(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}) \varphi_l(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}) \cos \omega_{n,l} \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}$$
$$\leq \mathrm{Re}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right) + \mathrm{Im}\left(\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}}\right)$$
$$= \boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{total}} = \mathrm{const}. \quad (102)$$

Therefore, the $\boldsymbol{E}_{\boldsymbol{r}_{\mathrm{p}},\mathrm{observe}}$ value is not constant, and can change with an increase in time (Fig. 9 (b)). Therefore, the energy conversion law cannot be applied at the transition state for very short time ($\Delta \boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}} \rightarrow 0$) (Fig. 9 (b)). This may be related to the dark energy and dark matter.

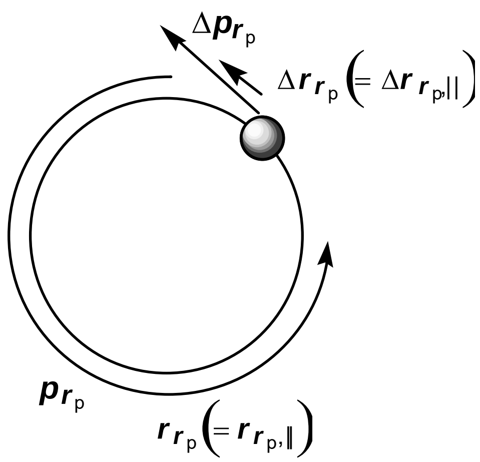



Fig. 10. Relationships between the position and the linear momentum.

## 4. Theoretical Background for the Origin of the Faraday's Electromagnetic Induction Law

In this article, we consider the 1-dimensional circular molecular systems for mathematical simplicity (Fig. 10). That is, we consider the case where the direction of the position is parallel to the linear momentum ($\Delta \boldsymbol{p}_{r_p}$ and $\Delta \boldsymbol{r}_{r_{p,||}}$) even though we denote this position as $\Delta \boldsymbol{r}_{r_p}$ instead of $\Delta \boldsymbol{r}_{r_{p,||}}$ (i.e., $\Delta \boldsymbol{r}_{r_p} || \Delta \boldsymbol{p}_{r_p}$) (Fig. 10). We can easily apply this discussion to the case in the solids.

The wavefunction for an electron $\left|\boldsymbol{k}_{\rm one}(T)\left((B_{\rm out},B_{\rm in});(E_{\rm out},E_{\rm in});B_{\rm one};I_{\rm one}\right)\right\rangle$ occupying the HOCO at the neutral material at any case (for wave, wave packet, and particle) can be expressed as (Figs. 8, 11),

$$\begin{aligned}
&\left|\boldsymbol{k}_{\rm one}(T)\left((B_{\rm out},B_{\rm in});(E_{\rm out},E_{\rm in});B_{\rm one};I_{\rm one}\right)\right\rangle \\
&\approx \sum_{l}^{\rm occupied} a_l\left|\boldsymbol{k}_l(T)\left((B_{\rm out},B_{\rm in});(E_{\rm out},E_{\rm in});B_l;I_l\right)\right\rangle \\
&\times\Theta\left(\frac{\Delta \boldsymbol{r}_{r_p}}{2}-\left|\boldsymbol{r}_{r_p}-\boldsymbol{r}_{r_p,\rm center}\right|\right)\Theta\left(\left|\Delta \boldsymbol{r}_{r_p}\right|\left|\Delta \boldsymbol{p}_{r_p}\right|-\frac{h}{2\pi}\right) \\
&\approx \sum_{l} a_l\left|\boldsymbol{k}_l(T)\left((B_{\rm out},B_{\rm in});(E_{\rm out},E_{\rm in});B_l;I_l\right)\right\rangle \\
&\times\Theta\left(V_{\rm excited}-(\varepsilon_l-\varepsilon_{\rm HO})\right)\Theta\left(\frac{\Delta \boldsymbol{r}_{r_p}}{2}-\left|\boldsymbol{r}_{r_p}-\boldsymbol{r}_{r_p,\rm center}\right|\right) \\
&\times\Theta\left(\left|\Delta \boldsymbol{r}_{r_p}\right|\left|\Delta \boldsymbol{p}_{r_p}\right|-\frac{h}{2\pi}\right), \qquad (103)
\end{aligned}$$

$$\begin{aligned}
V_{\rm excited} &= V_{\rm kin,HO,FT}\Theta\left(V_{\rm ext,HO}-\Delta E_{\rm BS\to FT}\right) \\
&+V_{\rm env,HO}(T), \qquad (104)
\end{aligned}$$

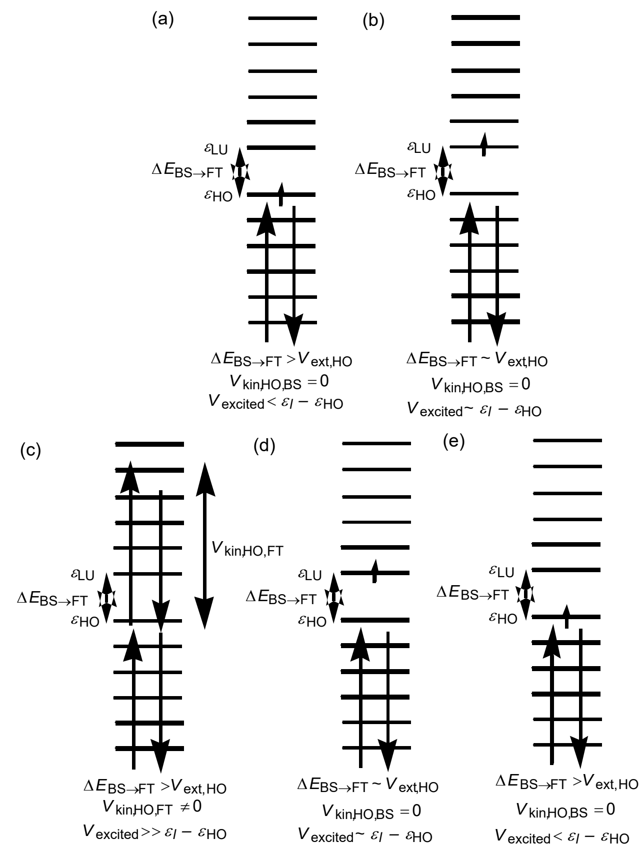



Fig. 11. Various electronic states in Faraday's electromagnetic induction law. (a) Bosonic standing wave state without inner magnetic field. (b) Transition from the bosonic standing wave state to the fermionic traveling wave state. (c) Fermionic traveling wave state. (d) Transition from the fermionic traveling wave state to the bosonic standing wave state. (e) Bosonic standing wave state with inner magnetic field.

$$\sum_{l}^{\rm occupied} a_l^2 = \sum_{l} a_l^2\Theta\left(V_{\rm excited}-(\varepsilon_l-\varepsilon_{\rm HO})\right)=1, \qquad (105)$$

$$\begin{aligned}
\Theta(x) &= 1 \ (x \geq 0) \\
&= 0 \ (x < 0), \qquad (106)
\end{aligned}$$

where the $V_{\rm excited}$ denotes the excited energy obtained by an electron, and can be expressed by the kinetic energy for the fermionic traveling waves ($V_{\rm kin,HO,FT}(\sim 35\ {\rm eV})$) occupying the HOCO and the energy change of electron occupying the HOCO by external energy at each environment around the material such as temperature at $T$ K ($V_{\rm env,HO}(T)$), the $\varepsilon_{\rm HO}$ denotes the energy level of the HOCO, the $\varepsilon_l$ denotes the energy level of the orbital $l$, the $V_{\rm ext,HO}$ denotes the energy such as external electric and magnetic field energy obtained by an electron which converts the bosonic standing wave state to the fermionic traveling wave state, the $\Delta E_{\mathrm{BS}\to\mathrm{FT}}$ denotes the necessary minimum energy for conversion from the bosonic standing wave state to the fermionic traveling wave state, the $\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}$ denotes the center position of the wave packet in a material, and the $\left|\boldsymbol{k}_l(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_l;I_l\right)\right\rangle$ denotes the wavefunction for an electron occupying the orbital $l$ in a material under the external applied field ($x_{\mathrm{in}}=B_{\mathrm{in}}$ or $E_{\mathrm{in}}$), as expressed as

$$\left|\boldsymbol{k}_l(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_l;I_l\right)\right\rangle = \sqrt{P_{\mathrm{ground},l}(T)}\left|\boldsymbol{k}_{l,\mathrm{ground},0}(x_{\mathrm{in}})\right\rangle + \sqrt{P_{\mathrm{excited},l}(T)}\left|\boldsymbol{k}_{l,\mathrm{excited},0}(x_{\mathrm{in}})\right\rangle, \quad (107)$$

where

$$\left|\boldsymbol{k}_{l,\mathrm{excited},0}(x_{\mathrm{in}})\right\rangle = c_{+\boldsymbol{k}_l\uparrow,0}(x_{\mathrm{in}})\left|+\boldsymbol{k}_l\uparrow\right\rangle + c_{-\boldsymbol{k}_l\downarrow,0}(x_{\mathrm{in}})\left|-\boldsymbol{k}_l\downarrow\right\rangle, \quad (108)$$

$$\left|\boldsymbol{k}_{l,\mathrm{ground},0}(x_{\mathrm{in}})\right\rangle = c_{-\boldsymbol{k}_l\uparrow,0}(x_{\mathrm{in}})\left|-\boldsymbol{k}_l\uparrow\right\rangle + c_{+\boldsymbol{k}_l\downarrow,0}(x_{\mathrm{in}})\left|+\boldsymbol{k}_l\downarrow\right\rangle, \quad (109)$$

$$P_{\mathrm{ground},l}(T)+P_{\mathrm{excited},l}(T)=1, \quad (110)$$

$$c^2_{+\boldsymbol{k}_l\downarrow,0}(x_{\mathrm{in}})+c^2_{-\boldsymbol{k}_l\uparrow,0}(x_{\mathrm{in}})=1, \quad (111)$$

$$c^2_{-\boldsymbol{k}_l\downarrow,0}(x_{\mathrm{in}})+c^2_{+\boldsymbol{k}_l\uparrow,0}(x_{\mathrm{in}})=1. \quad (112)$$

The each $\left|+\boldsymbol{k}_l\uparrow\right\rangle$, $\left|-\boldsymbol{k}_l\downarrow\right\rangle$, $\left|-\boldsymbol{k}_l\uparrow\right\rangle$, and $\left|+\boldsymbol{k}_l\downarrow\right\rangle$ state is the fermionic traveling wave component, and the pairs of the $\left|+\boldsymbol{k}_l\uparrow\right\rangle$ and $\left|-\boldsymbol{k}_l\downarrow\right\rangle$, and of the $\left|+\boldsymbol{k}_l\downarrow\right\rangle$ and $\left|-\boldsymbol{k}_l\uparrow\right\rangle$, are the bosonic standing wave components (Fig. 8).

In the case of $\Delta\boldsymbol{r}_{r_{\mathrm{p}}}\to\max.$ and $\Delta\boldsymbol{p}_{r_{\mathrm{p}}}\to\min.$, the $\left|\boldsymbol{k}_{\mathrm{one}}(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_{\mathrm{one}};I_{\mathrm{one}}\right)\right\rangle$ can be the bosonic standing wave $\Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)$, as usual, as expressed as (Figs. 8 (b), 11 (a)),

$$\Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)\to \lim_{\substack{\Delta\boldsymbol{r}_{r_{\mathrm{p}}}\to\max.\\ \Delta\boldsymbol{p}_{r_{\mathrm{p}}}\to\min.}}\left|\boldsymbol{k}_{\mathrm{one}}(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_{\mathrm{one}};I_{\mathrm{one}}\right)\right\rangle$$
$$\approx \sum_l a_l\left|\boldsymbol{k}_l(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_l;I_l\right)\right\rangle \times\Theta\left(V_{\mathrm{excited}}-(\varepsilon_l-\varepsilon_{\mathrm{HO}})\right)$$
$$=\sum_l^{\mathrm{occupied}} a_l\left|\boldsymbol{k}_l(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_l;I_l\right)\right\rangle, \quad (113)$$

$$\sum_l a_l^2\Theta\left(V_{\mathrm{excited}}-(\varepsilon_l-\varepsilon_{\mathrm{HO}})\right)=\sum_l^{\mathrm{occupied}} a_l^2=1. \quad (114)$$

In the case of the bosonic standing wave $\Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)$, any position $\boldsymbol{r}_{r_{\mathrm{p}}}$ in a material can be considered to be the center position $\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}$ in the wave,

$$\boldsymbol{r}_{r_{\mathrm{p}}}=\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}\ \left(\text{for any } \boldsymbol{r}_{r_{\mathrm{p}}}\right) \quad (115)$$

In the case of $\Delta\boldsymbol{r}_{r_{\mathrm{p}}}\to\min.$ and $\Delta\boldsymbol{p}_{r_{\mathrm{p}}}\to\max.$, the $\left|\boldsymbol{k}_{\mathrm{one}}(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_{\mathrm{one}};I_{\mathrm{one}}\right)\right\rangle$ can be the fermionic traveling wave packet $\phi_{\mathrm{w.p.electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)$ as expressed as (Figs. 8 (c), 11 (c)),

$$\phi_{\mathrm{w.p.electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)\to \lim_{\substack{\Delta\boldsymbol{r}_{r_{\mathrm{p}}}\to\min.\\ \Delta\boldsymbol{p}_{r_{\mathrm{p}}}\to\max.}}\left|\boldsymbol{k}_{\mathrm{one}}(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_{\mathrm{one}};I_{\mathrm{one}}\right)\right\rangle$$
$$\approx \lim_{\substack{\Delta\boldsymbol{r}_{r_{\mathrm{p}}}\to\min.\\ \Delta\boldsymbol{p}_{r_{\mathrm{p}}}\to\max.}}\sum_l a_l\left|\boldsymbol{k}_l(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_l;I_l\right)\right\rangle$$
$$\times\Theta\left(V_{\mathrm{excited}}-(\varepsilon_l-\varepsilon_{\mathrm{HO}})\right)\Theta\left(\frac{\Delta\boldsymbol{r}_{r_{\mathrm{p}}}}{2}-\left|\boldsymbol{r}_{r_{\mathrm{p}}}-\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}\right|\right)$$
$$\times\Theta\left(\left|\Delta\boldsymbol{r}_{r_{\mathrm{p}}}\right|\left|\Delta\boldsymbol{p}_{r_{\mathrm{p}}}\right|-\frac{h}{2\pi}\right), \quad (116)$$

$$\sum_l a_l^2\Theta\left(V_{\mathrm{excited}}-(\varepsilon_l-\varepsilon_{\mathrm{HO}})\right)=\sum_l^{\mathrm{occupied}} a_l^2=1. \quad (117)$$

In particular, in the limit state ($\Delta\boldsymbol{r}_{r_{\mathrm{p}}}\to 0$, $\Delta\boldsymbol{p}_{r_{\mathrm{p}}}\to\infty$), the $\left|\boldsymbol{k}_{\mathrm{one}}(T)\left((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_{\mathrm{one}};I_{\mathrm{one}}\right)\right\rangle$ becomes a particle (Figs. 8 (c), 11 (c)).

The kinetic energy $V_{\mathrm{kin,HO}}$ and the potential energy $V_{\mathrm{potential,HO}}$ for various wave states of an electron mainly occupying the HOCO can be expressed as,

$$V_{\mathrm{kin,HO}} = V_{\mathrm{kin,HO,FT}}\Theta\left(V_{\mathrm{ext,HO}} - \Delta E_{\mathrm{BS}\to\mathrm{FT}}\right) = V_{\mathrm{potential,HO,BS}}\Theta\left(V_{\mathrm{ext,HO}} - \Delta E_{\mathrm{BS}\to\mathrm{FT}}\right), \quad (118)$$

$$V_{\mathrm{potential,HO}} = V_{\mathrm{potential,HO,BS}}\Theta\left(\Delta E_{\mathrm{BS}\to\mathrm{FT}} - V_{\mathrm{ext,HO}}\right) = V_{\mathrm{kin,HO,FT}}\Theta\left(\Delta E_{\mathrm{BS}\to\mathrm{FT}} - V_{\mathrm{ext,HO}}\right), \quad (119)$$

$$V_{\mathrm{potential,HO,BS}} \leftrightarrow V_{\mathrm{kin,HO,FT}}, \quad (120)$$

where the $V_{\mathrm{potential,HO,BS}}$ value denotes the potential energy condensated into the vacuum in the bosonic standing wave state, the $V_{\mathrm{potential,HO,FT}}$ value denotes the potential energy in the fermionic standing wave state, and the $V_{\mathrm{kin,HO,BS}}$ and $V_{\mathrm{kin,HO,FT}}$ values denote the kinetic energies in the bosonic standing wave and the fermionic traveling wave states, respectively. The $V_{\mathrm{potential,HO,BS}}$ value of 35 eV originates from the fact that electron spreads and is located in the whole range of the material. That is, the $V_{\mathrm{potential,HO,BS}}$ value of 35 eV originates from the energy diffusion on whole of the material under consideration. The $V_{\mathrm{potential,HO,BS}}$ value of 35 eV is condensated into vacuum, and is not observed as a free energy. On the other hand, the $V_{\mathrm{kin,HO,BS}}$ value of 0 eV originates from the fact that the bosonic standing wave can be formed by two fermionic traveling waves $V_{\mathrm{kin,HO,FT}}$ with opposite momentum and spins. That is, the $V_{\mathrm{kin,HO,FT}}$ values for two fermionic traveling waves forming one bosonic standing wave can be canceled by each other (Fig. 8 (a), (b)). Furthermore, the $V_{\mathrm{potential,HO,BS}}$ value of 35 eV originates from such cancellations of the kinetic energy $V_{\mathrm{kin,HO,FT}}$ and is condensated into the vacuum ($V_{\mathrm{potential,HO,BS}} = V_{\mathrm{kin,HO,FT}} \approx 35$ eV).

The magnetic field ($B_l\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right)\left(= B_{\mathrm{in}}\right)$) at the condition of the external applied field $x_{\mathrm{out}}$ and the field felt by an electron $x_{\mathrm{in}}$ can be expressed as

$$B_l\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) = B_{\boldsymbol{k}_l\uparrow}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) - B_{\boldsymbol{k}_l\downarrow}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right), \quad (121)$$

where

$$B_{\boldsymbol{k}_l\uparrow}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) = P_{\mathrm{excited},l}(T)c^2_{+\boldsymbol{k}_l\uparrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right) + P_{\mathrm{ground},l}(T)c^2_{-\boldsymbol{k}_l\uparrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right), \quad (122)$$

$$B_{\boldsymbol{k}_l\downarrow}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) = P_{\mathrm{excited},l}(T)c^2_{-\boldsymbol{k}_l\downarrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right) + P_{\mathrm{ground},l}(T)c^2_{+\boldsymbol{k}_l\downarrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right). \quad (123)$$

The electric field ($I_l\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right)\left(= E_{\mathrm{in}}\right)$) at the condition of the external applied field $x_{\mathrm{out}}$ and the field felt by an electron $x_{\mathrm{in}}$ can be expressed as

$$I_l\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) = I_{+\boldsymbol{k}_l}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) - L_{\boldsymbol{k}_l}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right), \quad (124)$$

$$I_{+\boldsymbol{k}_l}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) = P_{\mathrm{excited},l}(T)c^2_{+\boldsymbol{k}_l\uparrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right) + P_{\mathrm{ground},l}(T)c^2_{+\boldsymbol{k}_l\downarrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right), \quad (125)$$

$$L_{\boldsymbol{k}_l}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) = P_{\mathrm{excited},l}(T)c^2_{-\boldsymbol{k}_l\downarrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right) + P_{\mathrm{ground},l}(T)c^2_{-\boldsymbol{k}_l\uparrow,x_{\mathrm{in}}}\left(x_{\mathrm{out}} - x_{\mathrm{in}}\right). \quad (126)$$

Let us look into the energy levels for various electronic states when the applied field increases from 0 to $x_{\mathrm{out}}$ at 0 K in superconductor, in which the orbital $l$ is partially occupied by an electron. The stabilization energy as a consequence of the electron–phonon interactions can be expressed as

$$E_{\mathrm{SC},l,\mathrm{electronic}}\left(x_{\mathrm{out}}, x_{\mathrm{in}}\right) - E_{\mathrm{NM},l,\mathrm{electronic}}(0,0) = -2V_{\mathrm{one},l}f_{l,0}\left(x_{\mathrm{in}}\right), \quad (127)$$

where the $-2V_{\mathrm{one},l}$ denotes the stabilization energy for the electron–phonon interactions between an electron occupying the orbital $l$ and the vibronically active modes [1–7] (Fig. 12).

The $f_{l,E_{\mathrm{in}}}(0)\left(= f_{l,0}\left(E_{\mathrm{in}}\right)\right)$ denotes the ratio of the bosonic property under the internal field $x_{\mathrm{in}}$ ($c_{+\boldsymbol{k}_l\downarrow,0}\left(x_{\mathrm{in}}\right) = c_{+\boldsymbol{k}_l\uparrow,0}\left(x_{\mathrm{in}}\right) = c_{+\boldsymbol{k}_l,0}\left(x_{\mathrm{in}}\right)$ and $c_{-\boldsymbol{k}_l\uparrow,0}\left(x_{\mathrm{in}}\right) = c_{-\boldsymbol{k}_l\downarrow,0}\left(x_{\mathrm{in}}\right) = c_{-\boldsymbol{k}_l,0}\left(x_{\mathrm{in}}\right)$), and can be estimated as

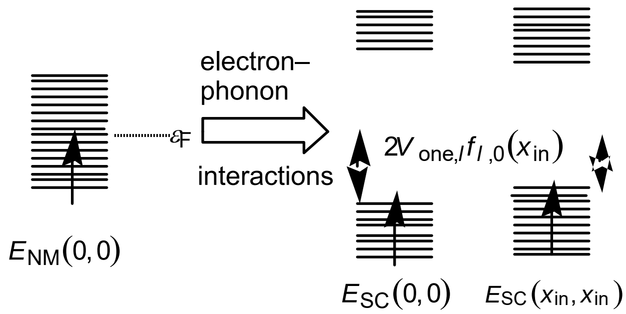



Fig. 12. Stabilization energy as a consequence of the electron–phonon interactions as a function of the external applied field.

$$f_{l,0}(x_{in}) = f_{l,x_{in}}(0)$$
$$= \frac{1}{2} + c_{-k_l,0}(x_{in})\sqrt{1 - c^2_{-k_l,0}(x_{in})}. \qquad (128)$$

The $f_{l,B_{in}}(0)$ ( $= f_{l,0}(B_{in})$ ) denotes the ratio of the bosonic property under the internal field $x_{in}$ ( $c_{+k_l\uparrow,0}(x_{in}) = c_{-k_l\uparrow,0}(x_{in}) = c_{k_l\uparrow,0}(x_{in})$ and $c_{+k_l\downarrow,0}(x_{in}) = c_{-k_l\downarrow,0}(x_{in}) = c_{k_l\downarrow,0}(x_{in})$ ), and can be estimated as

$$f_{l,0}(x_{in}) = f_{l,x_{in}}(0)$$
$$= \frac{1}{2} + c_{k_l\downarrow,0}(x_{in})\sqrt{1 - c^2_{k_l\downarrow,0}(x_{in})}. \qquad (129)$$

**5. New Interpretation of the Faraday's Electromagnetic Induction Law in the Normal Metallic States**

5.1 The Bosonic Standing Wave with No Electrical Current

Let us next apply the Higgs mechanism to the Faraday's electromagnetic induction law in the normal metallic states. Let us next consider the superconductor, the critical magnetic field of which is $B_c$. Below $T_c$, the bosonic Cooper pairs are in the superconducting states. We consider the case where the HOCO is partially occupied by an electron, and even the LUCO with energy level $\varepsilon_{LU}$ is not occupied by an electron ( $0 < V_{excited} < \varepsilon_{LU} - \varepsilon_{HO}$ ). That is, we consider an electron which occupies the only HOCO (Fig. 11 (a)). Therefore, we consider the normal metallic states in the superconductor material.

We consider that the magnetic field is quantized by $\Delta B_{unit}(= B_c / n_c)$. The $n_c$ value is very large and the quantization value of $B_c / n_c$ is very small ( $B_c / n_c \approx 0$ ) (Figs. 13, 14). That is, the $j$th quantized magnetic field

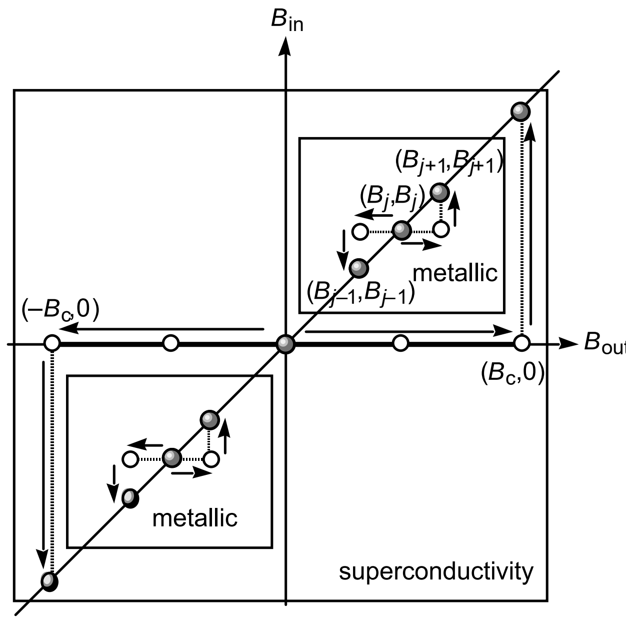



Fig. 13. $B_{out}$ versus $B_{in}$ in the normal metallic and superconducting states.

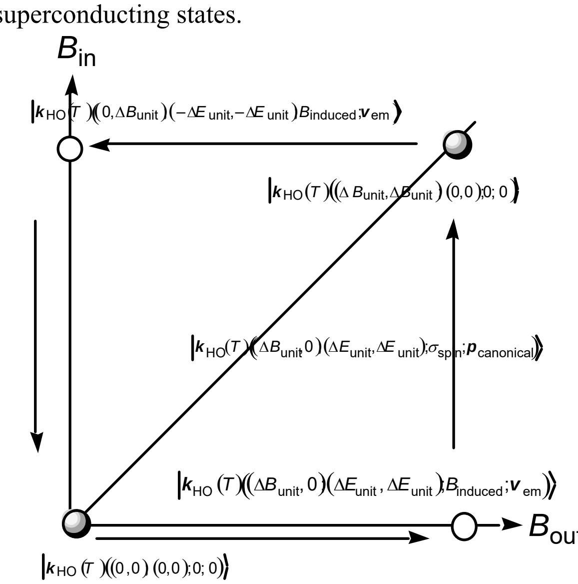



Fig. 14. The $B_{in}$ versus $B_{out}$ between $\gamma$=0 and $\gamma$= 1.

$B_j$ with respect to the zero magnetic field can be defined as

$$B_j = j\Delta B_{unit}. \qquad (130)$$

The ratio of the bosonic property for an electron occupying the orbital $l$ under the internal magnetic field $B_{excited}$ with respect to the ground state for the magnetic field $B_\gamma$ ( $B_{in} = B_\gamma + B_{excited}$ ) can be denoted as $f_{HO,B_\gamma}(B_{excited})$. In particular, the ratio of the bosonic property under the internal magnetic field $B_{\mathrm{in}}$ with respect to the ground state for the zero magnetic field can be denoted as $f_{\mathrm{HO},0}(B_{\mathrm{in}})$. We define the electronic $\left|\boldsymbol{k}_{\mathrm{HO}}(T)\left((B_{\mathrm{out}}, B_{\mathrm{in}}); (E_{\mathrm{out}}, E_{\mathrm{in}}); B_{\mathrm{HO}}; I_{\mathrm{HO}}\right)\right\rangle$ state, where the $E_{\mathrm{out}}$ denotes the induced electric field applied to the specimen, the $E_{\mathrm{in}}$ the induced electric field felt by the electron, the $B_{\mathrm{HO}}$ the induced magnetic moment from the electron (the induced magnetic field $B_{\mathrm{induced,HO}}$ or the change of the spin magnetic moment of an electron $\sigma_{\mathrm{spin,HO}}$ *from the each ground state*), and the $I_{\mathrm{HO}}$ the induced electric moment of an electron (canonical electric momentum $\boldsymbol{p}_{\mathrm{canonical,HO}}$ or the electric momentum of an electron $\boldsymbol{v}_{\mathrm{em,HO}}$).

In this article, the $\Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)$ denotes the independent bosonic standing electronic wave, the $\Psi_{\mathrm{photon}}\left(\boldsymbol{r}_{r_{\mathrm{p}}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)$ denotes the bosonic photonic wave, the $\Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right) \times \Psi_{\mathrm{photon}}\left(\boldsymbol{r}_{r_{\mathrm{p}}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)$ denotes the bosonic standing electronic wave interacting with the bosonic photonic wave, and the $\Psi_{\mathrm{photon}}\left(\boldsymbol{r}_{r_{\mathrm{p}}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right) \times \Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)$ denotes the bosonic photonic wave interacting with the bosonic standing electronic wave, at the position $\boldsymbol{r}_{r_{\mathrm{p}}}$ at the time $\boldsymbol{t}_{r_{\mathrm{p}}}$. The $\phi_{\mathrm{w.p.electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)$ denotes the fermionic traveling electronic wave packet, the $\phi_{\mathrm{w.p.photon}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)$ denotes the bosonic photonic wave packet, the $\phi_{\mathrm{w.p.electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)\left(\times \phi_{\mathrm{w.p.photon}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)\right)$ denotes the fermionic traveling electronic wave packet interacting with the bosonic photonic wave packet, and the $\phi_{\mathrm{w.p.photon}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)\left(\times \phi_{\mathrm{w.p.electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}, \boldsymbol{t}_{r_{\mathrm{p}}}\right)\right)$ denotes the bosonic photonic wave packet interacting with the fermionic traveling electronic wave packet, the center position of which are $\boldsymbol{r}_{r_{\mathrm{p}},\mathrm{center}}$ at the time $\boldsymbol{t}_{r_{\mathrm{p}}}$.

The $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)\left((0,0); (0,0); 0; 0\right)\right\rangle$ state is the ground bosonic standing wave which expels the external applied magnetic field (Figs. 8 (a), 11 (a), 15 (a)). The $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)\left((\Delta B_{\mathrm{unit}},0); (\Delta E_{\mathrm{unit}},0); 0; 0\right)\right\rangle$ state is the excited bosonic standing wave which expels the induced electromotive force (Figs. 8 (a), 11 (a), 15 (b)). The $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)\left((\Delta B_{\mathrm{unit}},0); (\Delta E_{\mathrm{unit}},0); B_{\mathrm{induced}}; 0\right)\right\rangle$ state is the excited bosonic standing wave which induces the magnetic field (Figs. 8 (a), 11 (a), 15 (c)). We discussed

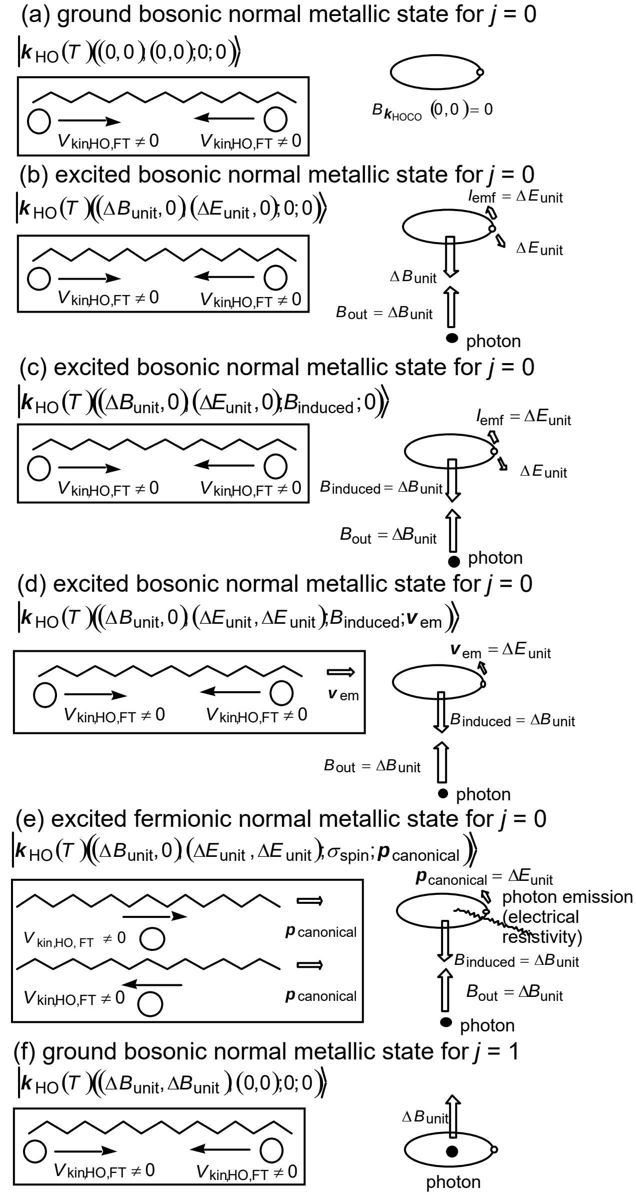



Fig. 15. The electronic states between $j$ =0 and $j$ = 1 in normal metallic states.

these electronic states in detail in the previous research [10].

5.2 The Excited Bosonic Standing Wave with the Supercurrent

The excited bosonic electronic state pairing of an electron with the induced magnetic fields $\left|\boldsymbol{k}_{\mathrm{HO}}(T)\left((\Delta B_{\mathrm{unit}},0); (\Delta E_{\mathrm{unit}},0); B_{\mathrm{induced}}; 0\right)\right\rangle$ can be immediately destroyed because the induced electric field penetrates into the normal metallic specimen, and the electronic state becomes another bosonic excited supercurrent state for $j=0$ ( $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ ) (Fig. 15 (d)). In the $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state, an electron receives the electromotive force $\Delta E_{\mathrm{unit}}$, and thus the superconducting current can be induced, and thus there is kinetic energy ( $E_{\mathrm{kinetic}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})$) of the supercurrent.

The $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state ($\Delta E_{\mathrm{BS}\to\mathrm{FT}} > V_{\mathrm{ext,HO}}$) is still in the bosonic standing wave state $\Psi_{\mathrm{electron}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})$ ( $\Delta\boldsymbol{k}_{\boldsymbol{r}_{\mathrm{p}}}=\min.$ , $\Delta\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}=\max.$ ) with $V_{\mathrm{kin,HO,BS}}=0$ eV and $V_{\mathrm{potential,HO,BS}}=35$ eV (Figs. 8 (b), 11 (a), (b), 15 (d)),

$$\Psi_{\mathrm{electron}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})\times\Psi_{\mathrm{photon}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})\to \lim_{\substack{\Delta\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}\to\max.\\ \Delta\boldsymbol{p}_{\boldsymbol{r}_{\mathrm{p}}}\to\min.}}\left|\boldsymbol{k}_{\mathrm{one}}(T)((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_{\mathrm{one}};I_{\mathrm{one}})\right\rangle$$
$$\approx\sum_{l}a_{l}\left|\boldsymbol{k}_{l}(T)((B_{\mathrm{out}},B_{\mathrm{in}});(E_{\mathrm{out}},E_{\mathrm{in}});B_{l};I_{l})\right\rangle$$
$$\times\Theta(V_{\mathrm{excited}}-(\varepsilon_{l}-\varepsilon_{\mathrm{HO}}))$$
$$=\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle. \quad (131)$$

The bosonic photonic wave interacting with the bosonic standing electronic wave can be expressed as $\Psi_{\mathrm{photon}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})\times\Psi_{\mathrm{electron}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})$ . This $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state induces the electrical current $\boldsymbol{v}_{\mathrm{em,HO}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})$ and the magnetic field $B_{\mathrm{induced,HO}}(\Delta E_{\mathrm{unit}},0)$ as a consequence of the fact that the bosonic electronic state tires not to receive the external applied magnetic field ( $(\times\Psi_{\mathrm{photon}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}}))$ ) and the induced electromotive forces. The $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state is in the excited bosonic standing wave state $\Psi_{\mathrm{electron}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})\times\Psi_{\mathrm{photon}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})$ , and can exist at various positions at the same time (Fig. 8 (b)).

The $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state is in the excited bosonic standing wave state $\Psi_{\mathrm{electron}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})\times\Psi_{\mathrm{photon}}(\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}},\boldsymbol{t}_{\boldsymbol{r}_{\mathrm{p}}})$ , and the bosonic standing wave $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ itself ( $\Delta\boldsymbol{k}_{\boldsymbol{r}_{\mathrm{p}}}=\min.$ , $\Delta\boldsymbol{r}_{\boldsymbol{r}_{\mathrm{p}}}=\max.$ ) with $V_{\mathrm{kin,HO,BS}}=0$ eV and $V_{\mathrm{potential,HO,BS}}=35$ eV , moves as a whole, according to the induced electric field (Fig. 15 (d)). As can be seen from the mathematical point of view, the bosonic standing wave $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state derived from the Schrödinger derivative equation can change smoothly and continuously as a function of the position and time, according to the principle of the causality under the conservation law of the angular momentum $\boldsymbol{l}_{\boldsymbol{r}_{\mathrm{p}}}$.

In such a case, the $B_{\mathrm{HO}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})$ and $I_{\mathrm{HO}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})$ values for the $\left|\boldsymbol{k}_{\mathrm{HO}}(T)((\Delta B_{\mathrm{unit}},0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state can be estimated as

$$\begin{aligned}
B_{\mathrm{HO}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}}) &= B_{\mathrm{HO}}(\Delta E_{\mathrm{unit}},0)\\
&=\{P_{\mathrm{excited,HO}}(T)c^{2}_{+\boldsymbol{k}_{\mathrm{HO}}\uparrow,0}(\Delta E_{\mathrm{unit}})\\
&\quad+P_{\mathrm{ground,HO}}(T)c^{2}_{-\boldsymbol{k}_{\mathrm{HO}}\uparrow,0}(\Delta E_{\mathrm{unit}})\}\\
&\quad-\{P_{\mathrm{excited,HO}}(T)c^{2}_{-\boldsymbol{k}_{\mathrm{HO}}\downarrow,0}(\Delta E_{\mathrm{unit}})\\
&\quad+P_{\mathrm{ground,HO}}(T)c^{2}_{+\boldsymbol{k}_{\mathrm{HO}}\downarrow,0}(\Delta E_{\mathrm{unit}})\}\\
&=2P_{\mathrm{excited,HO}}(T)\{c^{2}_{+\boldsymbol{k}_{\mathrm{HO}}\uparrow,0}(\Delta E_{\mathrm{unit}})\\
&\quad-c^{2}_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(\Delta E_{\mathrm{unit}})\}\\
&=B_{\mathrm{induced,HO}}(\Delta E_{\mathrm{unit}},0)=-\Delta B_{\mathrm{unit}}, \qquad (132)
\end{aligned}$$

$$\begin{aligned}
I_{\mathrm{HO}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}}) &= I_{\mathrm{HO}}(\Delta B_{\mathrm{unit}},0)\\
&=\{P_{\mathrm{excited,HO}}(T)c^{2}_{+\boldsymbol{k}_{\mathrm{HO}}\uparrow,0}(\Delta B_{\mathrm{unit}})\\
&\quad+P_{\mathrm{ground,HO}}(T)c^{2}_{+\boldsymbol{k}_{\mathrm{HO}}\downarrow,0}(\Delta B_{\mathrm{unit}})\}\\
&\quad-\{P_{\mathrm{excited,HO}}(T)c^{2}_{-\boldsymbol{k}_{\mathrm{HO}}\downarrow,0}(\Delta B_{\mathrm{unit}})\\
&\quad+P_{\mathrm{ground,HO}}(T)c^{2}_{-\boldsymbol{k}_{\mathrm{HO}}\uparrow,0}(\Delta B_{\mathrm{unit}})\}\\
&=2P_{\mathrm{excited,HO}}(T)\{c^{2}_{+\boldsymbol{k}_{\mathrm{HO}}\uparrow,0}(\Delta B_{\mathrm{unit}})\\
&\quad-c^{2}_{-\boldsymbol{k}_{\mathrm{HO}}\downarrow,0}(\Delta B_{\mathrm{unit}})\}\\
&=\boldsymbol{v}_{\mathrm{em,HO}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})=\Delta E_{\mathrm{unit}}. \qquad (133)
\end{aligned}$$

That is, the energy of the electromotive force $\Delta E_{\text{unit}}$ for the $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0\right)\right\rangle$ state is converted to the kinetic energy of the supercurrent for the $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$ state. Both the supercurrent ($\boldsymbol{v}_{\text{em,HO}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$) and the magnetic field ($B_{\text{induced,HO}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$) can be induced under the condition of the bosonic opened-shell electronic structure with zero spin magnetic momentum and canonical momentum ($\sigma_{\text{spin,HO}}=0$; $\boldsymbol{p}_{\text{canonical,HO}}=0$).

5.3 The Excited Fermionic Traveling Wave with the Normal Metallic Current

On the other hand, such excited bosonic normal metallic states with supercurrents ($\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$) can be immediately destroyed because of the unstable opened-shell electronic states subject to the external applied magnetic field, and the electronic state becomes another excited fermionic normal metallic states for $j=0$ ($\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$) (Fig. 15 (e)), as explained in detail below.

Interaction between the external applied wave $\Psi_{\text{photon}}\left(\boldsymbol{r}_{r_{\text{p}}},\boldsymbol{t}_{r_{\text{p}}}\right)$ and the electronic wave $\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_{\text{p}}},\boldsymbol{t}_{r_{\text{p}}}\right)$ can destroy the bosonic standing electronic wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$, and thus one bosonic standing wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$ with $V_{\text{kin,HO,BS}}=0$ eV and $V_{\text{potential,HO,BS}}=35$ eV can be changed to the two fermionic traveling waves $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ with $V_{\text{kin,HO,FT}}=35$ eV and $V_{\text{potential,HO,FT}}=0$ eV (Figs. 6, 8 (c), 11 (b), (c), 15 (e)), as shown below.

The necessary minimum energy $\Delta E_{\text{BS}\to\text{FT}}$ for destruction of the bosonic standing wave (conversion from the bosonic standing wave state to the fermionic traveling wave state) is usually very small (~ 0 eV) in solids and in the opened-shell electronic states in molecules,

$$\Delta E_{\text{BS}\to\text{FT}}\sim 0\text{ eV}. \tag{134}$$

Since the bosonic standing wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$ state has larger $\boldsymbol{v}_{\text{em,HO}}(=V_{\text{ext,HO}})$ ($\sim 10^{-2}$ eV) value than the $\Delta E_{\text{BS}\to\text{FT}}$ (~ 0 eV) value, the electronic states can be excited to the higher energy level (Fig. 11 (b)),

$$\boldsymbol{v}_{\text{em,HO}}(=V_{\text{ext,HO}})(\sim 10^{-2}\text{ eV}) > \Delta E_{\text{BS}\to\text{FT}}(\sim 0\text{ eV}). \tag{135}$$

Therefore, the bosonic standing $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$ wave with $V_{\text{kin,HO,BS}}=0$ eV and $V_{\text{potential,HO,BS}}=35$ eV can be immediately destroyed and the two components of the fermionic traveling waves ($\left|+\boldsymbol{k}_{\text{HO}}\uparrow\right\rangle$ and $\left|+\boldsymbol{k}_{\text{HO}}\downarrow\right\rangle$, and $\left|-\boldsymbol{k}_{\text{HO}}\downarrow\right\rangle$ and $\left|-\boldsymbol{k}_{\text{HO}}\uparrow\right\rangle$) with $V_{\text{kin,HO,FT}}=35$ eV and $V_{\text{potential,HO,FT}}=0$ eV can begin to explicitly appear. Since the fermionic traveling waves have the large $V_{\text{kin,HO,FT}}$ values of about 35 eV, the electronic states can be excited to the orbitals with much higher energy level (Figs. 6, 8 (c), 11 (c), 15 (e)).

Since the $V_{\text{kin,HO,FT}}$ value of about 35 eV is in general extremely large in solids, there are many orbital levels within the energy range between $\varepsilon_{\text{HO}}$ and $\varepsilon_{\text{HO}}+V_{\text{excited}}$ ($V_{\text{excited}}-(\varepsilon_l-\varepsilon_{\text{HO}})$). Therefore, the uncertainty of the linear and angular momentum ($\Delta\boldsymbol{p}_{r_{\text{p}}}\left(\sim\Delta\boldsymbol{k}_{r_{\text{p}}}\right)$) significantly increases. On the other hand, according to the Fourier transform, the localized fermionic traveling wave packet with $\Delta\boldsymbol{k}_{r_{\text{p}}}=$ max. and $\Delta\boldsymbol{r}_{r_{\text{p}}}=$ min. can be formed by various $\boldsymbol{p}_{r_{\text{p}}}\left(\sim\boldsymbol{k}_{r_{\text{p}}}\right)$ values. Therefore, the localized fermionic traveling wave packet can be formed (Figs. 6, 8 (c)). We observe it as an electronic particle.

In acoustics it corresponds to a short sound—we should hear a bang as a sharply defined pressure wave passed. Such a wave can be regarded as a collection of a large number of waves, each with a different wavelength, superimposed so that constructive interference creates the pressure zone but destructive interference occurs everywhere else [22]. In a similar way, such a fermionic wave packet can be regarded as a collection of a large number of waves, each with a different wavelength and momentum, superimposed so that constructive interference creates the central zone of the interaction ($\left|\boldsymbol{r}_{r_{\text{p}}}-\boldsymbol{r}_{r_{\text{p}},\text{center}}\right|\le\Delta\boldsymbol{r}_{r_{\text{p}}}/2$) but destructive interference occurs everywhere else ($\left|\boldsymbol{r}_{r_{\text{p}}}-\boldsymbol{r}_{r_{\text{p}},\text{center}}\right|>\Delta\boldsymbol{r}_{r_{\text{p}}}/2$) (Fig. 6). At $\boldsymbol{r}_{r_{\text{p}}}=\boldsymbol{r}_{r_{\text{p}},\text{center}}$, where the amplitude of the

$\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)\Psi_{\text{photon}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)$ as a consequence of the interference between the two waves $\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)$ and $\Psi_{\text{photon}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)$ becomes the maximum, density of energy which is needed for the destruction of the bosonic standing wave $\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)$ becomes the maximum. Therefore, the destruction of the bosonic standing wave $\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)$ can begin to occur at $\boldsymbol{r}_{r_\text{p}} = \boldsymbol{r}_{r_\text{p},\text{center}}$, similar to the center of the pressure zone in a bang in acoustics (Fig. 6) [22]. That is, the fermionic traveling wave packet $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ is located around the ranges as follows (Fig. 6),

$$\left|\boldsymbol{r}_{r_\text{p}} - \boldsymbol{r}_{r_\text{p},\text{center}}\right| \le \frac{1}{2}\Delta \boldsymbol{r}_{r_\text{p}}. \qquad (136)$$

The $\Delta \boldsymbol{r}_{r_\text{p}}$ and $\Delta \boldsymbol{p}_{r_\text{p}}$ values significantly rapidly (almost discontinuously) decrease and increase with an increase in the converting time from the bosonic standing wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$ state to the fermionic traveling wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state, according to principle of the causality (Fig. 6). The characteristics for the fermionic traveling wave increase when the $\Delta \boldsymbol{p}_{r_\text{p}}$ value increases and the $\Delta \boldsymbol{r}_{r_\text{p}}$ value decreases. Therefore, the fermionic traveling wave packet with $\Delta \boldsymbol{r}_{r_\text{p}} \to \text{min.}$ and $\Delta \boldsymbol{p}_{r_\text{p}} \to \text{max.}$ in the space axis can be expressed as follows,

$$\begin{aligned}
&\phi_{\text{w.p.electron}}\left(\boldsymbol{r}_{r_\text{p},\text{center}}, \boldsymbol{t}_{r_\text{p}}\right)\left(\times\phi_{\text{w.p.photon}}\left(\boldsymbol{r}_{r_\text{p},\text{center}}, \boldsymbol{t}_{r_\text{p}}\right)\right)\\
&\to \lim_{\substack{\Delta \boldsymbol{r}_{r_\text{p}} \to \text{min.}\\ \Delta \boldsymbol{p}_{r_\text{p}} \to \text{max.}}} \left\{\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p},\text{center}}, \boldsymbol{t}_{r_\text{p}}\right)\left(\times\Psi_{\text{photon}}\left(\boldsymbol{r}_{r_\text{p},\text{center}}, \boldsymbol{t}_{r_\text{p}}\right)\right)\right\}\\
&\to \lim_{\substack{\Delta \boldsymbol{r}_{r_\text{p}} \to \text{min.}\\ \Delta \boldsymbol{p}_{r_\text{p}} \to \text{max.}}} \left\{\max\left(\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)\times\left(\Psi_{\text{photon}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)\right)\right)\right\}\\
&\approx \lim_{\substack{\Delta \boldsymbol{r}_{r_\text{p}} \to \text{min.}\\ \Delta \boldsymbol{p}_{r_\text{p}} \to \text{max.}}} \sum_l a_l \left|\boldsymbol{k}_l(T)\left((B_{\text{out}},B_{\text{in}});(E_{\text{out}},E_{\text{in}});B_l;I_l\right)\right\rangle\\
&\times\Theta\left(V_{\text{excited}} - (\varepsilon_l - \varepsilon_{\text{HO}})\right)\Theta\left(\frac{\Delta \boldsymbol{r}_{r_\text{p}}}{2} - \left|\boldsymbol{r}_{r_\text{p}} - \boldsymbol{r}_{r_\text{p},\text{center}}\right|\right)\\
&\times\Theta\left(\left|\Delta \boldsymbol{r}_{r_\text{p}}\right|\left|\Delta \boldsymbol{p}_{r_\text{p}}\right| - \frac{h}{2\pi}\right)\\
&= \left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle,
\end{aligned} \qquad (137)$$

$$\lim_{\substack{\Delta \boldsymbol{r}_{r_\text{p}} \to \text{min.}\\ \Delta \boldsymbol{p}_{r_\text{p}} \to \text{max.}}} \left\{\Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p},\text{center}}, \boldsymbol{t}_{r_\text{p}}\right)\left(\times\Psi_{\text{photon}}\left(\boldsymbol{r}_{r_\text{p},\text{center}}, \boldsymbol{t}_{r_\text{p}}\right)\right)\right\} > \Psi_{\text{electron}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right)\times\Psi_{\text{photon}}\left(\boldsymbol{r}_{r_\text{p}}, \boldsymbol{t}_{r_\text{p}}\right) \ \left(\text{for any } \boldsymbol{r}_{r_\text{p}}\right) \qquad (138)$$

$$\sum_l a_l^2 \Theta\left(V_{\text{excited}} - (\varepsilon_l - \varepsilon_{\text{HO}})\right) = \sum_l^{\text{occupied}} a_l^2 = 1. \qquad (139)$$

The size and the figure ($\Delta \boldsymbol{r}_{r_\text{p}}$, $\Delta \boldsymbol{k}_{r_\text{p}}$) of a wave packet as expressed in Eq. (137) depends on the applied field energy, according to the principle of the causality. The $\Delta \boldsymbol{r}_{r_\text{p}}$ and $\Delta \boldsymbol{k}_{r_\text{p}}$ values for a wave packet rapidly (almost discontinuously) decreases and increases with an increase in the applied field energy, respectively. That is, the characteristic for the wave and particle almost discontinuously becomes less and more significant, respectively, with an increase in the applied field energy.

The $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state is in the excited fermionic traveling wave state $\phi_{\text{w.p.electron}}\left(\boldsymbol{r}_{r_\text{p},\text{center}}, \boldsymbol{t}_{r_\text{p}}\right)$, and the fermionic traveling wave components ($\Delta \boldsymbol{k}_{r_\text{p}} = \text{max.}$, $\Delta \boldsymbol{r}_{r_\text{p}} = \text{min.}$) with $V_{\text{kin,HO,FT}} = 35$ eV and $V_{\text{potential,HO,FT}} = 0$ eV, moves independently, according to the induced electric field. On the other hand, it should be noted that even though each fermionic traveling wave component of two fermionic traveling wave ($\left|+\boldsymbol{k}_{\text{HO}}\uparrow\right\rangle$ and $\left|+\boldsymbol{k}_{\text{HO}}\downarrow\right\rangle$, and $\left|-\boldsymbol{k}_{\text{HO}}\downarrow\right\rangle$ and $\left|-\boldsymbol{k}_{\text{HO}}\uparrow\right\rangle$) in an electron has large $V_{\text{kin,HO,FT}}$ value, which plays an essential role in the converting the bosonic standing wave to the fermionic traveling wave, the direction of the movement of the two fermionic traveling waves is opposite, and is still canceled by each other, and thus the total movement of the fermionic traveling wave of an electron ($\boldsymbol{p}_{\text{canonical,HO}}$) cannot change from the total movement of the bosonic standing wave ($\boldsymbol{v}_{\text{em,HO}}$) (Figs. 8 (b), (c), 11, 15 (d), (e)),

$$\boldsymbol{p}_{\text{canonical,HO}} = \boldsymbol{v}_{\text{em,HO}}. \quad (140)$$

In such a case, the $B_{\text{HO}}(\Delta E_{\text{unit}}, \Delta E_{\text{unit}})$ and $I_{\text{HO}}(\Delta E_{\text{unit}}, \Delta E_{\text{unit}})$ values for the $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\!\rangle$ state can be estimated as

$$\begin{aligned} B_{\text{HO}}(\Delta E_{\text{unit}}, \Delta E_{\text{unit}}) &= B_{\text{HO}}(\Delta E_{\text{unit}}, 0) \\ &= \left\{P_{\text{excited,HO}}(T)c^2_{+\boldsymbol{k}_{\text{HO}}\uparrow,0}(\Delta E_{\text{unit}}) + P_{\text{ground,HO}}(T)c^2_{-\boldsymbol{k}_{\text{HO}}\uparrow,0}(\Delta E_{\text{unit}})\right\} \\ &\quad - \left\{P_{\text{excited,HO}}(T)c^2_{-\boldsymbol{k}_{\text{HO}}\downarrow,0}(\Delta E_{\text{unit}}) + P_{\text{ground,HO}}(T)c^2_{+\boldsymbol{k}_{\text{HO}}\downarrow,0}(\Delta E_{\text{unit}})\right\} \\ &= 2P_{\text{excited,HO}}(T)\left\{c^2_{+\boldsymbol{k}_{\text{HO}}\uparrow,0}(\Delta E_{\text{unit}}) - c^2_{-\boldsymbol{k}_{\text{HO}}\downarrow,0}(\Delta E_{\text{unit}})\right\} \\ &= \sigma_{\text{spin,HO}}(\Delta E_{\text{unit}}, \Delta E_{\text{unit}}) = -\Delta B_{\text{unit}}, \end{aligned} \quad (141)$$

$$\begin{aligned} I_{\text{HO}}(\Delta E_{\text{unit}}, \Delta E_{\text{unit}}) &= I_{\text{HO}}(\Delta B_{\text{unit}}, 0) \\ &= \left\{P_{\text{excited,HO}}(T)c^2_{+\boldsymbol{k}_{\text{HO}}\uparrow,0}(\Delta B_{\text{unit}}) + P_{\text{ground,HO}}(T)c^2_{+\boldsymbol{k}_{\text{HO}}\downarrow,0}(\Delta B_{\text{unit}})\right\} \\ &\quad - \left\{P_{\text{excited,HO}}(T)c^2_{-\boldsymbol{k}_{\text{HO}}\downarrow,0}(\Delta B_{\text{unit}}) + P_{\text{ground,HO}}(T)c^2_{-\boldsymbol{k}_{\text{HO}}\uparrow,0}(\Delta B_{\text{unit}})\right\} \\ &= 2P_{\text{excited}}(T)\left\{c^2_{+\boldsymbol{k}_{\text{HO}}\uparrow,0}(\Delta B_{\text{unit}}) - c^2_{-\boldsymbol{k}_{\text{HO}}\downarrow,0}(\Delta B_{\text{unit}})\right\} \\ &= \boldsymbol{p}_{\text{canonical,HO}}(\Delta E_{\text{unit}}, \Delta E_{\text{unit}}) = \Delta E_{\text{unit}}. \end{aligned} \quad (142)$$

It should be noted that the electronic $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\!\rangle$ state is now fermionic traveling wave packet $\phi_{\text{w.p.electron}}\left(\boldsymbol{r}_{r_{\text{p}},\text{center}}, \boldsymbol{t}_{r_{\text{p}}}\right)$ because the $\boldsymbol{p}_{\text{canonical,HO}}$ value is not 0. In other words, the $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\!\rangle$ state is closely related to the normal conducting states in that the normal metallic current with $\boldsymbol{p}_{\text{canonical,HO}} \neq 0$ and $\boldsymbol{v}_{\text{em,HO}} = 0$ is induced by the induced electromotive forces (Fig. 15 (e)).

5.4 The Ground Bosonic Standing Wave under Applied Magnetic Field

Such excited fermionic normal metallic states with currents and the induced magnetic field ($\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\!\rangle$) can be immediately destroyed because of the unstable opened-shell electronic states subject to the external applied magnetic field, and the induced current and the magnetic field can be immediately destroyed, and thus the initially external applied magnetic field can start to penetrate into the normal metallic specimen. Therefore, the electronic state tries to become another ground bosonic metallic state for $j = 1$ ($\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},\Delta B_{\text{unit}});(0,0);0;0\right)\right\rangle\!\!\rangle$) (Figs. 15 (f), 16), as follows.

The $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state is in the excited fermionic traveling wave state $\phi_{\text{w.p.electron}}\left(\boldsymbol{r}_{r_{\text{p}},\text{center}}, \boldsymbol{t}_{r_{\text{p}}}\right)$. The energy for the fermionic traveling wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ is localized and confined within very small range. This localized fermionic traveling wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state is observed to be unstable because the $V_{\text{kin,HO,FT}}$ value is not condensated into vacuum and can usually be observed as a free energy. As described above, such two fermionic traveling wave state of an electron with the free energy $V_{\text{kin,HO,FT}}$ is unstable, and thus such two fermionic traveling wave state of an electron can be immediately destroyed as shown below.

The $\Delta \boldsymbol{r}_{r_{\text{p}}}$ and $\Delta \boldsymbol{p}_{r_{\text{p}}}$ values significantly rapidly (almost discontinuously) increase and decrease with an increase in the converting time from the fermionic traveling wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state to the bosonic standing wave $\left|\boldsymbol{k}_{\text{HO}}(T)\left((\Delta B_{\text{unit}},\Delta B_{\text{unit}});(0,0);0;0\right)\right\rangle$ state, according to principle of the causality (Fig. 16). In such a case, the fermionic traveling wave components $\left|+\boldsymbol{k}_{\text{HO}}\uparrow\right\rangle$, $\left|+\boldsymbol{k}_{\text{HO}}\downarrow\right\rangle$, $\left|-\boldsymbol{k}_{\text{HO}}\downarrow\right\rangle$, and $\left|-\boldsymbol{k}_{\text{HO}}\uparrow\right\rangle$, which moves

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independently, begin to interact with each other, and thus begin to form a pair formed by two components with

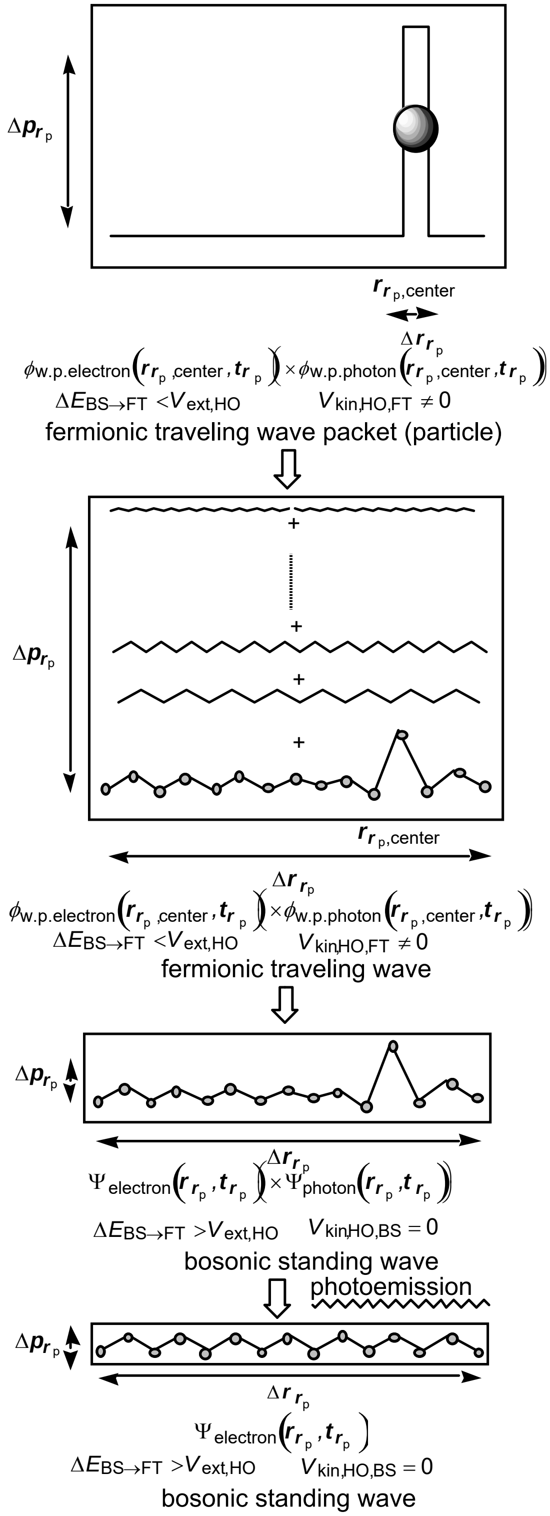


Fig. 16. Conversion from the fermionic traveling wave packet to the bosonic standing wave with photoemission.

opposite momentum and spins (between $\left|+\boldsymbol{k}_{\mathrm{HO}}\uparrow\right\rangle$ and $\left|+\boldsymbol{k}_{\mathrm{HO}}\downarrow\right\rangle$, and between $\left|-\boldsymbol{k}_{\mathrm{HO}}\downarrow\right\rangle$ and $\left|-\boldsymbol{k}_{\mathrm{HO}}\uparrow\right\rangle$).

During the conversion from the fermionic traveling wave state to the bosonic standing wave state, the entropy increases and the total energy in the 4-dimensional complex spacetime axis decreases. That is, the significant increase of the entropy and the significant decrease of the total energy in the 4-dimensional complex spacetime axis are the main reason why the bosonic standing wave state is much more stable than the fermionic traveling wave state in energy, and are the reason why the unstable fermionic traveling wave state is converted to the bosonic standing wave state significantly rapidly [20]. That is, the conversion from the fermionic traveling wave to the bosonic standing wave is closely related to the second law of the thermodynamics (increase of the entropy ). In such a case, the $V_{\mathrm{kin,HO,FT}}$ value of 35 eV for the fermionic traveling wave state, which can be observed as a free energy, is converted to the $V_{\mathrm{potential,HO,BS}}$ value of 35 eV for the bosonic standing wave state, which is condensated into the vacuum and cannot be observed. In the bosonic standing wave state, the $V_{\mathrm{kin,HO,BS}}$ value becomes 0 eV (Figs. 11 (c), 15 (e), 16).

At the same time, the bosonic standing wave with the $\boldsymbol{p}_{\mathrm{canonical,HO}}\left(=V_{\mathrm{ext,HO}}\right)\neq 0$ value is converted to the bosonic standing wave state $\left|\boldsymbol{k}_{\mathrm{HO}}(T)\left((\Delta B_{\mathrm{unit}},\Delta B_{\mathrm{unit}});(0,0);0;0\right)\right\rangle$ with the $V_{\mathrm{ext,HO}}=0$ value by the emission of the photon energy ($\boldsymbol{p}_{\mathrm{canonical,HO}}\left(=V_{\mathrm{ext,HO}}\right)\neq 0$) (Figs. 11 (d), (e), 15 (e), (f), 16).

This process depends on the environment of an electron. In other words, in principle, the movement and physical values of an electron during the conversion from the fermionic standing wave packet to the bosonic standing wave can be also precisely predicted at each position and each time if we know the physical values at the earlier time, according to the principle of the causality (Fig. 16), as in the second law of the classical thermodynamics.

The $\left|\boldsymbol{k}_{\mathrm{HO}}(T)\left((\Delta B_{\mathrm{unit}},\Delta B_{\mathrm{unit}});(0,0);0;0\right)\right\rangle$ state is in the bosonic standing wave state $\Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)$ ($\Delta\boldsymbol{k}_{r_{\mathrm{p}}}=\mathrm{min.}$, $\Delta\boldsymbol{r}_{r_{\mathrm{p}}}=\mathrm{max.}$) with $V_{\mathrm{kin,HO,BS}}=0$ eV and $V_{\mathrm{potential,HO,BS}}=35$ eV (Fig. 15 (f)),

$$\Psi_{\mathrm{electron}}\left(\boldsymbol{r}_{r_{\mathrm{p}}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)\times\Psi_{\mathrm{photon}}\left(\boldsymbol{r}_{r_{\mathrm{p}}},\boldsymbol{t}_{r_{\mathrm{p}}}\right)\rightarrow$$

$$\lim_{\substack{\Delta r_{r_p}\to\max.\\ \Delta p_{r_p}\to\min.}} \left|k_{one}(T)\big((B_{out},B_{in});(E_{out},E_{in});B_{one};I_{one}\big)\right\rangle$$
$$\approx \sum_l a_l \left|k_l(T)\big((B_{out},B_{in});(E_{out},E_{in});B_l;I_l\big)\right\rangle$$
$$\times\Theta\big(V_{excited}-(\varepsilon_l-\varepsilon_{HO})\big)$$
$$=\left|k_{HO}(T)\big((\Delta B_{unit},\Delta B_{unit});(0,0);0;0\big)\right\rangle. \qquad (143)$$

In such a case, the $B_{HO}(\Delta B_{unit},\Delta B_{unit})$ and $I_{HO}(\Delta B_{unit},\Delta B_{unit})$ values for the $\left|k_{HO}(T)\big((\Delta B_{unit},\Delta B_{unit});(0,0);0;0\big)\right\rangle$ state can be estimated as

$$I_{HO}(\Delta B_{unit},\Delta B_{unit})$$
$$=\left\{P_{excited}(T)c^2_{+k_{HO}\uparrow,\Delta B_{unit}}(0)+P_{ground}(T)c^2_{+k_{HO}\downarrow,\Delta B_{unit}}(0)\right\}$$
$$-\left\{P_{excited}(T)c^2_{-k_{HO}\downarrow,\Delta B_{unit}}(0)+P_{ground}(T)c^2_{-k_{HO}\uparrow,\Delta B_{unit}}(0)\right\}$$
$$=0, \qquad (144)$$

and thus

$$B_{HO}(\Delta B_{unit},\Delta B_{unit})$$
$$=\left\{P_{excited}(T)c^2_{+k_{HO}\uparrow,\Delta B_{unit}}(0)+P_{ground}(T)c^2_{-k_{HO}\uparrow,\Delta B_{unit}}(0)\right\}$$
$$-\left\{P_{excited}(T)c^2_{-k_{HO}\downarrow,\Delta B_{unit}}(0)+P_{ground}(T)c^2_{+k_{HO}\downarrow,\Delta B_{unit}}(0)\right\}$$
$$=2P_{excited}(T)\left\{c^2_{+k_{HO}\uparrow,\Delta E_{unit}}(0)-c^2_{-k_{HO}\downarrow,\Delta E_{unit}}(0)\right\}$$
$$=\sigma_{spin,HO}(\Delta B_{unit},\Delta B_{unit})=\Delta B_{unit}. \qquad (145)$$

**6. The Principle of the Causality in the Conversion between the Bosonic Standing Wave and the Fermionic Traveling Wave**

According to our research, the collapse of the wavefunction rapidly occurs under the principle of causality as follows (Fig. 6). When the bosonic standing wave $\Psi_{electron}(\boldsymbol{r}_{r_p},\boldsymbol{t}_{r_p})$ with $V_{ext,HO}=0$ and $V_{kin,HO,BS}=0$ eV (Fig. 11 (a)) interacts with the external field such as magnetic field (photon), the electronic $\Psi_{electron}(\boldsymbol{r}_{r_p},\boldsymbol{t}_{r_p})$ state finally receives the external energy for electrical current $(V_{ext,HO}(=\boldsymbol{v}_{em,HO}))$. Therefore, the electronic state is in the bosonic standing wave $\Psi_{electron}(\boldsymbol{r}_{r_p},\boldsymbol{t}_{r_p})$ with $V_{ext,HO}\neq 0$ and $V_{kin,HO}=0$ eV (Fig. 11 (b)). The $V_{ext,HO}(=\boldsymbol{v}_{em,HO})$ value is larger than the $\Delta E_{BS\to FT}$ value, and thus the bosonic standing electronic wave can be destroyed and the components of the two fermionic traveling wave with $V_{ext,HO}\neq 0$ and $V_{kin,HO}(=V_{kin,HO,FT})\neq 0$ begin to explicitly appear (Fig. 11 (c)). Since the $V_{kin,HO,FT}$ value of about 35 eV is in general extremely large in solids, there are many orbital levels within the energy range between $\varepsilon_{HO}$ and $\varepsilon_{HO}+V_{ext,HO}+V_{kin,HO,FT}$ (Fig. 11 (c)). Therefore, the uncertainty of the linear and angular momentum $(\Delta \boldsymbol{p}_{r_p}(\sim\Delta \boldsymbol{k}_{r_p}))$ significantly increases. Therefore, the localized fermionic traveling wave packet can be formed. We observe it as an electronic particle. This process occurs very rapidly (almost discontinuously) as a function of the time, according to the principle of the causality, as in the discontinuous process in the classical wave dynamics. Even in the non-relativistic classical dynamics, there are many phenomena which change discontinuously, according to the principle of the causality.

We can see from Eq. (137) that the fermionic traveling wave packet $\phi_{w.p.electron}(\boldsymbol{r}_{r_p,center},\boldsymbol{t}_{r_p})$ can be decided by the parameters, which are decided by the environment around the electron, according to the principle of the causality, and can change as a function of the time $\boldsymbol{t}_{r_p}$, as in the classical wave dynamics. In particular, the $\Delta \boldsymbol{r}_{r_p}$ and $\Delta \boldsymbol{p}_{r_p}$ values change as a function of the time, very rapidly and almost discontinuously.

The localized fermionic traveling wave state is observed to be unstable because the $V_{kin,HO,FT}$ value is not condensated into vacuum and can usually be observed as a free energy. Such two fermionic traveling wave state of an electron with the free energy $V_{kin,HO,FT}$ is unstable, and thus such two fermionic traveling wave state of an electron can be immediately destroyed.

During the conversion from the fermionic traveling wave state to the bosonic standing wave state, the entropy increases and the total energy in the 4-dimensional complex spacetime axis decreases. These are the reason why the unstable fermionic traveling wave state is converted to the bosonic standing wave state significantly rapidly. In such a case, the $V_{\mathrm{kin,HO,FT}}$ value of 35 eV for the fermionic traveling wave state, which can be observed as a free energy, is converted to the $V_{\mathrm{potential,HO,BS}}$ value of 35 eV for the bosonic standing wave state, which is condensed into the vacuum and cannot be observed.

At the same time, the bosonic standing wave with the $\boldsymbol{p}_{\mathrm{canonical,HO}}\left(=V_{\mathrm{ext,HO}}\right)\neq 0$ value is converted to the bosonic standing wave state $\left|\boldsymbol{k}_{\mathrm{HO}}(T)\left(\left(\Delta B_{\mathrm{unit}},\Delta B_{\mathrm{unit}}\right);(0,0);0;0\right)\right\rangle$ with the $V_{\mathrm{ext,HO}}=0$ value by the emission of the photon energy ($\boldsymbol{p}_{\mathrm{canonical,HO}}\left(=V_{\mathrm{ext,HO}}\right)\neq 0$) (Figs. 11 (d), (e), 15 (e), (f), 16).

This process depends on the environment of an electron. In other words, in principle, the movement and physical values during the conversion from the fermionic standing wave packet to the bosonic standing wave can be also precisely predicted at each position and each time if we know the physical values at the earlier time, according to the principle of the causality (Fig. 16), as in the second law of the classical thermodynamics.

Even though the movement of waves and wave packets at the each position and each time can be decided according to the principle of causality, it would be too complicated for us to analyze the movement of these waves precisely, for mathematical difficulty. This is the reason why we have thought that the principle of the causality is broken in the quantum mechanics, in particular, in the case of the collapse of the wavefunctions.

## 7. Summary

The superimposition between various wavefunctions with various momentum as a consequence of the appearance of large free energy such as kinetic energy is the origin of the collapse of the wavefunction from the bosonic standing wave to the fermionic traveling wave. The increase of the entropy and the total energy of the 4-dimensional complex spacetime world is the main reason why the conversion from the fermionic traveling wave state to the bosonic standing wave state occurs significantly rapidly.

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