On Intuitionistic Fuzzy Generalized γ closed sets

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Abstract: In this paper , we have introduced intuitionistic fuzzy generalized γ closed sets, and investigated some of their properties. Some characterization of the intuitionistic fuzzy generalized γ closed sets are also studied.

Keywords: *Intuitionistic fuzzy topology, intuitionistic fuzzy γ closed sets, intuitionistic fuzzy generalized γ closed sets.*

1. Introduction

In 1965, Zadeh[10] introduced fuzzy sets and in 1968, Chang[2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of these notions. The notion of intuitionistic fuzzy sets was introduced by Atanassov[1] as a generalization of fuzzy sets . In 1997, Coker[3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we have introduced intuitionistic fuzzy generalized γ closed sets and studied some of their properties.

2. Preliminaries

Definition 2.1[1]**:** An *intuitionistic fuzzy set*(IFS in short) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2[1]**:** Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

a) $A \subseteq B$ in and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
b) $A = B$ in and only if $A \subseteq B$ and $A \supseteq B$;
c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;
d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$;
e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3[3]**:** An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFS, in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4[6]**:** An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) *intuitionistic fuzzy γ closed set* (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$

(ii) *intuitionistic fuzzy γ open set* (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$

Definition 2.5[6]**:** Let A be an IFS in an IFTS (X, τ). Then the γ-interior and γ-closure of A are defined as

$\gamma int(A) = \cup$ {G / G is an IFγOS in X and $G \subseteq A$}
$\gamma cl(A) = \cap$ {K / K is an IFγCS in X and $A \subseteq K$}

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A)^c = (\gamma cl(A))^c$.

Result 2.6: Let A be an IFS in (X, τ), then

(i) $\gamma cl(A) \supseteq A \cup cl(int(A)) \cap int(cl(A))$

(ii) $\gamma int(A) \subseteq A \cap cl(int(A)) \cap int(cl(A))$

Proof: (i) Let A be an IFS in (X, τ). Now $cl(int(A)) \cap int(cl(A)) \subseteq cl(int(\gamma cl(A))) \cap int(cl(\gamma cl(A))) \subseteq \gamma cl(A)$, as γcl(A) is an IFγCS[6]. Since $A \subseteq \gamma cl(A)$ and γcl(A) is an IFγCS. Therefore $A \cup cl(int(A)) \cap int(cl(A)) \subseteq \gamma cl(A)$.

(ii) can be proved easily by taking complement in(i).

3. Intuitionistic fuzzy generalized γ closed sets

In this section we have introduced intuitionistic fuzzy generalized γ closed sets and studied some of their properties. Also we have provided some of the characterization.

Definition 3.1: An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy generalized γ closed set* (IFGγCS for short) if $\gamma cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). The family of all IFGγCSs of an IFTS (X, τ) is denoted by IFGγC(X).

Example 3.2: Let X = {a, b} and let τ = {0~, G₁, G₂, 1~} is an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ be an IFS in X. Then, IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1\}$. We have $A \subseteq G_1$. Now $\gamma cl(A) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle = G_1 \subseteq G_1$. Therefore A is an IFGγCS in X.

Theorem 3.3: Every *intuitionistic fuzzy closed* set(IFCS in short) in (X, τ) is an IFGγCS in (X, τ) but not conversely.

Proof: Let A be an IFCS in X and let $A \subseteq U$ and U be an IFOS in (X,τ). As $\gamma cl(A) \subseteq cl(A) = A \subseteq U$. We have $\gamma cl(A) \subseteq U$. Therefore A is an IFGγCS.

Example 3.4: Let X = {a, b} and let τ = {0~, G₁, G₂, 1~} is an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ is an IFS in X. Then, IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1\}$. We have $A \subseteq G_1$. Now $\gamma cl(A) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle = G_1 \subseteq G_1$. This implies that A is an IFGγCS in X. Now since $cl(A) = G_1^c \neq A$. Therefore A is not an IFCS in X.

Theorem 3.5: Every *intuitionistic fuzzy generalized closed set* (IFGCS in short) in (X, τ) is an IFGγCS in (X, τ) but not conversely.

Proof: Let A be an IFGCS[9] in X and let $A \subseteq U$ and U be an IFOS in (X,τ). As $\gamma cl(A) \subseteq cl(A) \subseteq U$. We have $\gamma cl(A) \subseteq U$. Therefore A is an IFGγCS.

Example 3.6: Let X = {a, b} and let τ = {0~, G₁, G₂, 1~} is an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ is an IFS in X. Then, IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1\}$. We have $A \subseteq G_1$. Now $\gamma cl(A) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle = G_1 \subseteq G_1$. Therefore A is an IFGγCS in X. Now since $cl(A) = G_1^c \nsubseteq G_1$. Therefore A is not an IFGCS in X.

Theorem 3.7: Every *intuitionistic fuzzy regular closed* set(IFRCS in short) in (X, τ) is an IFGγCS in (X, τ) but not conversely.

Proof: Since every IFRCS[9] is an IFCS, by theorem 3.3 A is an IFGγCS.

Example 3.8: Let X = {a, b} and let τ = {0~, G₁, G₂, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)\rangle$ is an IFS in X.

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Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. We have A ⊆ G_1. Now γcl(A) = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 ⊆ G_1. This implies that A is an IFGγCS in X. Now since cl(int(A)) = cl(G_2) = G_1^c ≠ A. Therefore A is not an IFRCS in X.

Theorem 3.9: Every *intuitionistic fuzzy semi closed* set(IFSCS in short) in (X, τ) is an IFGγCS in (X, τ) but not conversely.

Proof: Let A be an IFSCS[5] in X and let A ⊆ U and U be an IFOS in (X, τ). As γcl(A) ⊆ scl(A) =A, by hypothesis A ⊆ U. Hence γcl(A) ⊆ U. Therefore A is an IFGγCS.

Example 3.10: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩ G_2 = ⟨x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)⟩. Then the IFS A = ⟨x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)⟩ is an IFS in X. Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. We have A □ G_1. Now γcl(A) = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 ⊆ G_1. This implies that A is an IFGγCS in X. Now since int(cl(A)) = int(G_1^c) = G_1 ⊈ A. Therefore A is not an IFSCS in X.

Theorem 3.11: Every *intuitionistic fuzzy α closed set(*IFαCS in short) in (X, τ) is an IFGγCS in (X, τ) but not conversely.

Proof: Let A be an IFαCS[5] in X and let A □ U and U be an IFOS in (X, τ). Since γcl(A) □ αcl(A) = A by hypothesis γcl(A) □ U. Therefore A is an IFGγCS.

Example 3.12: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩ G_2 = ⟨x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)⟩. Then the IFS A = ⟨x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)⟩ is an IFS in X. Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. We have A □ G_1. Now γcl(A) = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 ⊆ G_1. This implies that A is an IFGγCS in X. Now since cl(int(cl(A))) = cl(int(G_1^c)) = cl(G_1) = G_1^c ⊈ A, A is not an IFαCS in X.

Theorem 3.13: Every *intuitionistic fuzzy pre closed set(*IFPCS in short) in (X, τ) is an IFGγCS in (X, τ) but not conversely.

Proof: Let A be an IFPCS[5] in X and let A □ U and U be an IFOS in (X, τ). As γcl(A) □ pcl(A) = A by hypothesis, γcl(A) □ U. Therefore A is an IFGγCS.

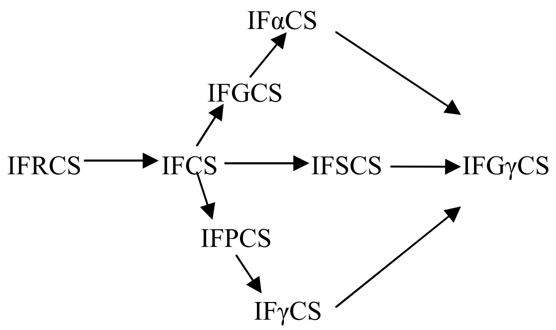
Example 3.14: Let X={a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩, G_2 = ⟨x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)⟩. Then the IFS A = ⟨x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)⟩ is an IFS in X. Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. We have A □ G_1. Now γcl(A) = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 ⊆ G_1. This implies that A is an IFGγCS in X. Now since cl(int(A)) = cl(G_2) = G_1^c ⊈ A, A is not an IFPCS in X.

Theorem 3.15: Every *intuitionistic fuzzy γ closed set*(IFγCS in short) in (X, τ) is an IFGγCS in (X, τ) but not conversely.

Proof: Let A be an IFγCS in X. Then γcl(A) = A[6]. Let A ⊆ U and U be an IFOS in (X, τ). By hypothesis γcl(A) ⊆ U. Therefore A is an IFGγCS.

Example 3.16: Let X={a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩, G_2 = ⟨x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)⟩. Then the IFS A = ⟨x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)⟩ is an IFS in X. Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. We have A ⊆ G_1. Now γcl(A) = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 ⊆ G_1. This implies that A is an IFGγCS in X. Now since cl(int(A)) ∩ int(cl(A)) = cl(G_2) ∩ int(G_1^c) = G_1^c ∩ G_1 = G_1 ⊈ A, A is not an IFγCS in X.

In the following diagram, we have provided relation between various types of intuitionistic fuzzy closedness.



The reverse implications are not true in general in the above diagram.

Remark 3.17: The union of any two IFGγCSs is not an IFGγCS in general as seen in following example.

Example 3.18: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, $(0.6_a, 0.8_b)$, $(0.4_a, 0.2_b)$⟩, G_2 = ⟨x, $(0.5_a, 0.5_b)$, $(0.4_a, 0.4_b)$⟩. Then the IFSs A = ⟨x, $(0.5_a, 0.4_b)$, $(0.4_a, 0.5_b)$⟩, B = ⟨x, $(0.4_a, 0.6_b)$, $(0.5_a, 0.2_b)$⟩ are IFGγCSs in (X, τ). We have IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a < 0.5$ whenever $\mu_b \geq 0.5$, $\mu_b < 0.5$ whenever $\mu_a \geq 0.5$ and $0 \leq \mu_a + \nu_a \leq 1$, and $0 \leq \mu_b + \nu_b \leq 1$}. Now γcl(A) = A. We have A ⊆ G_1 and A ⊆ G_2. We have γcl(A) =A. Therefore γcl(A) ⊆ G_1, whenever A ⊆ G_1 and γcl(A) ⊆ G_2, whenever A ⊆ G_2, where G_1 and G_2 are IFOS in X. This implies A is an IFGγCS in X. We have B ⊆ G_1, since γcl(B) = B. Therefore γcl(B) ⊆ G_1. Hence B is an IFGγCS in X. Now A ∪ B = ⟨x, $(0.5_a, 0.6_b)$, $(0.4_a, 0.2_b)$⟩ □ G_1 but γcl(A ∪ B) = 1~ ⊈ G_1.

Remark 3.19: The intersection of two IFGγCSs is not an IFGγCS in general as seen in the following example.

Examle 3.20: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, $(0.5_a, 0.7_b)$, $(0.5_a, 0.3_b)$⟩, G_2 = ⟨x, $(0.5_a, 0.6_b)$, $(0.5_a, 0.4_b)$⟩. Then the IFSs A = ⟨x, $(0.5_a, 0.9_b)$, $(0.5_a, 0.1_b)$⟩, B = ⟨x, $(0.5_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ are IFGγCSs in (X, τ) but A ∩ B is not an IFGγCS in (X, τ). IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}. Now A ⊆ 1~ and γcl(A) = 1~ ⊆ 1~, which implies A is an IFGγCS in X. We have B ⊆ 1~ and γcl(B) = 1~ ⊆ 1~. Therefore B is an IFGγCS in X. Now A ∩ B = ⟨x, $(0.5_a, 0.7_b)$, $(0.5_a, 0.3_b)$⟩ □ G_1 but γcl(A ∩ B) = 1~ ⊈ G_1.

Theorem 3.21: Let (X, τ) be an IFTS. Then for every A ϵ IFGγC(X) and for every B ϵ IFS(X), A □ B □ γcl(A) ⇒ B ϵ IFGγC(X).

Proof: Let B □ U and U be an IFOS. Then since A □ B, A □ U, by hypothesis B □ γcl(A). Therefore γcl(B) □ γcl(γcl(A)) = γcl(A) □ U, since A is an IFGγCS. Hence B ϵ IFGγC(X).

Theorem 3.22: An IFS A of an IFTS (X, τ) is an IFGγCS if and only if A_q ^{c} F ⇒ $γcl(A)_q$ ^{c} F for every IFCS F of X.

Proof: Necessity: Let F be an IFCS and A_q ^{c} F, then A □ F^c [8], where F^c is an IFOS. Then γcl(A) □ F^c , by hypothesis. Hence by definition, $γcl(A)_q$ ^{c} F.

Sufficiency: Let U be an IFOS such that A □ U. Then U^c is an IFCS and A □ $(U^c)^c$. By hypothesis, A_q ^{c} U^c ⇒ $γcl(A)_q$ ^{c} U^c. Hence γcl(A) □ $(U^c)^c$ = U. Therefore γcl(A) □ U. Hence A is an IFGγCS.

Theorem 3.23: If A is an IFOS and an IFGγCS in (X, τ) then A is an IFγCS in (X, τ).

Proof: Since A □ A and A is an IFOS, by hypothesis γcl(A) □ A. But A □ γcl(A). Therefore γcl(A) = A. Hence A is an IFγCS in (X, τ).

Theorem 3.24: Let F □ A □ X where A is an IFOS and an IFGγCS in X. Then F is an IFGγCS in A if and only if F is an IFGγCS in X.

Proof: Necessity: Let U be an IFOS in X and F □ U. Also let F be an IFGγCS in A. Then clearly F □ A ∩ U and A ∩ U is an IFOS in A. Hence gamma closure of F in A, $γcl_A(F)$ □ A ∩ U and by Theorem 3.23, A is an IFγCS. Therefore γcl(A) = A. Now gamma closure of F in X, γcl(F) □ γcl(F) ∩ γcl(A) = γcl(F) ∩ A = $γcl_A(F)$ □ A ∩ U □ U. That is γcl(F) □ U, whenever F □ U. Hence F is an IFGγCS in X.

Sufficiency: Let V be an IFOS in A. Such that F □ V. Since A is an IFOS in X, V is an IFOS in X. Therefore γcl(F) □ V. Since F is an IFGγCS in X. Thus, $\gamma cl_A(F)$ = γcl(F) ∩ A □ V ∩ A □ V. Hence F is an IFGγCS in A.

Theorem 3.25: An IFS A which is both an IFOS and an IFGγCS then A is an IFROS.

Proof: Let A be an IFOS and an IFGγCS. Then γcl(A) □ A and int(cl(A)) ∩ cl(int(A)) □ A by result 2.6. Since A is an IFOS, int(A) =A. Therefore int(cl(A)) ∩ cl(A) □ A ⇒ in(cl(A)) □ A. Since A is an IFOS, it is an IFPOS. Hence A □ int(cl(A)). Therefore A = int(cl(A)). Hence A is an IFROS[8].

Theorem 3.26: For an IFOS A in (X,τ), the following conditions are equivalent.

(i) A is an IFCS

(ii) A is an IFGγCS and an IFQ-set

Proof: (i) ⇒ (ii) Since A is an IFCS, it is an IFGγCS by Theorem 3.3. Now int(cl(A)) = int(A) = A = cl(A) = cl(int(A)), by hypothesis. Hence A is an IFQ-set[7].

(ii) ⇒ (i) Since A is an IFOS and an IFGγCS, by Theorem 3.25, A is an IFROS. Therefore A = int(cl(A)) = cl(int(A)) = cl(A), by hypothesis. Hence A is an IFCS.

Theorem 3.27: Let A be an IFGγCS in (X,τ) and $p_{(\alpha,\beta)}$ be an *intuitionistic fuzzy point* [5](IFP in short) in X such that $p_{(\alpha,\beta)}\ _q$ γcl(A), then $cl(p_{(\alpha,\beta)})\ _q$ A.

Proof: Let A be an IFGγCS and let $p_{(\alpha,\beta)}\ _q$ γcl(A). If $cl(p_{(\alpha,\beta)})\ _q^{\ c}$ A, then A □ $[cl(p_{(\alpha,\beta)})]^c$ [5] where $[cl(p_{(\alpha,\beta)})]^c$ is an IFOS. By hypothesis, γcl(A) □ $[cl(p_{(\alpha,\beta)})]^c$ □ $(p_{(\alpha,\beta)})^c$. This implies that $p_{(\alpha,\beta)}\ _q^{\ c}$ γcl(A), which is a contradiction to the hypothesis. Hence $cl(p_{(\alpha,\beta)})\ _q$ A.

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