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Upper Bounds For Odd and Relaxed Even Graceful Aztech Diamond Chain Graphs

M.Antony Arockiasamy [1], R.Poornima [2] and Lincy [3]
[1,2] Department of Mathematics,
Sacred Heart College,
Tirupattur - 635601, Vellore District
Tamil Nadu, S.India.

Abstract: Aztech Diamond graph of order n is a dual graph to the Aztech Diamond of order n in which the vertices are the squares and and edge is formed between two vertices if and only if the corresponding squares are adjacent. In this article we investigate the Aztech diamond graphs in the light of labelling and derived upper bounds for odd and relaxed even graceful Aztech diamond chain graphs that are a result of merging vertices and edges.
Key words: Aztech Diamond Graph, odd graceful, relaxed even graceful, chain of Aztech diamond graph.
Subject Class Verification: 05C78

1 Introduction

Graph labeling is one of the fastest developing areas of research. Graph labeling is a function defined on the vertex set or edge set subject to certain conditions enforced on the number of vertices p or the number of edges q or on both p and q. There are more than more than 200 different types of labeling techniques developed in the in the last 6 decades published in over 1500 research articles. All these labeling techniques have their origin in the labeling given in[8]. For a detailed list of labeling techniques one can look at dynamic survey updated yearly by [5]. The focus of our study is of two folded. Primarily our focus is on labeling techniques, but the family of graphs that we have chosen is something different. We have taken the famous Aztec diamond graphs for our study of labeling. Aztec diamond graphs are the popularly known for their well established role in tiling. There have been numerous studys conducted on Aztec diamonds and Aztec diamond graphs.

If the vertices are assigned values subject to certain conditions then it is known as graph labeling. Graph labelings is an active area of research in graph theory which has rigorous applications incoding theory, communication networks, optimal circuits layouts and graph decomposition problems.According to [1] graph labeling serves as a frontier between number theory and the structure of graphs. For a dynamic survey of various graph labeling problems along with an extensive bibliography one can refer to [2].

The principal focus of this paper is to look at Aztec diamond graphs in different and new perspective of labeling techniques. Moreover we do not take the Aztec diamond graphs as it is. But using the Aztec diamond graphs of order n, we construct new twin Aztec diamond graphs and triple Aztec diamond graphs and finally we generate the n-chain of Aztec diamond graphs. The labeling function that we have used generates mainly from the simple order of the Aztec diamond graphs. That is by mere order and number of vertices q and number edges p, we can get the

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graceful labeling pattern of the given Aztec diamond graphs. And finally the upper bounds on the number of Aztec diamond graphs that form the odd graceful and relaxed even graceful is found.

2 Preliminaries

A function f is called graceful labeling of a graph G if $f: V(G) \rightarrow \{0,1,2,\cdots,q\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1,2,\cdots,q\}$ defined as $f^*(e=uv) = |f(u) - f(v)|$ is bijective. The graph which admits graceful labeling is called a graceful graph. A function f is called odd graceful labeling of a graph G if $f: V(G) \rightarrow \{0,1,2,\cdots,2q-1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1,3,\cdots,2q-1\}$ defined as $f^*(e=uv) = |f(u) - f(v)|$ is bijective. The graph which admits odd graceful labeling is called an odd graceful graph. The concept of odd graceful labeling was introduced by [4]. It is possible to decompose the graph Kn,n with suitable odd graceful labeling of tree T of order n. The splitting graphs of path Pn and even cycle Cn are proved to be odd graceful by [4] while ladders and graphs obtained from them by subdividing each step exactly once are shown odd-graceful by [4]. [10]proved many results on odd graceful labeling. The following three types of problems are considered generally in the area of graph labeling. 1. How particular labeling is affected under various graph operations; 2. Investigation of new families of graphs which admit particular graph labeling; 3. Given a graph theoretic property P, characterizing the class/classes of graphs with property P that admit particular graph labeling.

From the literature survey, it is clear that the problems of second type are largely studied than the problems of first and third types. The present work is aimed to discuss some problems of the first kind in the context of graceful and odd graceful labeling A graph G = (V (G), E (G)) is said to admit even graceful labeling if $f: V(G) \rightarrow \{0,1,2,\cdots(2q-1)\}$ is injective and the induced function $f^*: E(G) \rightarrow \{2,4,6,\cdots(2q-2)\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. A graph which admits even graceful labeling is called an even graceful graph.Let n be a positive integer. The Aztec diamond of order n is the union Aztec diamond of all the unit squares with integral vertices (x, y) satisfying $|x| + |y| \leq n+1$. The Aztec diamond of order 1 consists of the 4 unit squares which have the origin (0,0) as one of their vertices.The Aztec diamond graph of order n is the dual graph to the Aztec diamond of order n in which the vertices are the squares and an edge joins two vertices if and only if the corresponding squares are adjacent in the Aztec diamond.

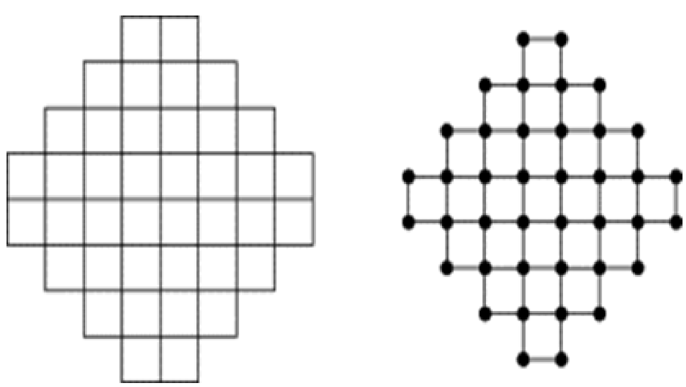

Figure 1: Aztec diamond And Aztec Diamond graph of order 4

We define a new labeling by relaxing a condition of even gracefulness, A graph $G=(V(G),E(G))$ is said to admit relaxed even graceful labeling if $f:V(G)\rightarrow\{0,1,2,\cdots,2q-1\}$ is injective and the induced function $f^*:E(G)\rightarrow\{2,4,6,\cdots,2q\}$ defined as $f^*(uv)=|f(u)-f(v)|$ is bijective. A graph which admits relaxed even graceful labeling is called an relaxed even graceful graph.

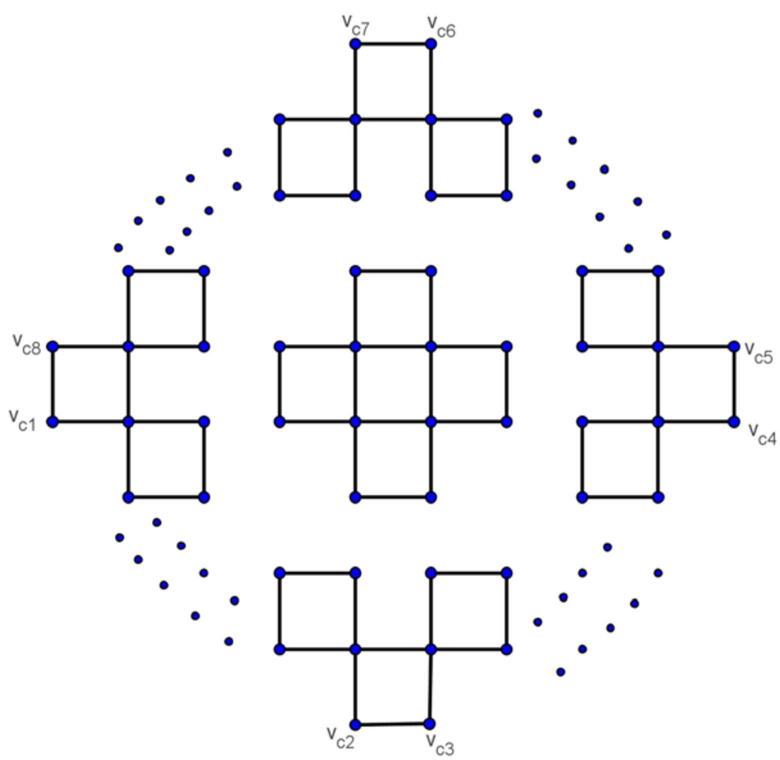

Figure 2: Aztec Diamond graph of order k $G(A,K)$

3 MAIN RESULTS

Theorem 3.1 *Any Aztec Diamond graph $G(A,k)$ is Odd graceful.*

Proof. The number of vertices in any Aztec diamond graph of order k is $(2k^2+k)$ and the number of edges in the Aztec diamond graph is $(4k^2)$.

It is found using finite differences relation. The $(2k)$ vertices in the shortest path $V_{C1}V_{C2}$ of G(A ,k) are labeled $V_{m1},V_{m2},\cdots,V_{m(2k)}$. For each vertex V_{mi} , $1\leq i\leq(2k)$, it follows that vertices diagonally opposite to V_{mi} and located upwards at distances $(2,4,6,\cdots 2k)$, are labeled as

$V_{mi+1}, V_{mi+3}, V_{mi+5}, \cdots, V_{mi+(2k)}$ respectively if i is even and $V_{mi-1}, V_{mi-3}, V_{mi-5}, \cdots, V_{mi\hat{a}\ (2k)}$ if i is odd respectively. For clarity and convenience, $V_{ma} = V_{m(2k-5)}, V_{mb} = V_{m(2k-3)}, V_{mc} = V_{m(2k-1)}$, $V_{md} = V_{m(2k)}$, $V_{me} = V_{m(2k+1)}$,are denoted in the following figure (3) .

A vertex labeling function $R: |V| \to 0,1,2,\cdots,2q-1$ on the vertex set of $G(A,k)$ is defined by,

$R(V_{m1}) = 8k^2$

$R(V_{m2}) = 0$

$R(V_{mi}) = R(V_{mj}) - 4k$ for $(i, j) = (3,1), (5,3), \cdots, (4k-1, 4k-3)$,

$R(V_{mi}) = R(V_{mj}) + (4k+2)$ for $(i, j) = (4,2), (6,4), (8,6), \cdots, 4k-2, 4k)$;

$R(V_{mi+a}) = R(V_{mi}) + a$,if i is even and $1 \le a \le 2k$;

$R(V_{mi-b}) = R(V_{mi}) - b$,if i is odd $1 \le b \le 2k$;

The induced edge set labeling $R(uv) = |R(u) - R(v)|$ for any edge $uv \in G(A,k)$ the resultant edge labels are distinct. Thus the Aztec diamond graph satisfies the conditions of odd graceful labeling. Hence any Aztec diamond graph $G(A,k)$ is odd graceful.

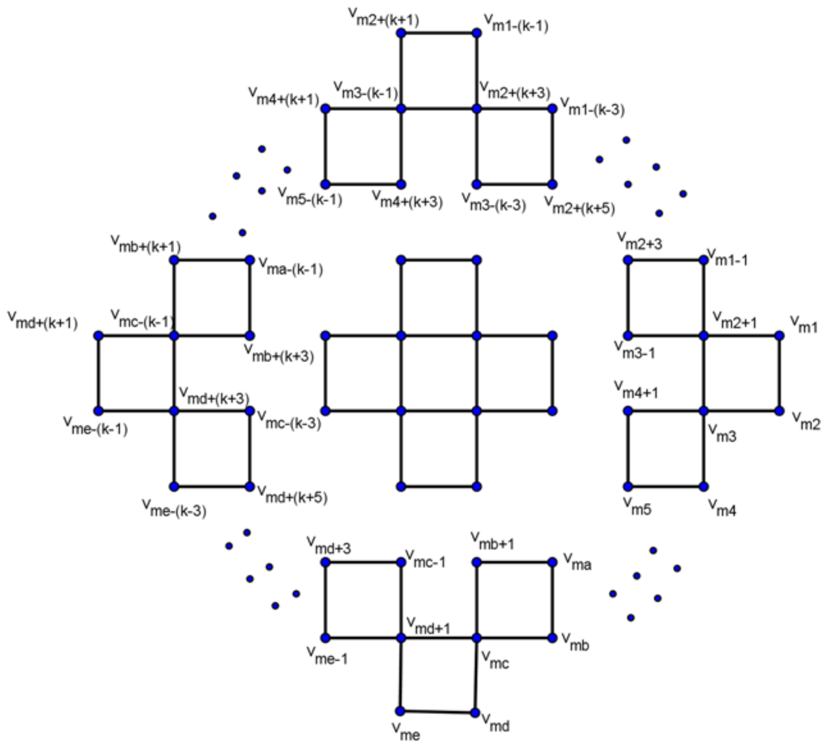

Figure 3: Odd graceful labeling pattern of $G(A,k)$

Example 3.2 *Aztec diamond graph of order 3 is shown to be odd graceful in fig 4.*

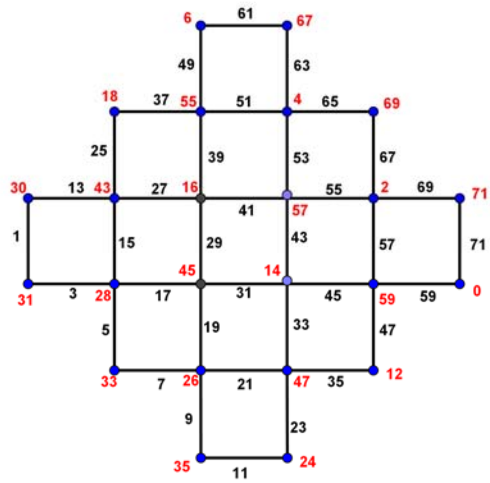

Figure 4: Odd graceful labeling of Aztec diamond graph of order 3

Theorem 3.3 *Any Aztec diamond graph G(A ,k) is relaxed even graceful.*

Proof. The number of vertices in any Aztec diamond graph of order k is $(2k^2+k)$ and the number of edges in the Aztec diamond graph is $(4k^2)$.

The $(2k)$ vertices in the shortest path $V_{C1}V_{C2}$ of $G(A,k)$ are labeled $V_{r1},V_{r2},\cdots,V_{w(2k)}$,.

For each vertex $V_{wi}, 1\le i\le(2k)$, it follows that vertices diagonally opposite to Vri and located upwards at distances $(2,4,6,\cdots 2k,)$ are labeled as $V_{wi+2},V_{wi+4},V_{wi+6},\cdots,V_(wi+2k)$ respectively if i is even and $V_{wi-2},V_{wi-4},V_{wi-6},\cdots,V_(wi-2k)$ if i is odd respectively.

For clarity and convenience, $V_{wa}=V_{w(2k-6)}, V_{wb}=V_{w(2k-4)}, V_{wc}=V_(w(2k-2)$, $V_{wd}=V_{w(2k)}$, $V_{we}=V_{r(2k+2)}$,are denoted in following figure (5).

A vertex labeling function $T:|V|\to 0,1,2,\cdots,2q-1$ on the vertex set of $G(A,k)$ is defined by,

$T(V_(w1))=8k^2$,

$T(V_{w2})=0$

$T(V_{wi})=T(V_{wj})-4k$ for $(i,j)=(3,1),(5,3),\cdots(4k-1,4k-3)$,

$T(V_{wi})=T(V_{wj})+(4k+2)$ for $(i,j)=(4,2),(6,4),(8,6)$â¦$(4k-2,4k)$;

$T(V_{wi+a})=T(V_{wi})+a$, if i is even and $1\le a\le 2k$;

$T(V_{wi-b})=T(V_{wi})-b$, if i is odd $1\le b\le 2k$.

By defining the edge set labeling $T(uv)=|T(u)$â $T(v)|$ for any edge $uv\in G(A,k)$, the resultant edge labels are distinct. Thus the diamond graph satisfies the conditions of relaxed even graceful labeling. Hence any Aztec diamond graph $G(A,k)$ is relaxed even graceful.

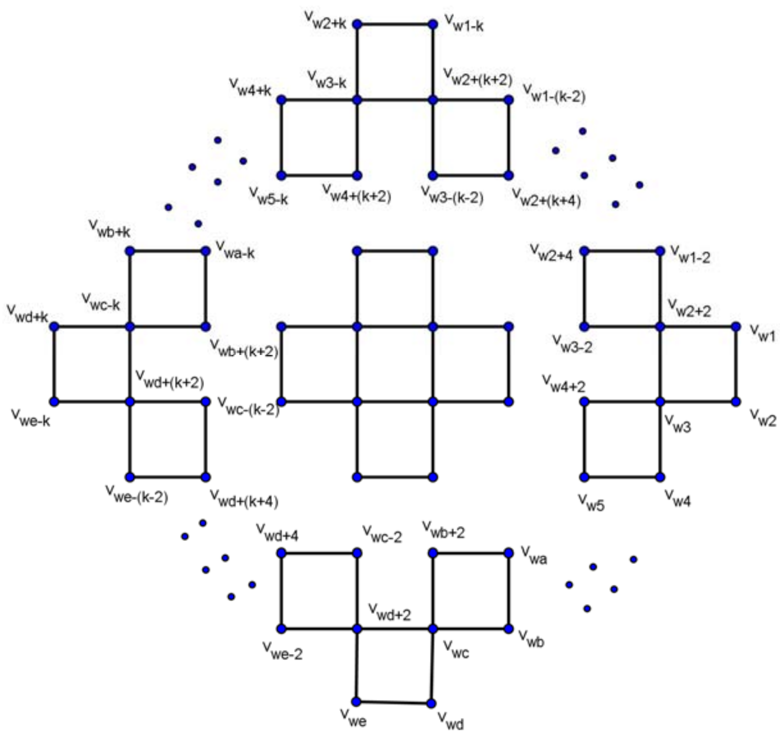

Figure 5: Relaxed even graceful labeling pattern of $G(A,k)$

Example 3.4 *Aztec Diamond graph of order 2 is relaxed even graceful.*

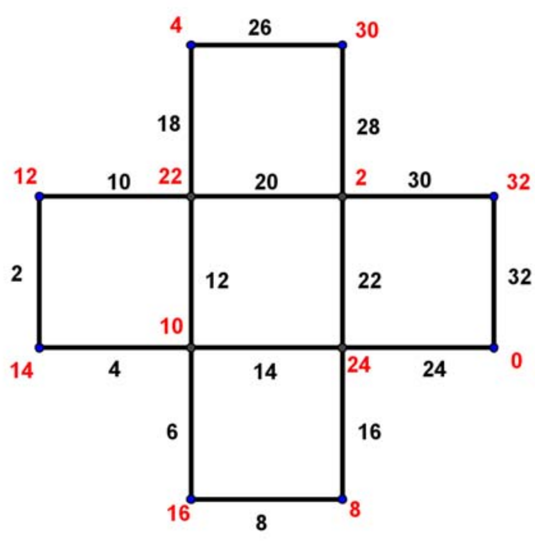

Figure 6: Relaxed even graceful labeling of $G(A,2)$

Definition 3.5 *Twin Aztec diamond graph*

Let $G_1(A,k)$ and $G_2(A,k)$ be any two Aztec diamond graphs of same order. A twin-diamond graph is the graph, obtained by joining any one of the corner vertices of $G_1(A,k)$, with any one of the corner vertices of $G_2(A,k)$ by an edge. The resulting twin-diamond graph is denoted by $G(A,k)$. The number of corner vertices in $G(A,k)$ is 6. The ends of the link edge are called corners of axis and they are denoted by V_{A1} and V_{A2}.Any $G(A,k)$ has 2 corners of axis and $4(k^2+k)$ vertices and $8k^2+1$ edges.

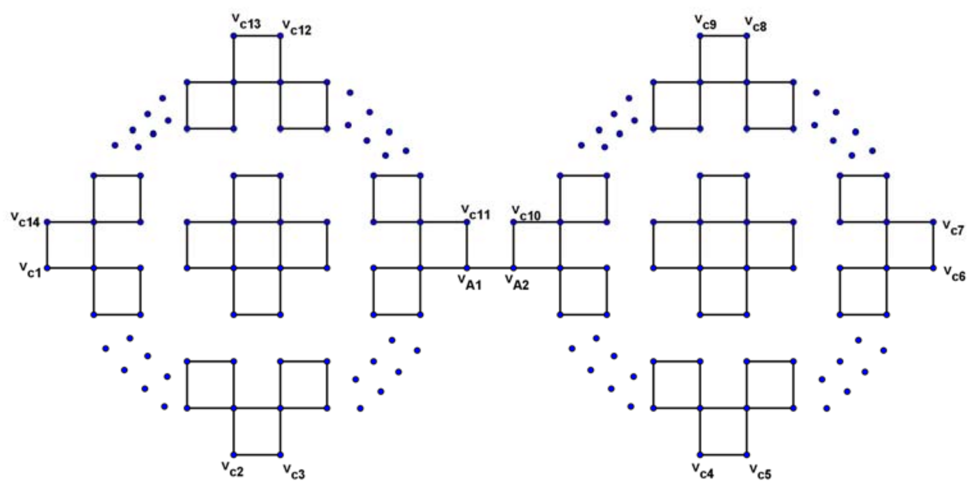

Figure 7: Twin Aztec Diamond graph

Theorem 3.6 *Any twin-Aztec diamond graph G(2A , k) is odd- graceful.*

Proof. The pattern of labeling we propose is of two fold. We take each diamond separately and give a pattern of labeling. A minor change of pattern is necessary in the connecting vertices. For the (2k) number of vertices in the shortest path $V_{C1}V_{C2}$ of G(2A ,k) and the labels $V_{m1},V_{m2},\cdots,V_{m(2k)}$,are assigned in order. For each vertex V_{mi}, $1\le i\le(2k+1)$, it follows that vertices diagonally opposite to V_{mi} and located upwards at distances $2,4,6,\cdots,(2k+1)$ from V_{mi} , are labeled as $V_{mi+1},V_{mi+3},V_{mi+5},\cdots,V_{mi+(2k)}$ respectively if i is even; and $V_{mi-1},V_{mi-3},V_{mi-5},\cdots,V_{mi-(k-1)}$ if i is odd respectively. consider the $(2k)$ vertices in the shortest path $V_{A2}V_{C4}$ (V_{A2} refers to corner of axis) of $G(2A,k)$ the labels $V_{n1},V_{n2},\cdots,V_{n(2k)}$ are given in order. For each vertex, V_{ni}, $1\le i\le(2k)$, it gives that vertices diagonally opposite to V_{ni} and located upwards at distances $2,4,6,\cdots,2k$ are allocated the labels $V_{ni+1},V_{ni+3},V_{ni+5},\cdots,V_{ni+2k}$ respectively if i is odd; and $V_{ni-1},V_{ni-3},V_{ni-5},\cdots,V_{ni-(2k-1)}$ if i is even respectively. For the sake of clarity and convenience, the following shorter forms are used in figure(8).

$V_{ma}=V_{n(2k-5)}$, $V_{mb}=V_{m(2k-3)}$, $V_{mc}=V_{m(2k-1)}$, $V_{md}=V_{m(2k)}$, $V_{me}=V_{m(2k+1)}$, $V_{na}=V_{n(2k-5)}$, $V_{nb}=V_{n(2k-3)}$, $V_{nc}=V_{n(2k-1)}$, $V_{nd}=V_{n(2k)}$, $V_{ne}=V_{n(2k+1)}$. A vertex labeling injective function

$R:|V| \to 0,1,2,\cdots,q$ is defined by $R(V_{m1}) = 8k^2$, $R(V_{m2}) = 0$

$R(V_{mi}) = R(V_{mj}) - (2k+2)$ for $(i, j) = (3,1), (5,3), \cdots, (4k-1, 4k-3)$;

$R(V_{mi}) = R(V_{mj}) + (2k+2)$ for $(i, j) = (4,2), (6,4), (8,6) \cdots, (4k, 4k-2)$;

$R(V_{mi+a}) = R(V_{mi}) + a$, if i is even, $1 \le a \le 2k$;

$R(V_{mi-b}) = R(V_{mi}) - b$, if i is odd, $1 \le b \le 2k$

with the exception that the range of a or b value extends upto 4k if â€˜iâ€™ = 4k . $R(V_{n1}) = 2k^2 + 5k$, $R(V_{n2}) = 6k^2 + 11k$

$R(V_{ni}) = R(V_{nj}) + (2k+2)$ for $(i, j) = (3,1), (5,3), \cdots (4k-1, 4k-3)$;

$R(V_{ni}) = R(V_{nj}) - (2k+2)$ for $(i, j) = (4,2), (6,4), (8,6), \cdots, (4k, 4k-2)$;

$R(V_{ni+a}) = R(V_{ni}) + a$, if i is odd, $1 \le a \le 2k$;

$R(V_{ni-b}) = R(V_{ni}) - b$, if i is even, $1 \le b \le 2k$.

The above pattern and vertex labeling function assigns labels to all but the 2-degree vertices in the path $V_{C1}V_{C3}$. These k vertices are labeled as $0,1,2,\cdots,2k-1$. This completes the entire set of vertices. By defining the edge set labeling $R(uv) = |R(u) - R(v)|$ for any edge $uv \in G(2A, k)$, the resultant edge labels are distinct. Thus the twin-Aztec diamond graph becomes odd-graceful.

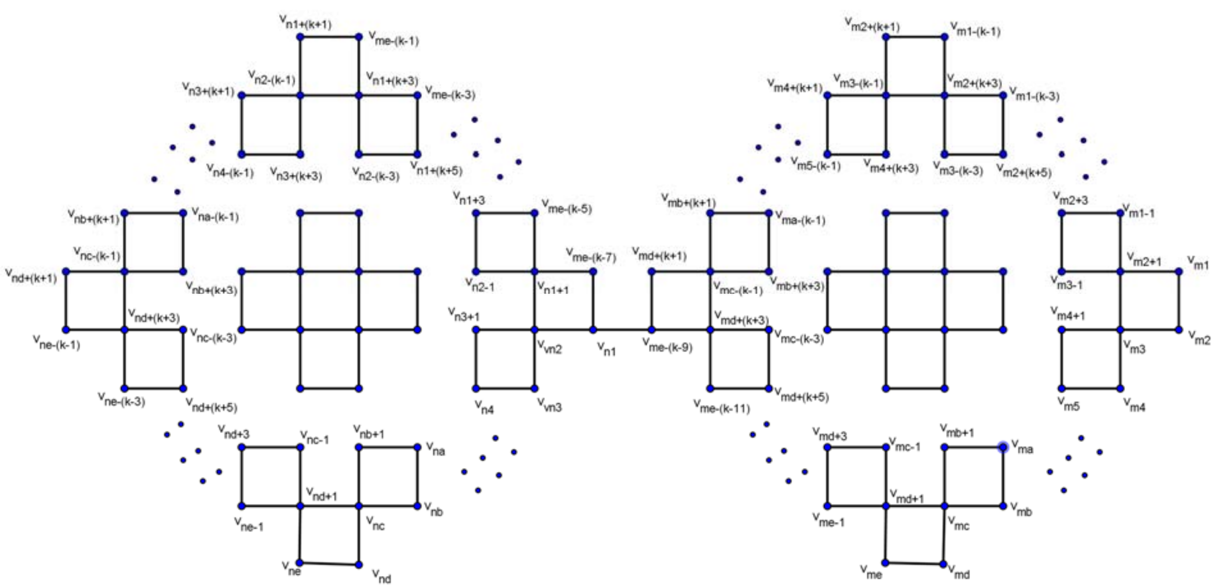

Figure 8: Vertex labeling pattern for Twin Aztec diamond graph

Example 3.7 *Twin Aztec diamond graph is odd graceful*

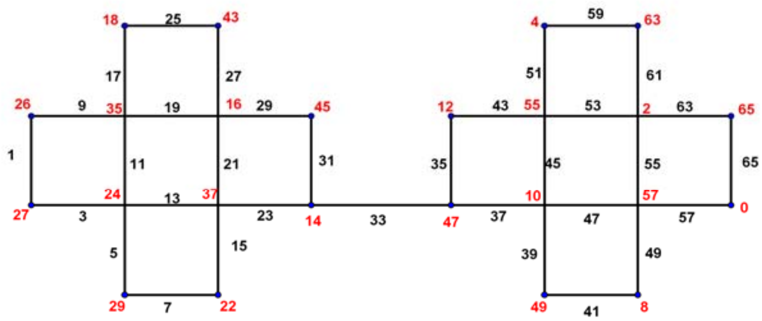

Figure 9: Odd Graceful Labeling of G(2A, 2)

Theorem 3.8 *Any twin-Aztec diamond graph G(2A , k) is relaxed even graceful.*

Proof. The pattern of labeling we propose is of two fold. We take each diamond separately and give a pattern of labeling. A minor change of pattern is necessary in the connecting vertices. For the $(2k)$ number of vertices in the shortest path $V_{C1}V_{C2}$ of $G(2A,k)$ and the labels $V_{w1},V_{w2},\cdots,V_{w(2k)}$,are assigned in order. For each vertex V_{wi} , $1\le i\le(2k+1)$, it follows that vertices diagonally opposite to V_{wi} and located upwards at distances $2,4,6,\cdots,(2k+1)$ from V_{wi} , are labeled as $V_{wi+2},V_{wi+4},V_{wi+6},\cdots,V_{wi+2k}$ respectively if i is even; and $V_{wi-2},V_{wi-4},V_{wi-6},\cdots,V_{wi-2k}$ if i is odd respectively. Consider the $(2k)$ vertices in the shortest path $V_{A2}V_{C4}$ (V_{A2} refers to corner of axis) of $G(2A,k)$ the labels $V_{z1},V_{z2},\cdots,V_{z(2k)}$ are given in order. For each vertex, V_{zi} , $1\le i\le(2k)$, it gives that vertices diagonally opposite to V_{zi} and located upwards at distances $2,4,\cdots 2k$ are allocated the labels $V_{zi-2},V_{zi-4},V_{zi-6},\cdots,V_{zi+2k}$ respectively if i is odd; and $V_{zi+2},V_{zi+4},V_{zi+6},\cdots,V_{zi-(2k)}$ if i is even respectively. For the sake of clarity and convenience, the following shorter forms are used in $V_{wa}=V_{w(2k-6)}$, $V_{wb}=V_{w(2k-4)}$, $V_{wc}=V_{w(2k-2)}$, $V_{wd}=V_{w(2k)}$, $V_{we}=V_{w(2k+2)}$, $V_{za}=V_{z(2k-6)}$, $V_{zb}=V_{z(2k-4)}$, $V_{zc}=V_{z(2k-2)}$, $V_{zd}=V_{z(2k)}$, $V_{ze}=V_{z(2k+2)}$. A vertex labeling injective function $T:|V|\rightarrow\{0,1,2,\cdots,q\}$ is defined by $T(V_{w1}) = 8k^2$

$T(V_{w2})=0$

$T(V_{wi}) = T(V_{wj})-(2k+2)$ for $(i,j)=(3,1),(5,3),$â¦$(4k-1,4k-3)$;

$T(V_{wi}) = T(V_{wj})+(2k+2)$ for $(i,j)=(4,2),(6,4),(8,6)$â¦$(4k,4k-2)$;

$T(V_{wi+a}) = T(V_{wi})+a$, if i is even, $1\le a\le 2k$;

$T(V_{wi-b}) = T(V_{wi})-b$, if i is odd, $1\le b\le 2k$ with the exception that the range of a or b value extends upto 4k if i = 4k . $T(V_{z1})=2k^2+5k$,

$T(V_{z2}) = 6k^2 + 11k$

$T(V_{zi}) = T(V_{zj}) + (2k+2)$ for $(i, j) = (3,1),(5,3),\cdots,(4k-1,4k-3)$; $T(V_{zi}) = T(V_{zj}) - (2k+2)$ for $(i, j) = (4,2),(6,4),(8,6),\cdots,(4k,4k-2)$; $T(V_{zi+a}) = T(V_{zi}) + a$,if i is odd, $1 \le a \le 2k$; $T(V_{zi-b}) = T(V_{zi}) - b$, if i is even, $1 \le b \le 2k$. The above pattern and vertex labeling function assigns labels to all but the 2-degree vertices in the path $V_{C1}V_{C3}$. These k vertices are labeled as $0,1,2,\cdots,2k-1$.This completes the entire set of vertices. By defining the edge set labeling $T(uv) = |T(u) - T(v)|$ for any edge $uv \in G(2A,k)$, the resultant edge labels are distinct.Thus the twin-Aztec diamond graph becomes relaxed even-graceful.

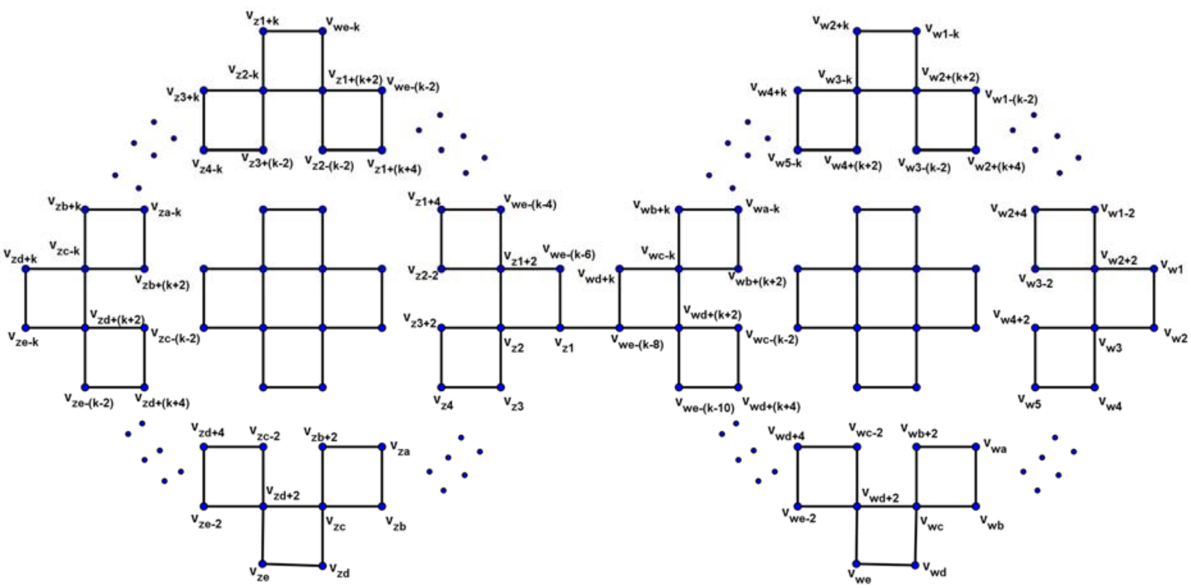

Figure 10: Relaxed Even Graceful Vertex labeling pattern of G(2A, k)

Example 3.9 *Twin Aztec diamond graph is relaxed even graceful.*

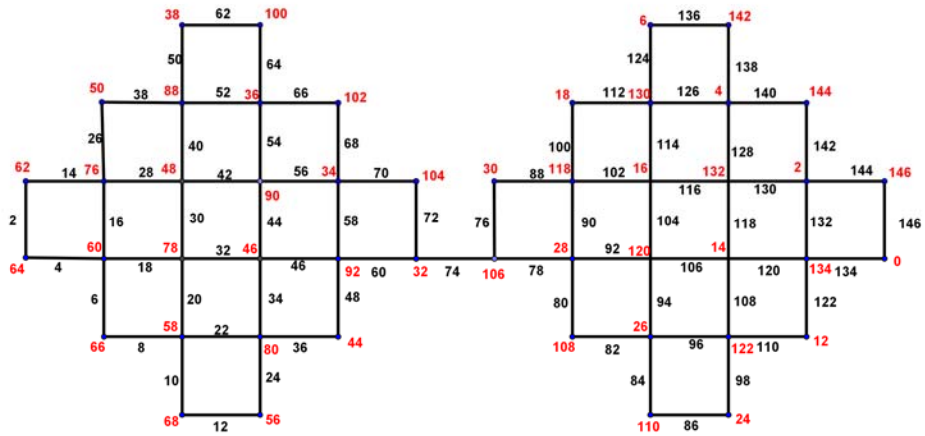

Figure 11: Relaxed Even Graceful Labeling of G(2A, 3)

Definition 3.10 *Triple Aztech Diamond Graph*

By joining an edge between any one of the corner vertices of the second diamond, of a twin-Aztec diamond graph, with any one of the corner vertices of a Aztec diamond graph, a triple-Aztec diamond graph is obtained and it is denoted by $G(3A,k)$. Any $G(3A,k)$ has 4 corners of axis and 8 corner vertices. It contains $(6k^2+6k)$ vertices and $(12k^2+2)$ edges.

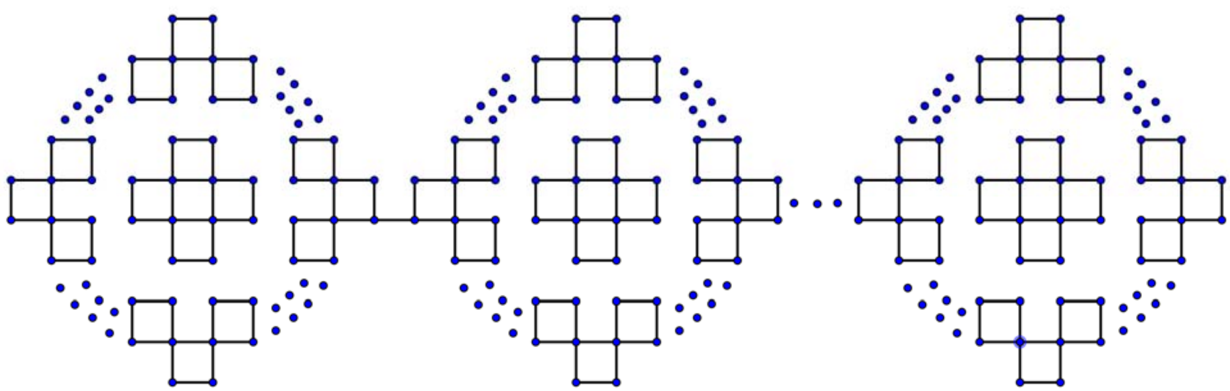

Figure 12: Triple chain Aztec diamond graph $G(3A,k)$

Theorem 3.11 *Any triple-Aztec diamond graph $G(3A,k)$ is odd graceful*

Proof. Using the proof similar to the one given in 3.6, the triple Aztec Diamond Graph can be shown to be Odd Graceful.

Example 3.12 *Triple Aztec diamond graph is odd graceful.*

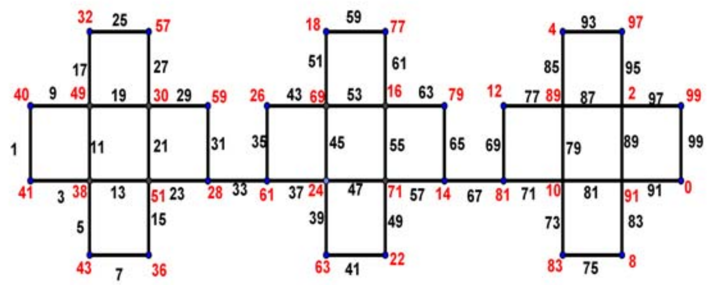

Figure 13: Odd graceful triple Aztec diamond graph

Theorem 3.13 *Any triple-Aztec diamond graph* $G(3A,k)$ *is relaxed even graceful*

Proof. Similar to a proof given for Twin Aztec Diamond Graph in 3.8, the triple Aztec diamond graph can be shown to be Relaxed Even Graceful.

Example 3.14 *Triple Aztec diamond graph* G(3A,2)*is relaxed even graceful.*

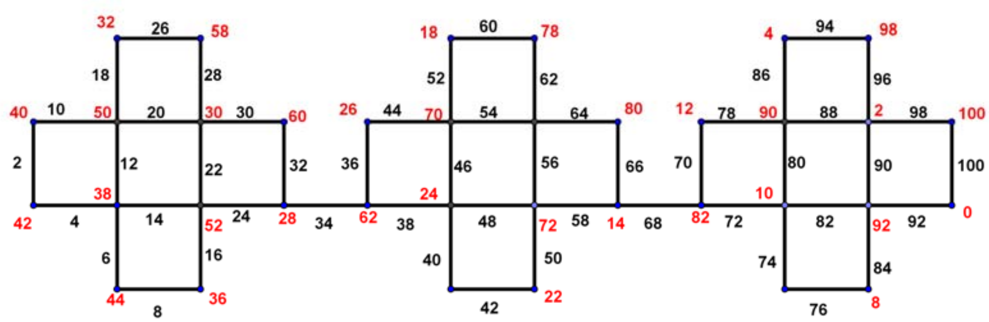

Figure 14: Relaxed even graceful labeling of G(3A,2)

Definition 3.15 ***n-Aztec diamond chain graph****:*

An n-Aztec diamond chain graph consists of n Aztec diamond graphs, each Aztec diamond graph linked by an edge in at least one corner vertex and at most two corner vertices.(First and nth Aztec diamonds graphs are linked in one corner vertex and the rest of the Aztec diamond graphs are linked in two corner vertices). The number of corner vertices in n-Aztec diamond chain graph is 2n+2. The order of the graph is k. It is denoted by G (nA ,k) and defined . It contains $(2nk^2 + 2k)$ vertices and $(nk^2 + n - 1)$ edges.

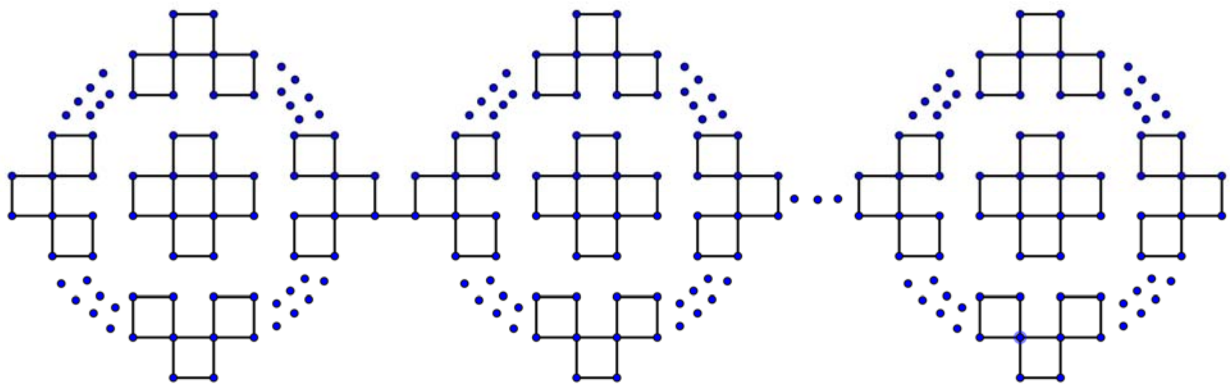

Figure 15: n - chain Aztec diamond graph

Theorem 3.16 *Any n- diamond chain graph G (nA ,k) is odd and relaxed even graceful.*

Proof.

Case-1: n is odd:

If the number of Aztec diamond graph in G (nA,k) is odd then the vertex labeling will be similar to triple-diamond graphs.

Case-2 :**n is even**:

when the number of diamonds in the n- diamond chain graph is even, the vertex labeling will be similar to twin-diamond graphs. And the edge labeling will be the same as defined for the lemmas and thus the odd gracefulness and relaxed even gracefulness of n-diamond chain graph can easily be verified.

Theorem 3.17 *Any n-chain Aztec diamond graph G(nA,k) of order k is m-odd graceful.*

Proof. By changing the vertex labeling function defined in theorem into $R:|V| \rightarrow \{m, m+1, m+2, \cdots, m+q+1\}$ the theorem can easily be verified.

Upperbounds:

For any chain of Aztec diamond graceful graph, the upper bound of the number of Aztec diamonds in the n-chain Aztec diamond graph is countably infinite.

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