

## **Fuzzy Transportation Problem of Hexagon Number with $\alpha$ - cut and Ranking Technique**

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### **Abstract**

In this paper we are presenting a ranking technique with alpha cut optimal solution for solving transportation problem, where fuzzy demand and supply are all in the form of Hexagonal fuzzy number. The main aim of this paper is to introduce a new operation for addition, subtraction, multiplication of Hexagonal fuzzy numbers on the basis of alpha cuts sets of fuzzy numbers. In an organisation, where a number of alternatives and variables such as production, inventory, financial management, costing and various other parameters are involved. This ranking procedure serves as an efficient method wherein a numerical example is also taken and the inference is given.

### **Keywords**

Robust Ranking method, Hexagonal fuzzy numbers,  $\alpha$ -optimal solution, Fuzzy transportation problem.

### **1. Introduction**

The Fuzzy Transportation Problem (FTP) is one of the special kinds of fuzzy linear programming problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. To deal quantitatively with imprecise

information in making decisions Bellman and Zadeh and Zadeh introduced the notion of fuzziness. Fuzzy transportation is the transportation of fuzzy quantity from the fuzzy origin to fuzzy destination in such a way that the total fuzzy transportation cost is minimum. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost, while satisfying fuzzy supply and demand limits.

The Fuzzy set Theory has been applied in many fields such as Management, Engineering etc. In this paper a new operation on Hexagonal Fuzzy number is defined where the methods of addition, subtraction, and multiplication has been modified with some conditions.

## **2. Preliminaries**

### **2.1 Definition**

Let  $X$  be a nonempty set. A fuzzy set  $A$  in  $X$  is characterized by its membership function  $A : X \rightarrow [0, 1]$ , where  $A(x)$  is interpreted as the degree of membership of element  $x$  in fuzzy  $A$  for each other  $x \in X$ .

### **2.2 Interval Number**

Let  $R$  be the set of real numbers. Then closed interval  $[a, b]$  is said to be an interval number, where  $a, b \in R, a \leq b$ .

### **2.3 Fuzzy Number**

A Fuzzy set  $A$  of the real line  $R$  with membership function

$\mu_{\tilde{A}}(X) : R \rightarrow [0, 1]$  is called fuzzy number if

- i.  $A$  must be normal and convex fuzzy set;

- ii. The support of  $\tilde{A}$ , must be bounded
- iii.  $\alpha_A$  must be closed interval for every  $\alpha \in [0, 1]$

### 3. Hexagon Fuzzy Number

The Fuzzy number H is a hexagonal fuzzy  $\tilde{A}_H$  is a hexagonal fuzzy number denoted  $\tilde{A}_H(a, b, c, d, e, f; 1)$  and its member function  $\mu_{\tilde{A}_H}(x)$  is give below.

$\mu_{\tilde{A}_H}(x) =$	$y - a / b - a$	$a \leq y \leq b$
	$1$	$b \leq y \leq c$
	$d - y / d - c$	$c \leq y \leq d$
	$0$	Otherwise
	$y - c / d - c$	$c \leq y \leq d$
	$1$	$d \leq y \leq e$
	$f - y / f - e$	$e \leq y \leq f$
	$0$	Otherwise

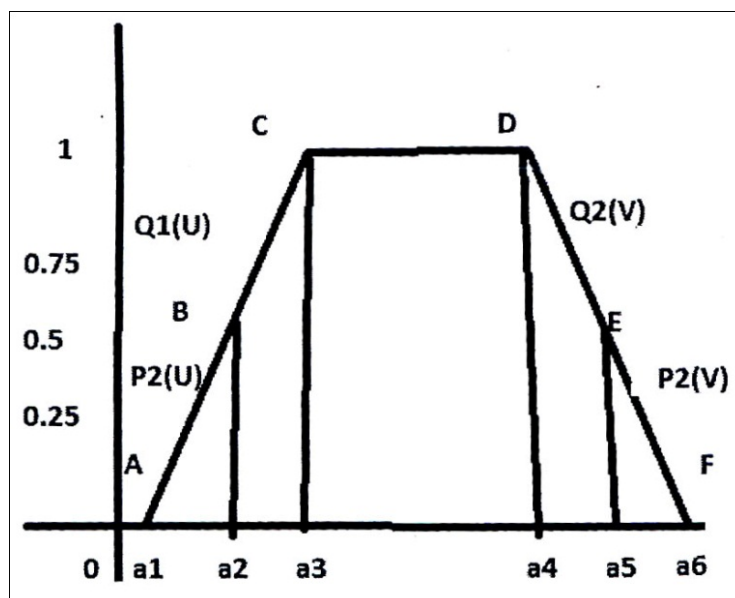


Figure 1 : Graphical representation of a Hexagonal fuzzy number

### 3.1 Arithmetic operations on Hexagonal fuzzy number

Let  $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$  and  $\tilde{N}_H = (n_1, n_2, n_3, n_4, n_5, n_6)$  be two hexagonal fuzzy numbers, then

- i.  $\tilde{A}_H (+)\tilde{N}_H = (a_1 + n_1, a_2 + n_2, a_3 + n_3, a_4 + n_4, a_5 + n_5, a_6 + n_6)$
- ii.  $\tilde{A}_H (-)\tilde{N}_H = (a_1 + n_1, a_2 + n_2, a_3 + n_3, a_4 + n_4, a_5 + n_5, a_6 + n_6)$
- iii.  $\tilde{A}_H (*)\tilde{N}_H = (a_1 + n_1, a_2 + n_2, a_3 + n_3, a_4 + n_4, a_5 + n_5, a_6 + n_6)$

### 3. Robust Ranking Technique

Robust Ranking Technique satisfies the following properties,

- i. Compensation
- ii. Linearity
- iii. Additivity

It provides results which are consist human intuition . If  $\tilde{A}$  is a fuzzy number then the Robust ranking is defined by

$$R(\tilde{A}_H) = \int_0^1 (0.5) (a_{h\alpha}^L, a_{h\alpha}^U) d\alpha \text{ where } (a_{h\alpha}^L, a_{h\alpha}^U)$$

Is the  $\alpha$  level cut of the fuzzy number  $\tilde{A}_H$ . In this paper we find the rank of the objective

### Numerical Example

A Company has six sources  $S_1, S_2, S_3, S_4, S_5$  and  $S_6$  and six destination  $D_1, D_2, D_3, D_4, D_5$  and  $D_6$ ; the fuzzy transportation cost for unit quantity of the product from  $i^{th}$  source  $j^{th}$  destination is

$B_{ij}$  Where  $[b_{ij}]_{3 \times 4} = (1,2,3,4,5,6) (1,3,4,6,7,8) (8,9,7,8,6,5,4) (2,6,5,4,3,2)$   
 $(3, 6, 5, 4, 3, 2) (2,3,5,6,7,5) (4,7,6,5,2,1) (3,4,5,6,7,5)$   
 $(1,5,6,7,6,2) (1,8,7,6,5,6) (5,9,4,6,7,6) (8,7,1,0,6,5)$

And fuzzy availability of the product at source are  $(2,3,5,6,2,1)$ ,  $(5,10,12,17,11,10)$ ,  $(8,10,12,12,6,4)$  and the fuzzy demand of the product at destination are  $(5,8,8,7,5,4)$ ,  $(5,1,6,7,5,4,2)$ ,  $(2,3,1,3,5,7)$  and  $(3,6,9,12,15,13,18)$ .

Then the problem becomes as

**Table – 1**

	<b>FD1</b>	<b>FD2</b>	<b>FD3</b>	<b>FD4</b>	<b>Supply</b>
$FS_1$	(1,2,3,4,5,6)	(1,3,4,6,7,8)	(8,9,7,6,5,4)	(2,6,5,4,3,2)	(2,3,5,6,2,1)
$FS_2$	(3,6,5,4,3,2)	(2,3,5,6,7,5)	(4,7,6,5,2,1)	(3,4,5,6,7,5)	(5,10,12,17,11,10)
$FS_3$	(1,5,6,7,6,2)	(1,8,7,6,5,6)	(5,9,4,6,7,6)	(8,7,1,0,6,5)	(8,10,12,12,6,4)
Demand	(5,8,8,7,5,4)	(5,1,6,7,5,2)	(2,3,1,3,5,7)	(3,6,9,12,15,18)	

### Solution

The fuzzy transportation problem can be formulated in the following mathematical programming from

$$\text{Min } x = R(1,2,3,4,5,6) y_{11} + R(1,3,4,6,7,8) y_{12} + R(8,9,7,6,5,4) y_{13} + R(2,6,5,6,7,5) y_{14} + R(3,6,5,4,3,2) y_{21} + R(2,3,5,6,7,5) y_{22} + R(4,7,6,5,2,1) y_{23} + R(3,4,5,6,7,5) y_{24} + R(1,5,6,7,6,2) y_{31} + R(1,8,7,6,5,6) y_{32} + R(5,9,4,6,7,6) y_{33} + R(8,7,0,6,5) y_{34}$$

$$R(H) = \int_0^1 (0.5) (a_{h\alpha}^L, a_{h\alpha}^U) d\alpha$$

$$\text{Where } = \{(b-a)\alpha + a, d - (d-c)\alpha\} + \{(d-c)\alpha + c, f - (f-e)\alpha\}$$

$$R(1, 2, 3, 4, 5, 6) = \int_0^1 (0.5) (\alpha + 1 + 4 - \alpha + \alpha + 3 + 6 - \alpha) d\alpha$$

Similarly

$$R(1,3,4,6,7,8) = 9.75; R(8,9,7,6,5,4) = 13; R(2,6,5,4,3,2) = 7.75; R(3,6,5,4,3,2) = 8$$

$$R(2,3,5,6,7,8) = 9.75; R(4,7,6,5,2,1) = 9; R(3,4,5,6,7,5) = 10.25; R(1,5,6,7,6,2) = 10; R(1,8,7,6,5,6) = 11.5; R(5,9,4,6,7,6) = 11.75; R(8,7,1,0,6,5) = 7;$$

Rank of all supply

$$R(8,10,12,12,6,4) = 19; R(2,3,5,6,2,1) = 9.75; R(2,3,1,3,5,7) = 6.25;$$

$$R(3, 6, 9, 12, 15, 18) = 21;$$

Table after ranking

**Table 2**

	<b>FD<sub>1</sub></b>	<b>FD<sub>2</sub></b>	<b>FD<sub>3</sub></b>	<b>FD<sub>4</sub></b>	<b>Supply</b>
FS <sub>1</sub>	7	9.75	13	7.75	19
FS <sub>2</sub>	8	9.75	9	10.25	7.5
FS <sub>3</sub>	10	11.5	11.25	7	23.5
Demand	13	9.75	6.25	21	

Table after applying Matrix Minima method

**Table 3**

	<b>FD<sub>1</sub></b>	<b>FD<sub>2</sub></b>	<b>FD<sub>3</sub></b>	<b>FD<sub>4</sub></b>	<b>Supply</b>
FS <sub>1</sub>			<b>7</b>		
	7	9.75	13	7.75	19
FS <sub>2</sub>		<b>6</b>		<b>9.75</b>	
	8	9.75	9	10.25	7.5
FS <sub>3</sub>		<b>2.5</b>		<b>21</b>	
	10	11.5	11.25	7	23.5
Demand	13	9.75	6.25	21	

The Transportation cost is

$$\begin{aligned}
 & (13)(7) + (9.75)(6) + (10.25)(9.75) + (11.5)(205) + (7)(21) \\
 & = 393.6875 \\
 & = 393.7
 \end{aligned}$$

### Conclusion

We have thus obtained an optimal solution for fuzzy transportation problem using hexagon fuzzy numbers. The new arithmetic operations of hexagon fuzzy numbers are employed to get the fuzzy optimal solutions. Moreover the fuzzy transportation problem using Robust's ranking indices, numerical example show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using Robust's [10] ranking method we have shown that the total cost obtained is optimal. We can conclude that the solution of fuzzy transportation problems can be obtained by Robust's ranking method effectively.

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