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A Markov Model for Prediction of Annual Rainfall

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Abstract
Markov modelling is one of the tools that can be utilized to assist planners in assessing the rainfall. The first-order Markov chain model was used to predict annual rainfall intervals using transitional probability matrices and Monte Carlo simulation. The class intervals were treated as probability states and the transition probability matrices were using the frequency distribution of the class intervals. Six probability states were used to construct first-order Markov chain. To illustrate the applicability of the model, Owerri city in Nigeria, was used as a case study. Model parameters were estimated from 1976-2005 historical rainfall records. When Monte Carlo scheme was used to simulate the probability of occurrence of annual rainfall intervals, the results demonstrated that the model can be applied successfully and provided forecasts of high accuracy.
Keywords: *Annual rainfall, Markov Chain, Model, Monte Carlo, transition probability,*

1. Introduction

Water plays an important role for agriculture, industry and daily domestics. It is one of the vital natural resources. The amount of rainfall received over an area is an important factor in assessing the amount of water available to meet the various demands of agriculture, industry and other human activities [1]. The analysis of rainfall records for long periods provides information about rainfall patterns and variability [2].Prediction of the probability of occurrence of annual rainfall has remained neglected despite its great relevance in hydrologic risk and reliability studies. The variability of rainfall is very important for agriculture. The estimation of annual rainfall occurrence from available time series helps to obtain predictions for annual rainfall statistical parameters such as the averages, standard deviations and the coefficient of variation. Several methods ([1];[2];[3];[4]&[5]) have been proposed by various researchers for modelling rainfall data. Mandal et al. [6] analyzed the annual rainfall of Daspalla region in Odisha, eastern India for prediction of monsoon and post-monsoon rainfall and the results revealed that Log Pearson Type-III and Gumbel distribution were the best fit probability distribution functions. Shadeed et al. [7] evaluated the distribution characteristics of the rainfall in semi-arid regions and Gumbel distribution was proved to simulate the annual rainfalls of the six stations of Faria catchment in India. Yet, most of these models do not account for the year to year variations in the probability of occurrence of rainfall. These variations may either be in the form too much water, which will lead to flooding or too little water, which will lead to draught.

A stochastic model can be used to predict the probability of occurrence of annual rainfall. According to Taha [8], a stochastic process consisting of a random variable X_n, characterizing the state of the system at discrete points in time t=1,2,.. is a Markov process if the occurrence of a future state depends only on the immediately preceding state. The Markov chain models provide quick forecasts immediately after any observations have been made because they use only the local information as predictors and they need minimal computation after the data have been processed [8]; [9].The main purpose of the present study is to show the use of first-order Markov Chain model for the prediction of the occurrence of annual rainfall intervals in Owerri, Nigeria.

2. Materials and Method

2.1 Study Area

As shown in Fig 1, the study area is Owerri. Owerri is the capital city of Imo State located in South-East of Nigeria. Owerri situates between 50 20'N, 6055'E in the south-western corner and 50 34'N, 70 08'E in the north-eastern corner [10]. It falls within the rainforest zone of 2290mm of rainfall per annum, relative humidity of 55-85% and temperature of 27^{0}C.

It has a subequatorial climate. The two prevalent seasons, the dry and rainy seasons occur from October to March and April to September respectively [10]. Owerri covers a land mass of 5200km^2 and lies entirely within coastal plain sand stones.

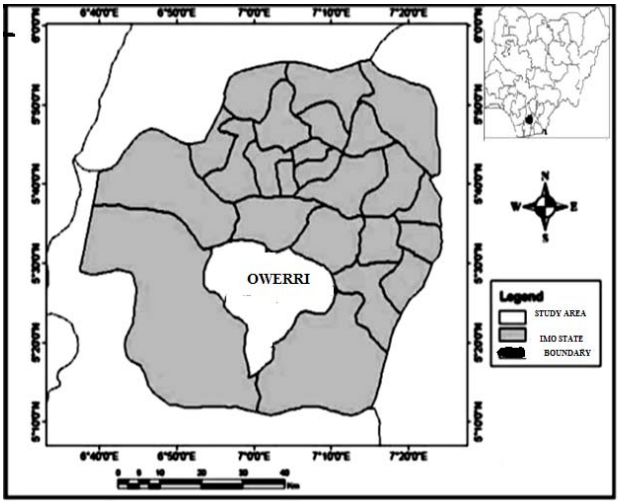

Fig. 1 Location Map of Owerri in Imo State, Nigeria

2.2 Data Collection and Processing

1976 to 2005monthly rainfall amount of Owerri were obtained from the Nigerian Meteorological Agency (NIMET) office at Imo Airport, Owerri. The daily data were summed to yield the annual rainfall amount.

2.3 Determination of Annual Rainfall States

A frequency table of annual rainfall is obtained by dividing the range of data into non-overlapping bins. According to Taha [8], given the boundaries (I_{i-1}, I_i) for bin i, the corresponding frequency is determined as the count of all the raw data, x, that satisfy $I_{i-1} < x \le I_i$. Bin i are then converted into state i such that $i = A, B \ldots$

2.4 Model Formulation

Given the chronological times $t_0, t_1 \ldots t_n$, the family of random variables $x_0, x_1, \ldots, x_{n+1} \in \{0, \quad 1\}$ is said to be a Markov process if it possesses the following property:

$$P\{X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \ldots, X_0 = x_0\} = P\{X_{n+1} = x_{n+1} | X_n = x_n\} \quad (1)$$

This is known as the one-step transition probability of moving from state i at t-1 to state j at t i.e P_{ij}. By definition,

$$0 < P_{ij} < 1.0 \quad \text{where } i,j = 1,2,\ldots,n$$
$$\sum_{j=1}^{n} P_{ij} = 1 \quad (2)$$

2.5 Parameter Estimation

The estimates of P_{ij} was obtained by

$$P_{ij} = f_{ij} + \sum^{n} f_{ij}\, i, j = 1,2,\ldots n \quad (3)$$

where f_{ij} is the number of times the observed data went from state i to state j.
Wilks [11] defined the transition probability matrix P for n-step first-order Markov chain as

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{n1} & P_{n2} & P_{n3} & \cdots & P_{nn} \end{bmatrix}$$

2.6 Model Simulation

For generating the sequences of annual rainfall states, the initial state, say state i, was selected randomly. Then random values between 0 and 1 were produced by using a uniform random number generator [9]. This is called Monte Carlo simulation. For next annual rainfall state in first-order Markov process, the value of the random number was compared with the elements of the ith row of the cumulative probability transition matrix. According to Piantadosi et al. [9], if the random number value was greater than the cumulative probability of the previous state but less than or equal to the cumulative probability of the following state, the following state was adopted. The cumulative probability transition matrix was obtained by successive multiplication of P matrix by itself, until a stabilization of the transition probabilities led to the transition probability matrix.

Thus if the transition probability in the i^{th} row at the k^{th} state was P_{ik}, then the cumulative probability, P_{ik}, could be expressed as

$$P_{ik} = \sum_{j=1}^{k} P_{ij} \qquad (5)$$

3. Results and Discussion

Fig. 1 shows the variation of annual rainfall in Owerri between 1976 and 2005. The total annual rainfall ranged between 2889.9 to 527.6 mm with a mean of 2319.7 mm and the standard deviation is 320.765 (Table 1).The Kurtosis and coefficient of skewness indicate platykurtic (kurtosis < 3) and negative skewness, which means that the probability plot has a flat tail at the left side. The data has 15% variability indicated by the coefficient of variation.

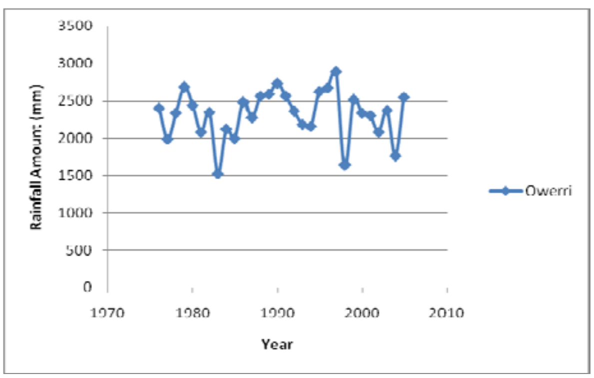

Fig. 2 Annual Variation of Owerri Rainfall from 1976 to 2005

A, B, C, D, E and F are the rainfall states obtained from the frequency distribution table shown in Table 2.

Table 1: Descriptive Statistics of Annual Rainfall Data of Owerri

Statistic	Mean	Standard Deviation	Skewness	Range	Sample Size
Value	2319.7	320.765	-0.7283	1362.3	30

Table 2: Frequency Distribution of Annual Rainfall Data

Class	Frequency	State
0-1527.6	1	A
1527.6-1800.06	2	B
1800.06-2072.52	2	C
2072.52-2344.98	9	D
2344.98-2617.44	11	E
>2617.44	5	F

Probability transition matrix P defines the Markov Chain presenting the six states of annual rainfall was obtained from equation (4) as

$$P = \begin{bmatrix} 0.4 & 0.4 & 0 & 0 & 0.2 & 0 \\ 0.1818 & 0.1818 & 0.3636 & 0.0909 & 0.0909 & 0.0909 \\ 0.2222 & 0.4444 & 0.2222 & 0.1111 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The cumulative probability transition matrix, P_c obtained from equation (3) as the cumulative summation of each row as

$$P_c = \begin{bmatrix} 0.4 & 0.8 & 0.8 & 0.8 & 1 & 1 \\ 0.1818 & 0.3636 & 0.3636 & 0.7272 & 0.8181 & 0.909 \\ 0.2222 & 0.6666 & 0.8888 & 1 & 1 & 1 \\ 0 & 0.5 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The transition probabilities show that if this year's rainfall interval is 0-1527.6mm(state A), there is a 40% chance that it will not vary next year, 40% chance that it will vary between 1527.6-1800.06mm (state B), and 40% chance that it will range between2344.98-2617.44 (state E).If this year's annual rainfall interval is 1800.06-2072.52 mm (state B), there is 18% chance that it will be in state A next year, 18% chance that it will not vary, 36% chance that it will be in state A, and 9% chance that it will be in states D, E or F. If this year's annual rainfall interval 1800.06-2072.52 mm (state C), there is 22% chance that it will vary between 0-1527.6 mm (state A) next year, 44% chance that it will vary in range of1800.06-2072.52 mm (state B), 22% chance that it will not vary, and 11% chance that it will range between 2072.52-2344.98mm (state D).

In Table 3, by Monte Carlo simulation, the change of annual rainfall intervals where generated as the random number changes values and assumes the various states of rainfall data. The synthetic annual rainfall series were obtained from the states by decomposing the states; where u is a random number from a uniform distribution u

$$\text{State} = \begin{cases} A \text{ if } u < 0.1818 \\ B \text{ if } 0.1818 < u < 0.3636 \\ C \text{ if } 0.3636 < u < 0.7272 \\ D \text{ if } 0.7272 < u < 0.8181 \\ E \text{ if } 0.8181 < u < 0.909 \\ F \text{ if } 0.909 < u < 1.0 \end{cases}$$

4. Conclusions

The results of the study show that Markov chain is useful in simulating future rainfall intervals. This type of simulation is useful in generating intervals of rainfall when the raingauge malfunctioned during small interval of time. The analysis of yearly rainfall shows that Markov Chain approach provides one alternative of modelling future variation in rainfall.

Table 3: Synthetic Generation of Annual Rainfall

Year	Random Number	State
1976	2401.7	B
1977	1988.8	D
1978	2336.7	C
1979	2685.9	A
1980	2440	B

References

[1] C.W. Richardson (1981). Stochastic simulation of daily precipitation, temperature, and solar radiation, Water Resour. Res. Vol. 17, No. 1, pp.182-190.

[2] A.O. Nnaji. Forecasting seasonal rainfalls for agricultural decision making in northern Nigeria. J. of Agric. and Forest Met. Vol. 107, 2001, pp. 193-205.

[3] R. Lazaro, F.S. Rodrigo, L. Gutierrez, J. Puigdefafregas. Analysis of a 30-year rainfall record (1967-1997) in Semi-arid SE Spain for implications on vegetation, J. Arid Environ.. Vol. 48, 2001, pp. 373 - 395.

[4] S. Tamil Selvi and R. Samuel Selvaraj. Stochastic Modelling of Annual Rainfall at Tamil Nadu. Universal Journal of Environmental Research and Technology, Volume 1, No. 4, 2011, pp. 566-570.

[5] M.Thyer and G. Kuzcera. Modelling long-term persistence in rainfall time series: Sydney rainfall case study. Hydrology and Water Resources Symposium, Institution of Engineer, Australia, 1999, pp.550-555.

[6] K. G. Mandal, J. Padhi, A. Kumar, D. K. Sahoo, P. Majhi, S. Ghosh, R. K. Mohanty and M. Roychaudhuri. Analyzing rainfall events and soil characteristics for water resources management in a canal irrigated area. Journal of Water Resources and Ocean Science Vol. 2, No. 1, 2013, pp.1-8.
doi: 10.11648/j. wros.20130201.11

[7] S. Shadeed, A. Jayyousi and H. Shaheen. Probability Distribution of Faria-Catchment Rainfall 2015

[8] H. A. Taha. Operations Research: An Introduction, 8th Ed, Prentice Hall of India, India, 2008.

[9] J. Piantadosi, J. W. Boland and P.G. Howlett. Generating synthetic rainfall on various timescales daily, monthly and yearly. J. Environ. Model Assess. Vol.14, 2008, pp. 431-438.

[10]C. Akajiaku, J. Chukwuocha and I. Igbokwe. Delineation and Characterization of Sub-catchments of Owerri, South East Nigeria, Using GIS. American Journal of Geographic Information System, Vol. 3, No.1, 2014, pp.1-9 doi:10.5923/j.ajgis.20140301.01

[11]D. S. Wilks. Adopting stochastic weather generation algorithms for climate change studies. Climatic Change, Vol. 22, 1992, pp. 67–84.