Theorem 4.1. Let $P_B = P_1 \cup P_2$, $Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ be three $m \times m$ square $PFBM$ iff $P_1 + Q_1 + R_1 \neq P_2 + Q_2 + R_2$.

Proof. Given $P_B = P_1 \cup P_2$, $Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ are three $m \times m$ square $PFBM$ and $P_B + Q_B + R_B$ is a $PFBM$.
ie, $P_B + Q_B + R_B = (P_1 \cup P_2) + (Q_1 \cup Q_2) + (R_1 \cup R_2)$
$= P_1 + Q_1 + R_1 \cup P_2 + Q_2 + R_2$ is a $PFBM$,
Hence $P_1 + Q_1 + R_1 \neq P_2 + Q_2 + R_2$
Clearly, if $P_1 + Q_1 + R_1 \neq P_2 + Q_2 + R_2$, then
$P_B + Q_B + R_B = P_1 + Q_1 + R_1 \cup P_2 + Q_2 + R_2$ is a $m \times m$ square $PFBM$.

Theorem 4.2. Let $P_B = P_1 \cup P_2$, $Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ be three $m \times m$ square $PFBM$, $P_B \cdot Q_B \cdot R_B = P_1 \cdot Q_1 \cdot R_1 \cup P_2 \cdot Q_2 \cdot R_2$ is a $m \times m$ square $PFBM$ iff $P_1 \cdot Q_1 \cdot R_1 \neq P_2 \cdot Q_2 \cdot R_2$.

Proof. Given $P_B = P_1 \cup P_2$, $Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ are three $m \times m$ square $PFBM$ and $P_B \cdot Q_B \cdot R_B$ is a $PFBM$.
ie, $P_B \cdot Q_B \cdot R_B = (P_1 \cup P_2) \cdot (Q_1 \cup Q_2) \cdot (R_1 \cup R_2)$
$= P_1 \cdot Q_1 \cdot R_1 \cup P_2 \cdot Q_2 \cdot R_2$ is a $PFBM$,
Hence $P_1 \cdot Q_1 \cdot R_1 \neq P_2 \cdot Q_2 \cdot R_2$
Clearly, if $P_1 \cdot Q_1 \cdot R_1 \neq P_2 \cdot Q_2 \cdot R_2$, then
$P_B \cdot Q_B \cdot R_B = P_1 \cdot Q_1 \cdot R_1 \cup P_2 \cdot Q_2 \cdot R_2$ is a $m \times m$ square $PFBM$.

Theorem 4.3. Let $P_B = P_1^{m\times n} \cup P_2^{r\times s}$ be mixed $PFBM$, $Q_B = Q_1^{n\times m} \cup Q_2^{s\times r}$ and $R_B = R_1^{n\times m} \cup R_2^{s\times r}$ be another two mixed $PFBM$, then the Addition is defined and is a mixed square $PFBM$ as,
$P_B + Q_B + R_B = Z_1^{m\times m} \cup Z_2^{r\times r} \cup Z_3^{s\times s}$
$Z = Z_1^{m\times m} \cup Z_2^{r\times r} \cup Z_3^{s\times s}$,
where, $Z_1^{m\times m} = P_1^{m\times m} + Q_1^{n\times m} + R_1^{n\times m}$,
$Z_2^{r\times r} = P_2^{r\times s} + Q_2^{s\times r} + R_2^{s\times r}$,
and $Z_3^{s\times s} = P_3^{s\times r} + Q_3^{r\times s} + R_3^{r\times s}$,

In general, the Addition of three mixed $PFBM$ must not be a square mixed $PFBM$.

Theorem 4.4. Let $P_B = P_1^{m\times n} \cup P_2^{r\times s}$ be mixed $PFBM$, $Q_B = Q_1^{n\times m} \cup Q_2^{s\times r}$ and $R_B = R_1^{n\times m} \cup R_2^{s\times r}$ be another two mixed $PFBM$, then the Product is defined and is a mixed square $PFBM$ as,
$P_B \cdot Q_B \cdot R_B = Z_1^{m\times m} \cup Z_2^{r\times r} \cup Z_3^{s\times s}$
with $Z = Z_1^{m\times m} \cup Z_2^{r\times r} \cup Z_3^{s\times s}$,
where, $Z_1^{m\times m} = P_1^{m\times n} \cdot Q_1^{n\times m} \cdot R_1^{n\times m}$,
$Z_2^{r\times r} = P_2^{r\times s} \cdot Q_2^{s\times r} \cdot R_2^{s\times r}$,
and $Z_3^{s\times s} = P_3^{s\times r} \cdot Q_3^{r\times s} \cdot R_3^{s\times r}$,
In general, the Product of three mixed $PFBM$ must not be a square mixed $PFBM$.

Theorem 4.5. Let P_B, Q_B and R_B are any three $PFBM$ of same order $'m'$, then $(P_B + Q_B) \vee (Q_B + R_B) \geq (P_B + Q_B) \wedge (Q_B + R_B)$, whenever $P_B \leq Q_B$, $R_B \leq Q_B$.

Proof. Let $P_B = P_1 \cup P_2, Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ are any three $PFBM$ of same order $'m'$, then
Given $(P_B + Q_B) \vee (Q_B + R_B) \geq (P_B + Q_B) \wedge (Q_B + R_B)$
$\Rightarrow [(P_1 \cup P_2) + (Q_1 \cup Q_2) \vee (Q_1 \cup Q_2) + (R_1 \cup R_2)] \geq [(P_1 \cup P_2) + (Q_1 \cup Q_2) \wedge (Q_1 \cup Q_2) + (R_1 \cup R_2)]$
$\Rightarrow [(P_1 + Q_1) \cup (P_2 + Q_2) \vee (Q_1 + R_1) \cup (Q_2 + R_2)] \geq [(P_1 + Q_1) \cup (P_2 + Q_2) \wedge (Q_1 + R_1) \cup (Q_2 + R_2)]$
It is clear that,
Hence, $(P_B + Q_B) \vee (Q_B + R_B) \geq (P_B + Q_B) \wedge (Q_B + R_B)$.

Theorem 4.6. Let P_B, Q_B and R_B are any three $PFBM$ of same order $'n'$, then $(P_B + Q_B) \wedge (Q_B + R_B) \geq (P_B + Q_B) \wedge (P_B + R_B)$, whenever $P_B \leq Q_B$, $R_B \leq Q_B$.

Proof. Let $P_B = P_1 \cup P_2, Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ are any three $PFBM$ of same order $'n'$, then
Given $(P_B + Q_B) \wedge (Q_B + R_B) \geq (P_B + Q_B) \wedge (P_B + R_B)$

$\Rightarrow [(P_1 \cup P_2) + (Q_1 \cup Q_2) \wedge (Q_1 \cup Q_2) + (R_1 \cup R_2)] \geq [(P_1 \cup P_2) + (Q_1 \cup Q_2) \wedge (P_1 \cup P_2) + (R_1 \cup R_2)]$

$\Rightarrow [(P_1 + Q_1) \cup (P_2 + Q_2) \wedge (Q_1 + R_1) \cup (Q_2 + R_2)] \geq [(P_1 + Q_1) \cup (P_2 + Q_2) \wedge (P_1 + R_1) \cup (P_2 + R_2)]$

It is clear that,
Hence, $(P_B + Q_B) \wedge (Q_B + R_B) \geq (P_B + Q_B) \wedge (P_B + R_B)$.

Theorem 4.7. Let P_B, Q_B and R_B are any three $PFBM$ of same order $'m'$, then $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \geq (P_B \cdot Q_B) \wedge (Q_B \cdot R_B)$, whenever $P_B \geq Q_B,\ R_B \geq Q_B$.

Proof. Let $P_B = P_1 \cup P_2, Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ be three $PFBM$ of order $'m'$, then
Given $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \geq (P_B \cdot Q_B) \wedge (Q_B \cdot R_B)$

$\Rightarrow [(P_1 \cup P_2) \cdot (Q_1 \cup Q_2) \vee (Q_1 \cup Q_2) \cdot (R_1 \cup R_2)] \geq [(P_1 \cup P_2) \cdot (Q_1 \cup Q_2) \wedge (Q_1 \cup Q_2) \cdot (R_1 \cup R_2)]$

$\Rightarrow [(P_1 \cdot Q_1) \cup (P_2 \cdot Q_2) \vee (Q_1 \cdot R_1) \cup (Q_2 \cdot R_2)] \geq [(P_1 \cdot Q_1) \cup (P_2 \cdot Q_2) \wedge (Q_1 \cdot R_1) \cup (Q_2 \cdot R_2)]$

It is clear that,
Hence, $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \geq (P_B \cdot Q_B) \wedge (Q_B \cdot R_B)$.

Theorem 4.8. Let P_B, Q_B and R_B are three $PFBM$ of same order $'n'$, then $(P_B + Q_B) \vee (Q_B + R_B) \leq (P_B + Q_B) \vee (P_B + R_B)$, whenever $P_B \geq Q_B,\ Q_B \leq R_B$.

Proof. Let $P_B = P_1 \cup P_2, Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ are three $PFBM$ of order $'n'$.
Given, $(P_B + Q_B) \vee (Q_B + R_B) \leq (P_B + Q_B) \vee (P_B + R_B)$

$\Rightarrow [(P_1 \cup P_2) + (Q_1 \cup Q_2) \vee (Q_1 \cup Q_2) + (R_1 \cup R_2)] \leq [(P_1 \cup P_2) + (Q_1 \cup Q_2) \vee (P_1 \cup P_2) + (R_1 \cup R_2)]$

$\Rightarrow [(P_1 + Q_1) \cup (P_2 + Q_2) \vee (Q_1 + R_1) \cup (Q_2 + R_2)] \leq [(P_1 + Q_1) \cup (P_2 + Q_2) \vee (P_1 + R_1) \cup (P_2 + R_2)]$

It is clear that,
Hence, $(P_B + Q_B) \vee (Q_B + R_B) \leq (P_B + Q_B) \vee (P_B + R_B)$.

Theorem 4.9. Let P_B, Q_B and R_B are three $PFBM$ of same order $'n'$, then $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \geq (P_B \cdot Q_B) \wedge (Q_B \cdot R_B)$, whenever $P_B \geq Q_B,\ R_B \geq Q_B$.

Proof. Let $P_B = P_1 \cup P_2, Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ be three $PFBM$ of order $'m'$, then
Given $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \geq (P_B \cdot Q_B) \wedge (Q_B \cdot R_B)$

$\Rightarrow [(P_1 \cup P_2) \cdot (Q_1 \cup Q_2) \vee (Q_1 \cup Q_2) \cdot (R_1 \cup R_2)] \geq [(P_1 \cup P_2) \cdot (Q_1 \cup Q_2) \wedge (Q_1 \cup Q_2) \cdot (R_1 \cup R_2)]$

$\Rightarrow [(P_1 \cdot Q_1) \cup (P_2 \cdot Q_2) \vee (Q_1 \cdot R_1) \cup (Q_2 \cdot R_2)] \geq [(P_1 \cdot Q_1) \cup (P_2 \cdot Q_2) \wedge (Q_1 \cdot R_1) \cup (Q_2 \cdot R_2)]$

It is clear that,
Hence, $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \geq (P_B \cdot Q_B) \wedge (Q_B \cdot R_B)$.

Theorem 4.10. Let P_B, Q_B and R_B are three $PFBM$ of same order $'n'$, then $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \leq (P_B \cdot Q_B) \vee (P_B \cdot R_B)$, whenever $P_B \geq Q_B,\ Q_B \leq R_B$.

Proof. Let $P_B = P_1 \cup P_2, Q_B = Q_1 \cup Q_2$ and $R_B = R_1 \cup R_2$ are three $PFBM$ of order $'n'$, then
Given, $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \leq (P_B \cdot Q_B) \vee (P_B \cdot R_B)$

$\Rightarrow [(P_1 \cup P_2) \cdot (Q_1 \cup Q_2) \vee (Q_1 \cup Q_2) \cdot (R_1 \cup R_2)] \leq [(P_1 \cup P_2) \cdot (Q_1 \cup Q_2) \vee (P_1 \cup P_2) \cdot (R_1 \cup R_2)]$

$[(P_1 \cdot Q_1) \cup (P_2 \cdot Q_2) \vee (Q_1 \cdot R_1) \cup (Q_2 \cdot R_2)] \leq [(P_1 \cdot Q_1) \cup (P_2 \cdot Q_2) \vee (P_1 \cdot R_1) \cup (P_2 \cdot R_2)]$

It is clear that,
Hence, $(P_B \cdot Q_B) \vee (Q_B \cdot R_B) \leq (P_B \cdot Q_B) \vee (P_B \cdot R_B)$.

Theorem 4.11. Let P_B, Q_B and R_B are three $PFBM$ in the Field F, then the following Matrix set conditions are holds,

(i) $[Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] = [Z(Q_B) + Z(R_B)]$ iff $[Z(P_B) + Z(Q_B)] \leq [Z(Q_B) + Z(R_B)]$, whenever $Z(P_B) \leq Z(Q_B),\ Z(R_B) \leq Z(Q_B)$.

(ii) $[Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] = [Z(P_B) + Z(Q_B)]$ iff $[Z(P_B) + Z(Q_B)] \leq [Z(Q_B) + Z(R_B)]$, whenever $Z(P_B) \leq Z(Q_B),\ Z(R_B) \leq Z(Q_B)$.

(iii) $[Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] = [Z(P_B) + Z(Q_B)]$ iff $[Z(P_B) + Z(Q_B)] \geq [Z(Q_B) + Z(R_B)]$, whenever $Z(P_B) \geq Z(Q_B),\ Z(Q_B) \geq Z(R_B)$.

(iv) $[Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] = [Z(Q_B) + Z(R_B)]$ iff $[Z(P_B) + Z(Q_B)] \geq [Z(Q_B) + Z(R_B)]$, whenever $Z(P_B) \geq Z(Q_B),\ Z(Q_B) \geq Z(R_B)$.

Proof. (i) Let $F \in [Z(P_B) + Z(Q_B)]$, then $F \in [Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)]$
Hence $[Z(P_B) + Z(Q_B)] \leq [Z(Q_B) + Z(R_B)]$
Conversely, Let $Z(P_B) + Z(Q_B) \leq Z(Q_B) + Z(R_B)$
Let $F \in [Z(P_B) + Z(Q_B)] \leq [Z(Q_B) + Z(R_B)]$
Since $[Z(P_B) + Z(Q_B)] \leq [Z(Q_B) + Z(R_B)]$,
$F \in [Z(P_B) + Z(Q_B)] F \in [Z(Q_B) + Z(R_B)]$
Thus, $F \in [Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] F \in [Z(Q_B) + Z(R_B)]$
Hence, $[Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] \leq [Z(Q_B) + Z(R_B)]$
$[Z(Q_B) + Z(R_B)] \geq [Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)]$ is always true.
Therefore, $[Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] = [Z(Q_B) + Z(R_B)]$.

(ii) Let $F \in [Z(P_B) + Z(Q_B)]$, then $F \in [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)]$
Hence $[Z(P_B) + Z(Q_B)] \leq [Z(Q_B) + Z(R_B)]$
Conversely, Let $Z(P_B) + Z(Q_B) \leq Z(Q_B) + Z(R_B)$
Let $F \in [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)]$
Since $[Z(P_B) + Z(Q_B)] \leq [Z(Q_B) + Z(R_B)]$,
$F \in [Z(P_B) + Z(Q_B)] F \in [Z(Q_B) + Z(R_B)]$
Thus, $F \in [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] F \in [Z(P_B) + Z(Q_B)]$
Hence, $[Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] \leq [Z(P_B) + Z(Q_B)]$
$[Z(P_B) + Z(Q_B)] \geq [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)]$ is always true.
Therefore, $[Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] = [Z(P_B) + Z(Q_B)]$.

(iii) Let $F \in [Z(Q_B) + Z(R_B)]$, then $F \in [Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)]$
Hence $[Z(P_B) + Z(Q_B)] \geq [Z(Q_B) + Z(R_B)]$
Conversely, Let $Z(P_B) + Z(Q_B) \geq Z(Q_B) + Z(R_B)$
Let $F \in [Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)]$
Since $[Z(P_B) + Z(Q_B)] \geq [Z(Q_B) + Z(R_B)]$,
$F \in [Z(Q_B) + Z(R_B)], F \in [Z(P_B) + Z(Q_B)]$
Thus, $F \in [Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] F \in [Z(P_B) + Z(Q_B)]$
Hence, $[Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] \geq [Z(P_B) + Z(Q_B)]$
$[Z(P_B) + Z(Q_B)] \geq [Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)]$ is always true.
Therefore, $[Z(P_B) + Z(Q_B)] \vee [Z(Q_B) + Z(R_B)] = [Z(P_B) + Z(Q_B)]$.

(iv) Let $F \in [Z(Q_B) + Z(R_B)]$, then $F \in [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)]$
Hence $[Z(P_B) + Z(Q_B)] \geq [Z(Q_B) + Z(R_B)]$
Conversely, Let $Z(P_B) + Z(Q_B) \geq Z(Q_B) + Z(R_B)$
Let $F \in [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)]$
Since $[Z(P_B) + Z(Q_B)] \geq [Z(Q_B) + Z(R_B)]$,
$F \in [Z(Q_B) + Z(R_B)] F \in [Z(P_B) + Z(Q_B)]$
Thus, $F \in [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] F \in [Z(Q_B) + Z(R_B)]$
Hence, $[Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] \geq [Z(Q_B) + Z(R_B)]$
ie, $[Z(Q_B) + Z(R_B)] \geq [Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)]$ is always true.
Therefore, $[Z(P_B) + Z(Q_B)] \wedge [Z(Q_B) + Z(R_B)] = [Z(Q_B) + Z(R_B)]$.

Theorem 4.12. For any three $PFBM$ in the field F, then the following Matrix set conditions are holds,
(i) $[Z(P_B)] \vee [Z(Q_B) \wedge Z(R_B)] = [Z(P_B) \vee Z(Q_B)] \wedge [Z(P_B) \vee Z(R_B)]$ iff
$Z(P_B) \leq Z(Q_B),\ Z(P_B) \leq Z(R_B)$.
(ii) $[Z(P_B)] \wedge [Z(Q_B) \vee Z(R_B)] = [Z(P_B) \wedge Z(Q_B)] \vee [Z(P_B) \wedge Z(R_B)]$ iff
$Z(P_B) \geq Z(Q_B),\ Z(P_B) \geq Z(R_B)$.

Proof. (i) Let $F \in Z(P_B)$, then $F \in Z(P_B) \vee [Z(Q_B) \wedge Z(R_B)]$ and
$F \in [Z(P_B) \vee Z(Q_B)] \wedge [Z(P_B) \vee Z(R_B)]$
Hence $Z(P_B) \leq Z(Q_B), Z(R_B)$
Conversely, Let $Z(P_B) \leq Z(Q_B), Z(R_B)$
Let $F \in Z(P_B) \vee [Z(Q_B) \wedge Z(R_B)]$
Since $Z(P_B) \leq Z(Q_B), Z(R_B)$,
$F \in Z(P_B) F \in [Z(P_B) \vee Z(Q_B)] \wedge [Z(P_B) \vee Z(R_B)]$
Thus, $F \in Z(P_B) \vee [Z(Q_B) \wedge Z(R_B)] F \in [Z(P_B) \vee Z(Q_B)] \wedge [Z(P_B) \vee Z(R_B)]$
Hence, $Z(P_B) \vee [Z(Q_B) \wedge Z(R_B)] \leq [Z(P_B) \vee Z(Q_B)] \wedge [Z(P_B) \vee Z(R_B)]$
$[Z(P_B) \vee Z(Q_B)] \wedge [Z(P_B) \vee Z(R_B)] \leq Z(P_B) \vee [Z(Q_B) \wedge Z(R_B)]$ is always true.
Therefore, $Z(P_B) \vee [Z(Q_B) \wedge Z(R_B)] = [Z(P_B) \vee Z(Q_B)] \wedge [Z(P_B) \vee Z(R_B)]$.

(ii) Let $F \in Z(P_B)$, then $F \in Z(P_B) \wedge [Z(Q_B) \vee Z(R_B)]$ and
$F \in [Z(P_B) \wedge Z(Q_B)] \vee [Z(P_B) \wedge Z(R_B)]$
Hence $Z(P_B) \geq Z(Q_B), Z(R_B)$
Conversely, Let $Z(P_B) \geq Z(Q_B), Z(R_B)$
Let $F \in Z(P_B) \wedge [Z(Q_B) \vee Z(R_B)]$
Since $Z(P_B) \geq Z(Q_B), Z(R_B)$,
$F \in Z(P_B) F \in [Z(P_B) \wedge Z(Q_B)] \vee [Z(P_B) \wedge Z(R_B)]$
Thus, $F \in Z(P_B) \wedge [Z(Q_B) \vee Z(R_B)] F \in [Z(P_B) \wedge Z(Q_B)] \vee [Z(P_B) \wedge Z(R_B)]$
Hence, $Z(P_B) \wedge [Z(Q_B) \vee Z(R_B)] \geq [Z(P_B) \wedge Z(Q_B)] \vee [Z(P_B) \wedge Z(R_B)]$
$[Z(P_B) \wedge Z(Q_B)] \vee [Z(P_B) \wedge Z(R_B)] \geq Z(P_B) \wedge [Z(Q_B) \vee Z(R_B)]$ is always true.
Therefore, $Z(P_B) \wedge [Z(Q_B) \vee Z(R_B)] = [Z(P_B) \wedge Z(Q_B)] \vee [Z(P_B) \wedge Z(R_B)]$.

Theorem 4.13. Let P_B, Q_B and R_B are any three $PFBM$ in the field F, then the following Matrix set conditions are holds,
(i) $[Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] = [Z(Q_B^c) + Z(R_B^c)]$ iff
$[Z(P_B^c) + Z(Q_B^c)] \leq [Z(Q_B^c) + Z(R_B^c)]$, whenever $Z(P_B^c) \leq Z(Q_B^c),\ Z(R_B^c) \leq Z(Q_B^c)$.
(ii) $[Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] = [Z(P_B^c) + Z(Q_B^c)]$ iff
$[Z(P_B^c) + Z(Q_B^c)] \leq [Z(Q_B^c) + Z(R_B^c)]$, whenever $Z(P_B^c) \leq Z(Q_B^c),\ Z(R_B^c) \leq Z(Q_B^c)$.
(iii) $[Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] = [Z(P_B^c) + Z(Q_B^c)]$ iff
$[Z(P_B^c) + Z(Q_B^c)] \geq [Z(Q_B^c) + Z(R_B^c)]$, whenever $Z(P_B^c) \geq Z(Q_B^c),\ Z(Q_B^c) \geq Z(R_B^c)$.
(iv) $[Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] = [Z(Q_B^c) + Z(R_B^c)]$ iff
$[Z(P_B^c) + Z(Q_B^c)] \geq [Z(Q_B^c) + Z(R_B^c)]$, whenever $Z(P_B^c) \geq Z(Q_B^c),\ Z(Q_B^c) \geq Z(R_B^c)$.

Proof. (i) Let $F \in [Z(P_B^c) + Z(Q_B^c)]$, then $F \in [Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)]$
Hence $[Z(P_B^c) + Z(Q_B^c)] \leq [Z(Q_B^c) + Z(R_B^c)]$
Conversely, Let $Z(P_B^c) + Z(Q_B^c) \leq Z(Q_B^c) + Z(R_B^c)$
Let $(F \in [Z(P_B^c) + Z(Q_B^c)]) \leq ([Z(Q_B^c) + Z(R_B^c)])$
Since $[Z(P_B^c) + Z(Q_B^c)] \leq [Z(Q_B^c) + Z(R_B^c)]$,
$F \in [Z(P_B^c) + Z(Q_B^c)] F \in [Z(Q_B^c) + Z(R_B^c)]$
Thus, $F \in [Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] F \in [Z(Q_B^c) + Z(R_B^c)]$
Hence, $[Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] \leq [Z(Q_B^c) + Z(R_B^c)]$
$[Z(Q_B^c) + Z(R_B^c)] \geq [Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)]$ is always true.
Therefore, $[Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] = [Z(Q_B^c) + Z(R_B^c)]$.

(ii) Let $F \in [Z(P_B^c) + Z(Q_B^c)]$, then $F \in [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)]$
Hence $[Z(P_B^c) + Z(Q_B^c)] \leq [Z(Q_B^c) + Z(R_B^c)]$
Conversely, Let $Z(P_B^c) + Z(Q_B^c) \leq Z(Q_B^c) + Z(R_B^c)$

Let $F \in [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)]$
Since $[Z(P_B^c) + Z(Q_B^c)] \leq [Z(Q_B^c) + Z(R_B^c)]$,
$$F \in [Z(P_B^c) + Z(Q_B^c)] F \in [Z(Q_B^c) + Z(R_B^c)]$$
Thus, $F \in [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] F \in [Z(P_B^c) + Z(Q_B^c)]$
Hence, $[Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] \leq [Z(P_B^c) + Z(Q_B^c)]$
$[Z(P_B^c) + Z(Q_B^c)] \geq [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)]$ is always true.
Therefore, $[Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] = [Z(P_B^c) + Z(Q_B^c)]$.

(iii) Let $F \in [Z(Q_B^c) + Z(R_B^c)]$, then $F \in [Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)]$
Hence $[Z(P_B^c) + Z(Q_B^c)] \geq [Z(Q_B^c) + Z(R_B^c)]$
Conversely, Let $Z(P_B^c) + Z(Q_B^c) \geq Z(Q_B^c) + Z(R_B^c)$
Let $F \in [Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)]$
Since $[Z(P_B^c) + Z(Q_B^c)] \geq [Z(Q_B^c) + Z(R_B^c)]$,
$$F \in [Z(Q_B^c) + Z(R_B^c)], F \in [Z(P_B^c) + Z(Q_B^c)]$$
Thus, $F \in [Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] F \in [Z(P_B^c) + Z(Q_B^c)]$
Hence, $[Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] \geq [Z(P_B^c) + Z(Q_B^c)]$
$[Z(P_B^c) + Z(Q_B^c)] \geq [Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)]$ is always true.
Therefore, $[Z(P_B^c) + Z(Q_B^c)] \vee [Z(Q_B^c) + Z(R_B^c)] = [Z(P_B^c) + Z(Q_B^c)]$.

(iv) Let $F \in [Z(Q_B^c) + Z(R_B^c)]$, then $F \in [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)]$
Hence $[Z(P_B^c) + Z(Q_B^c)] \geq [Z(Q_B^c) + Z(R_B^c)]$
Conversely, Let $Z(P_B^c) + Z(Q_B^c) \geq Z(Q_B^c) + Z(R_B^c)$
Let $F \in [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)]$
Since $[Z(P_B^c) + Z(Q_B^c)] \geq [Z(Q_B^c) + Z(R_B^c)]$,
$$F \in [Z(Q_B^c) + Z(R_B^c)] F \in [Z(P_B^c) + Z(Q_B^c)]$$
Thus, $F \in [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] F \in [Z(Q_B^c) + Z(R_B^c)]$
Hence, $[Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] \geq [Z(Q_B^c) + Z(R_B^c)]$
ie, $[Z(Q_B^c) + Z(R_B^c)] \geq [Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)]$ is always true.
Therefore, $[Z(P_B^c) + Z(Q_B^c)] \wedge [Z(Q_B^c) + Z(R_B^c)] = [Z(Q_B^c) + Z(R_B^c)]$.

Theorem 4.14. For any three $PFBM$ in the field F, then the following Matrix set conditions are holds,
(i) $[Z(P_B^c)] \vee [Z(Q_B^c) \wedge Z(R_B^c)] = [Z(P_B^c) \vee Z(Q_B^c)] \wedge [Z(P_B^c) \vee Z(R_B^c)]$ iff $Z(P_B^c) \leq Z(Q_B^c)$, $Z(P_B^c) \leq Z(R_B^c)$.
(ii) $[Z(P_B^c)] \wedge [Z(Q_B^c) \vee Z(R_B^c)] = [Z(P_B^c) \wedge Z(Q_B^c)] \vee [Z(P_B^c) \wedge Z(R_B^c)]$ iff $Z(P_B^c) \geq Z(Q_B^c)$, $Z(P_B^c) \geq Z(R_B^c)$.

Proof. (i) Let $F \in Z(P_B^c)$, then $F \in Z(P_B^c) \vee [Z(Q_B^c) \wedge Z(R_B^c)]$ and
$$F \in [Z(P_B^c) \vee Z(Q_B^c)] \wedge [Z(P_B^c) \vee Z(R_B^c)]$$
Hence $Z(P_B^c) \leq Z(Q_B^c), Z(R_B^c)$
Conversely, Let $Z(P_B^c) \leq Z(Q_B^c), Z(R_B^c)$
Let $F \in Z(P_B^c) \vee [Z(Q_B^c) \wedge Z(R_B^c)]$
Since $Z(P_B^c) \leq Z(Q_B^c), Z(R_B^c)$,
$$F \in Z(P_B^c) F \in [Z(P_B^c) \vee Z(Q_B^c)] \wedge [Z(P_B^c) \vee Z(R_B^c)]$$
Thus, $F \in Z(P_B^c) \vee [Z(Q_B^c) \wedge Z(R_B^c)] F \in [Z(P_B^c) \vee Z(Q_B^c)] \wedge [Z(P_B^c) \vee Z(R_B^c)]$
Hence, $Z(P_B^c) \vee [Z(Q_B^c) \wedge Z(R_B^c)] \leq [Z(P_B^c) \vee Z(Q_B^c)] \wedge [Z(P_B^c) \vee Z(R_B^c)]$
$[Z(P_B^c) \vee Z(Q_B^c)] \wedge [Z(P_B^c) \vee Z(R_B^c)] \leq Z(P_B^c) \vee [Z(Q_B^c) \wedge Z(R_B^c)]$ is always true.
Therefore, $Z(P_B^c) \vee [Z(Q_B^c) \wedge Z(R_B^c)] = [Z(P_B^c) \vee Z(Q_B^c)] \wedge [Z(P_B^c) \vee Z(R_B^c)]$.

(ii) Let $F \in Z(P_B^c)$, then $F \in Z(P_B^c) \wedge [Z(Q_B^c) \vee Z(R_B^c)]$ and
$$F \in [Z(P_B^c) \wedge Z(Q_B^c)] \vee [Z(P_B^c) \wedge Z(R_B^c)]$$
Hence $Z(P_B^c) \geq Z(Q_B^c), Z(R_B^c)$
Conversely, Let $Z(P_B^c) \geq Z(Q_B^c), Z(R_B^c)$
Let $F \in Z(P_B^c) \wedge [Z(Q_B^c) \vee Z(R_B^c)]$
Since $Z(P_B^c) \geq Z(Q_B^c), Z(R_B^c)$,
$$F \in Z(P_B^c) F \in [Z(P_B^c) \wedge Z(Q_B^c)] \vee [Z(P_B^c) \wedge Z(R_B^c)]$$

Thus, $F \in Z(P_B^c) \wedge [Z(Q_B^c) \vee Z(R_B^c)]F \in [Z(P_B^c) \wedge Z(Q_B^c)] \vee [Z(P_B^c) \wedge Z(R_B^c)]$
Hence, $Z(P_B^c) \wedge [Z(Q_B^c) \vee Z(R_B^c)] \geq [Z(P_B^c) \wedge Z(Q_B^c)] \vee [Z(P_B^c) \wedge Z(R_B^c)]$
$[Z(P_B^c) \wedge Z(Q_B^c)] \vee [Z(P_B^c) \wedge Z(R_B^c)] \geq Z(P_B^c) \wedge [Z(Q_B^c) \vee Z(R_B^c)]$ is always true.
Therefore, $Z(P_B^c) \wedge [Z(Q_B^c) \vee Z(R_B^c)] = [Z(P_B^c) \wedge Z(Q_B^c)] \vee [Z(P_B^c) \wedge Z(R_B^c)]$.

Theorem 4.15. Let P_B, Q_B and R_B are any three $PFBM$, then $(P_B^c), (Q_B^c)$ and (R_B^c) are also $PFBM$ of same order ``m", then the following condition are holds, $(P_B^c + Q_B^c) \vee (Q_B^c + R_B^c) \geq (P_B^c + Q_B^c) \wedge (Q_B^c + R_B^c)$,whenever $P_B^c \geq Q_B^c$, $R_B^c \geq Q_B^c$.

Proof. Let $P_B^c = P_1^c \cup P_2^c$, $Q_B^c = Q_1^c \cup Q_2^c$ and $R_B^c = R_1^c \cup R_2^c$ be three $PFBM$ of order `m',
Given $(P_B^c + Q_B^c) \vee (Q_B^c + R_B^c) \geq (P_B^c + Q_B^c) \wedge (Q_B^c + R_B^c)$

$\Rightarrow [(P_1^c \cup P_2^c) + (Q_1^c \cup Q_2^c) \vee (Q_1^c \cup Q_2^c) + (R_1^c \cup R_2^c)] \geq [(P_1^c \cup P_2^c) + (Q_1^c \cup Q_2^c) \wedge (Q_1^c \cup Q_2^c) + (R_1^c \cup R_2^c)]$

$\Rightarrow [(P_1^c + Q_1^c) \cup (P_2^c + Q_2^c) \vee (Q_1^c + R_1^c) \cup (Q_2^c + R_2^c)] \geq [(P_1^c + Q_1^c) \cup (P_2^c + Q_2^c) \wedge (Q_1^c + R_1^c) \cup (Q_2^c + R_2^c)]$
It is clear that,
Hence $(P_B^c + Q_B^c) \vee (Q_B^c + R_B^c) \geq (P_B^c + Q_B^c) \wedge (Q_B^c + R_B^c)$.

Theorem 4.16. Let P_B, Q_B and R_B are any three $PFBM$, then $(P_B^c), (Q_B^c)$ and (R_B^c) are also $PFBM$ of same order `n', then the following condition are true,
$(P_B^c + Q_B^c) \wedge (Q_B^c + R_B^c) \geq (P_B^c + Q_B^c) \wedge (P_B^c + R_B^c)$,whenever $P_B^c \leq Q_B^c$ and $R_B^c \leq Q_B^c$.

Proof. Let $P_B^c = P_1^c \cup P_2^c$, $Q_B^c = Q_1^c \cup Q_2^c$ and $R_B^c = R_1^c \cup R_2^c$ are three $PFBM$ of order `n',
Given $(P_B^c + Q_B^c) \wedge (Q_B^c + R_B^c) \geq (P_B^c + Q_B^c) \wedge (P_B^c + R_B^c)$

$\Rightarrow [(P_1^c \cup P_2^c) + (Q_1^c \cup Q_2^c) \wedge (Q_1^c \cup Q_2^c) + (R_1^c \cup R_2^c)] \geq [(P_1^c \cup P_2^c) + (Q_1^c \cup Q_2^c) \wedge (P_1^c \cup P_2^c + (R_1^c \cup R_2^c)]$

$\Rightarrow [(P_1^c + Q_1^c) \cup (P_2^c + Q_2^c) \wedge (Q_1^c + R_1^c) \cup (Q_2^c + R_2^c)] \geq [(P_1^c + Q_1^c) \cup (P_2^c + Q_2^c) \wedge (P_1^c + R_1^c) \cup (P_2^c + R_2^c)]$
It is clear that,
Hence $(P_B^c + Q_B^c) \wedge (Q_B^c + R_B^c) \geq (P_B^c + Q_B^c) \wedge (P_B^c + R_B^c)$.

Theorem 4.17. Let P_B, Q_B and R_B are any three $PFBM$, then $(P_B^c), (Q_B^c)$ and (R_B^c) are also $PFBM$ of same order `m', then the following condition are holds, $(P_B^c \cdot Q_B^c) \vee (Q_B^c \cdot R_B^c) \geq (P_B^c \cdot Q_B^c) \wedge (Q_B^c \cdot R_B^c)$,whenever $P_B^c \geq Q_B^c$, $R_B^c \geq Q_B^c$.

Theorem 4.18. . Let $P_B^c = P_1^c \cup P_2^c$, $Q_B^c = Q_1^c \cup Q_2^c$ and $R_B^c = R_1^c \cup R_2^c$ be three $PFBM$ of order `m',
Given $(P_B^c \cdot Q_B^c) \vee (Q_B^c \cdot R_B^c) \geq (P_B^c \cdot Q_B^c) \wedge (Q_B^c \cdot R_B^c)$

$\Rightarrow [(P_1^c \cup P_2^c) \cdot (Q_1^c \cup Q_2^c) \vee (Q_1^c \cup Q_2^c) \cdot (R_1^c \cup R_2^c)] \geq [(P_1^c \cup P_2^c) \cdot (Q_1^c \cup Q_2^c) \wedge (Q_1^c \cup Q_2^C) \cdot (R_1^c \cup R_2^c)]$

$\Rightarrow [(P_1^c \cdot Q_1^c) \cup (P_2^c \cdot Q_2^c) \vee (Q_1^c \cdot R_1^c) \cup (Q_2^c \cdot R_2^c)] \geq [(P_1^c \cdot Q_1^c) \cup (P_2^c \cdot Q_2^c) \wedge (Q_1^c \cdot R_1^c) \cup (Q_2^c \cdot R_2^c)]$
It is clear that,
Hence $(P_B^c \cdot Q_B^c) \vee (Q_B^c \cdot R_B^c) \geq (P_B^c \cdot Q_B^c) \wedge (Q_B^c \cdot R_B^c)$.

Theorem 4.19. Let P_B, Q_B and R_B are any three $PFBM$, then $(P_B^c), (Q_B^c)$ and (R_B^c) are also $PFBM$ of same order `n', then the following condition are true,
$(P_B^c + Q_B^c) \vee (Q_B^c + R_B^c) \leq (P_B^c + Q_B^c \vee (P_B^c + R_B^c)$,whenever $P_B^c \geq Q_B^c$ and $R_B^c \geq Q_B^C$.

Proof. Let $P_B^c = P_1^c \cup P_2^c$, $Q_B^c = Q_1^c \cup Q_2^c$ and $R_B^c = R_1^c \cup R_2^c$ are three $PFBM$ of order `n',
Given $(P_B^c + Q_B^c) \vee (Q_B^c + R_B^c) \leq (P_B^c + Q_B^c) \vee (P_B^c + R_B^c)$

$\Rightarrow [(P_1^c \cup P_2^c) + (Q_1^c \cup Q_2^c) \vee (Q_1^c \cup Q_2^c) + (R_1^c \cup R_2^c)] \leq [(P_1^c \cup P_2^c) + (Q_1^c \cup Q_2^c) \vee (P_1^c \cup P_2^c) + (R_1^c \cup R_2^c)]$

$\Rightarrow [(P_1^c + Q_1^c) \cup (P_2^c + Q_2^c) \vee (Q_1^c + R_1^c) \cup (Q_2^c + R_2^c)] \leq [(P_1^c + Q_1^c) \cup (P_2^c + Q_2^c) \vee (P_1^c + R_1^c) \cup (P_2^c + R_2^c)]$
It is clear that,
Hence $(P_B^c + Q_B^c) \vee (Q_B^c + R_B^c) \leq (P_B^c + Q_B^c) \vee (P_B^c + R_B^c)$.

5. Multi - Criteria Decision Making(MCDM) using PFSBMs

Definition 5.1. The Score Function (SF) helps to integrate the elements of Picture Fuzzy Soft Bimatrices (PFSBM)into real numbers. It is significant to integrate the positive, nuetral and negative membership value into a single real numbers to arrive at a numerical value that can be used a measure of the property.
Let $Q = [q_{ij}^1] \cup [q_{ij}^2] =< \mu_{ij}^1, \eta_{ij}^1, \gamma_{ij}^1 > \cup < \mu_{ij}^2, \eta_{ij}^2, \gamma_{ij}^2 >$ $i = (1,2,3,\ldots,m)$ and $j = (1,2,3,\ldots,n)$.
Then the Score Function for PFSBM,
$S(Q) = [q_{ij}]$ is given as

$$S(Q) = \left[\frac{\mu_{ij}^1 + \mu_{ij}^2 + 2\eta_{ij}^1 + 2\eta_{ij}^2 + \gamma_{ij}^1 + \gamma_{ij}^2}{4}\right]$$

$S(Q)$ is an mxn matrix, having the same dimension as Q.

Statement of the Problem

Let $P = p_1, p_2, p_3, \ldots, p_m$ be alternatives and $E = e_1, e_2, e_3, \ldots, e_m$ be criteria each alternatives can be expressed in Picture Fuzzy soft Matrices(PFSMs). Construct two PFSMs (P_1^i and P_2^i), $i = (1,2,\ldots,m)$ for each alternatives p_i, corresponding to the criteria e_j, $j = (1,2,\ldots,n)$. Now combine (P_1^i and P_2^i) to form PFSBM p_i as below:
$p_i = p_1^i \cup p_2^i$, where $p_1^i \neq p_2^i$ with

$$p_1^i = \begin{bmatrix} p_{11}^1 & p_{12}^1 & \ldots p_{1n}^1 \\ p_{21}^1 & p_{22}^1 & \ldots p_{2n}^1 \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ p_{m1}^1 & p_{m2}^1 & \ldots p_{mn}^1 \end{bmatrix} \text{ and } p_2^i = \begin{bmatrix} p_{11}^2 & p_{12}^2 & \ldots p_{1n}^2 \\ p_{21}^2 & p_{22}^2 & \ldots p_{2n}^2 \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ p_{m1}^2 & p_{m2}^2 & \ldots p_{mn}^2 \end{bmatrix}$$

where $p_{ij}^1 =< \mu_{ij}^1, \eta_{ij}^1, \gamma_{ij}^1 >$ and $p_{ij}^2 =< \mu_{ij}^2, \eta_{ij}^2, \gamma_{ij}^2 >$ $i = (1,2,3,\ldots,m)$ and $j = (1,2,3,\ldots,n)$.
Here, μ_{ij}^1 and μ_{ij}^2 represent the positive membership values given by the experts for the elements p_{ij}^1 and p_{ij}^2 corresponding to the criteria e_j, η_{ij}^1 and η_{ij}^2 represent the neutral membership values given by the expert for the elements p_{ij}^1 and p_{ij}^2 corresponding to the criteria e_j. Similarly, γ_{ij}^1 and γ_{ij}^2 represent the negative membership values given by the expert for the elements p_{ij}^1 and p_{ij}^2 corresponding to the criteria e_j. calculate the score value S of p^i. Combine the corresponding elements of the experts and determine the average of each alternative corresponding to the criteria e_j. finally, to know the outcome of the results, find the maximum value of each alternative corresponding to the criteria e_j.

Algorithm for computing MCDM:

step 1: Construct a PFSM (p_1^i) with the help of experts.
step 2: Construct a PFSM (p_2^i) with the help of experts.
step 3: Combine (p_1^i) and (p_2^i) form PFSBM, $p_i = (p_1^i) \cup (p_2^i)$.
step 4: Calculate the score value S of p_i using Definition 1.
step 5: Combine the corresponding elements of each experts and find the average of
the alternatives corresponding to the criteria.
step 6: Determine the maximum value and conclude the results.

Case study:

In this section, we determine the combined side effects of two shots of Covid-19 Vaccine given to health and safety representatives(HSRs) pf different age group by using PFSBM and SF.

Case (i)

The aim of this study is to find the combined side effects of two shots of covid-19 vaccine given to HSR of same age group (30-40). Let $p_i, (i = 1,2)$, represent the experts who observe the HSRs, $R = (r_1, r_2, \ldots, r_5)$ after each shots of covid -19 vaccine and present the results in PFSM. Let the criteria be $e_1 = mild\ \ fever, e_2 = muscle/joint\ \ aches, e_3 = mild\ \ headache, e_4 = nausea\ \ and\ \ e_5 = rashes\ \ over\ \ the\ \ body$.

The Method:

To find out the combined side effects of two shots of covid -19 vaccine, we use the following method.

step 1:
The experts observe HSRS and present the results of first shot in PFSMs as below,

$$p_1^1 = \begin{bmatrix} \langle 0.2,0.5,0.3\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.1,0.3,0.6\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.1,0.5,0.4\rangle \\ \langle 0.7,0.3,0.0\rangle & \langle 0.1,0.2,0.7\rangle & \langle 0.2,0.1,0.6\rangle & \langle 0.3,0.5,0.1\rangle & \langle 0.2,0.7,0.1\rangle \\ \langle 0.2,0.2,0.6\rangle & \langle 0.4,0.3,0.3\rangle & \langle 0.1,0.1,0.0\rangle & \langle 0.2,0.3,0.3\rangle & \langle 0.1,0.2,0.4\rangle \\ \langle 0.1,0.3,0.5\rangle & \langle 0.4,0.4,0.2\rangle & \langle 0.1,0.7,0.2\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.2,0.5,0.2\rangle \\ \langle 0.6,0.1,0.1\rangle & \langle 0.0,0.7,0.3\rangle & \langle 0.1,0.5,0.4\rangle & \langle 0.3,0.6,0.1\rangle & \langle 0.1,0.2,0.7\rangle \end{bmatrix}$$

and

$$p_1^2 = \begin{bmatrix} \langle 0.3,0.1,0.5\rangle & \langle 0.7,0.2,0.1\rangle & \langle 0.1,0.3,0.6\rangle & \langle 0.2,0.2,0.5\rangle & \langle 0.1,0.1,0.0\rangle \\ \langle 0.5,0.5,0.0\rangle & \langle 0.2,0.2,0.2\rangle & \langle 0.4,0.5,0.1\rangle & \langle 0.1,0.2,0.4\rangle & \langle 0.6,0.1,0.1\rangle \\ \langle 0.1,0.1,0.1\rangle & \langle 0.7,0.1,0.1\rangle & \langle 0.0,0.9,0.1\rangle & \langle 0.3,0.5,0.2\rangle & \langle 0.3,0.1,0.5\rangle \\ \langle 0.2,0.1,0.7\rangle & \langle 0.1,0.1,0.3\rangle & \langle 0.2,0.8,0.0\rangle & \langle 0.0,0.7,0.2\rangle & \langle 0.4,0.5,0.1\rangle \\ \langle 0.4,0.2,0.4\rangle & \langle 0.3,0.3,0.3\rangle & \langle 0.2,0.1,0.7\rangle & \langle 0.4,0.3,0.1\rangle & \langle 0.3,0.5,0.1\rangle \end{bmatrix}$$

p_1^1 and p_1^2 represent the observation of HSRs by two experts and present the result in PFSMs form for the first shot.

step 2:
The experts observe HSRs and present the results of second shot in PFSM as below

$$p_2^1 = \begin{bmatrix} \langle 0.3,0.2,0.5\rangle & \langle 0.7,0.1,0.1\rangle & \langle 0.3,0.6,0.1\rangle & \langle 0.1,0.2,0.5\rangle & \langle 0.0,0.3,0.3\rangle \\ \langle 0.7,0.3,0.0\rangle & \langle 0.2,0.5,0.2\rangle & \langle 0.1,0.5,0.2\rangle & \langle 0.4,0.4,0.2\rangle & \langle 0.3,0.3,0.4\rangle \\ \langle 0.2,0.3,0.2\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.3,0.7,0.0\rangle & \langle 0.9,0.1,0.0\rangle & \langle 0.1,0.2,0.4\rangle \\ \langle 0.1,0.3,0.5\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.0,0.9,0.1\rangle & \langle 0.4,0.2,0.4\rangle & \langle 0.7,0.2,0.1\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.0,0.1,0.9\rangle & \langle 0.5,0.1,0.1\rangle & \langle 0.2,0.3,0.5\rangle & \langle 0.1,0.2,0.7\rangle \end{bmatrix}$$

and

$$p_2^2 = \begin{bmatrix} \langle 0.2,0.3,0.5\rangle & \langle 0.1,0.2,0.7\rangle & \langle 0.2,0.2,0.6\rangle & \langle 0.1,0.1,0.8\rangle & \langle 0.5,0.1,0.3\rangle \\ \langle 0.1,0.3,0.6\rangle & \langle 0.3,0.1,0.3\rangle & \langle 0.9,0.0,0.0\rangle & \langle 0.6,0.1,0.1\rangle & \langle 0.1,0.2,0.6\rangle \\ \langle 0.7,0.1,0.0\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.0,0.5,0.5\rangle & \langle 0.8,0.1,0.1\rangle & \langle 0.3,0.2,0.2\rangle \\ \langle 0.3,0.6,0.0\rangle & \langle 0.1,0.2,0.3\rangle & \langle 0.1,0.9,0.0\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.2,0.2,0.6\rangle \\ \langle 0.1,0.9,0.0\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.4,0.2,0.4\rangle & \langle 0.0,0.9,0.0\rangle & \langle 0.1,0.2,0.7\rangle \end{bmatrix}$$

p_2^1 and p_2^2 represent the observation of HSRs by two experts and present the result in PFSMs form for the first shot.

step 3:
Combine PFSMs of $p_1^1 \cup p_1^2$ and $p_2^1 \cup p_2^2$ to form PFSBM as below $p_1 = p_1^1 \cup p_1^2$

$$p_1 = \begin{bmatrix} \langle 0.2,0.5,0.3\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.1,0.3,0.6\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.1,0.5,0.4\rangle \\ \langle 0.7,0.3,0.0\rangle & \langle 0.1,0.2,0.7\rangle & \langle 0.02,0.1,0.6\rangle & \langle 0.3,0.5,0.1\rangle & \langle 0.2,0.7,0.1\rangle \\ \langle 0.2,0.2,0.6\rangle & \langle 0.4,0.3,0.3\rangle & \langle 0.1,0.1,0.0\rangle & \langle 0.2,0.3,0.3\rangle & \langle 0.1,0.2,0.4\rangle \\ \langle 0.1,0.3,0.5\rangle & \langle 0.4,0.4,0.2\rangle & \langle 0.1,0.7,0.2\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.2,0.5,0.2\rangle \\ \langle 0.6,0.1,0.1\rangle & \langle 0.0,0.7,0.3\rangle & \langle 0.1,0.5,0.4\rangle & \langle 0.3,0.6,0.1\rangle & \langle 0.1,0.2,0.7\rangle \end{bmatrix} \cup$$

$$\begin{bmatrix} \langle 0.3, 0.1, 0.5\rangle & \langle 0.7,0.2,0.1\rangle & \langle 0.1,0.3,0.6\rangle & \langle 0.2,0.2,0.5\rangle & \langle 0.1, 0.1, 0.5\rangle \\ \langle 0.5,0.5,0.0\rangle & \langle 0.2,0.2,0.2\rangle & \langle 0.4,0.5,0.1\rangle & \langle 0.1,0.2,0.4\rangle & \langle 0.6,0.1,0.1\rangle \\ \langle 0.1,0.1,0.1\rangle & \langle 0.7,0.1,0.1\rangle & \langle 0.0,0.9,0.1\rangle & \langle 0.3,0.5,0.2\rangle & \langle 0.3,0.1,0.5\rangle \\ \langle 0.2,0.1,0.7\rangle & \langle 0.1,0.1,0.3\rangle & \langle 0.2,0.8,0.0\rangle & \langle 0.0,0.7,0.2\rangle & \langle 0.4,0.5,0.1\rangle \\ \langle 0.4,0.2,0.4\rangle & \langle 0.3,0.3,0.3\rangle & \langle 0.2,0.1,0.7\rangle & \langle 0.4,0.3,0.1\rangle & \langle 0.3,0.5,0.1\rangle \end{bmatrix}$$

Similarly $p_2 = p_2^1 \cup p_2^2$

$$p_2 = \begin{bmatrix} \langle 0.3, 0.2, 0.5\rangle & \langle 0.7, 0.1, 0.1\rangle & \langle 0.3, 0.6, 0.1\rangle & \langle 0.1, 0.2, 0.5\rangle & \langle 0.0, 0.3, 0.3\rangle \\ \langle 0.7, 0.3, 0.0\rangle & \langle 0.2, 0.5, 0.2\rangle & \langle 0.1, 0.5, 0.2\rangle & \langle 0.4, 0.4, 0.2\rangle & \langle\ 0.3, 0.3, 0.4\rangle \\ \langle 0.2, 0.3, 0.2\rangle & 0.5, 0.2, 0.2 & \langle 0.3, 0.7, 0.0\rangle & \langle 0.9, 0.1, 0.0\rangle & \langle 0.1, 0.2, 0.4\rangle \\ \langle 0.1, 0.3, 0.5\rangle & \langle\ 0.6, 0.1, 0.2\rangle & \langle 0.0, 0.9, 0.1\rangle & \langle 0.4, 0.2, 0.4\rangle & \langle 0.7, 0.2, 0.1\rangle \\ \langle 0.6, 0.1, 0.2\rangle & \langle 0.0, 0.1, 0.9\rangle & 0.5, 0.1, 0.1 & \langle 0.2, 0.3, 0.5\rangle & \langle 0.1, 0.2, 0.7\rangle \end{bmatrix}$$

$$\cup$$

$$\begin{bmatrix} \langle 0.2,0.3,0.5\rangle & \langle 0.1,0.2,0.7\rangle & \langle 0.2,0.2,0.6\rangle & \langle 0.1,0.1,0.8\rangle & \langle 0.5,0.1,0.3\rangle \\ \langle 0.1,0.3,0.6\rangle & \langle 0.3,0.1,0.3\rangle & \langle 0.9,0.0,0.0\rangle & \langle 0.6,0.1,0.1\rangle & \langle 0.1,0.2,0.6\rangle \\ \langle 0.7,0.1,0.0\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.0,0.5,0.5\rangle & \langle 0.8,0.1,0.1\rangle & \langle 0.3,0.2,0.2\rangle \\ \langle 0.3,0.6,0.0\rangle & \langle 0.2,0.2,0.3\rangle & \langle 0.1,0.9,0.0\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.2,0.2,0.6\rangle \\ \langle 0.1,0.9,0.0\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.4,0.2,0.4\rangle & \langle 0.0,0.9,0.0\rangle & \langle 0.1,0.2,0.7\rangle \end{bmatrix}$$

step 4: By using definition 1, we determine the score value of each element.

$$S(p_1) = \begin{bmatrix} 0.625 & 0.55 & 0.65 & 0.55 & 0.45 \\ 0.7 & 0.5 & 0.625 & 0.575 & 0.65 \\ 0.4 & 0.575 & 0.55 & 0.65 & 0.38 \\ 0.576 & 0.5 & 0.875 & 0.65 & 0.725 \\ 0.525 & 0.725 & 0.65 & 0.675 & 0.65 \end{bmatrix}$$

and

$$S(p_2) = \begin{bmatrix} 0.625 & 0.55 & 0.7 & 0.525 & 0.475 \\ 0.65 & 0.55 & 0.55 & 0.575 & 0.6 \\ 0.475 & 0.5 & 0.8 & 0.55 & 0.45 \\ 0.675 & 0.45 & 0.95 & 0.6 & 0.6 \\ 0.725 & 0.625 & 0.5 & 0.775 & 0.6 \end{bmatrix}$$

step 5:
Combine the corresponding elements of p_1 and p_2 and find the average of each HSR corresponding to the criteria e_j.

$$A = \begin{bmatrix} 0.625 & 0.55 & 0.675 & 0.537 & 0.462 \\ 0.675 & 0.525 & 0.587 & 0.575 & 0.625 \\ 0.437 & 0.537 & 0.675 & 0.6 & 0.415 \\ 0.625 & 0.475 & 0.912 & 0.625 & 0.662 \\ 0.625 & 0.675 & 0.575 & 0.725 & 0.625 \end{bmatrix}$$

step 6:
Determine the maximum value of each HSR and conclude the side effects of covid-19 vaccine, also

$$maximum\ \ value = \begin{bmatrix} 0.675 \\ 0.675 \\ 0.675 \\ 0.912 \\ 0.725 \end{bmatrix}$$

The maximum value for r_1, r_3 and r_4 HSR are $0.675, 0.675$ and 0.912 respectively. This shows that they had mild headaches as a side effect towards covid-19 vaccine. similarly for r_2 and r_5 the maximum values are 0.675 and 0.725. This shows that r_2 has mild fever as a side effect and r_5 had nausea as a side effect towards covid-19 vaccine. From the above observation, we conclude that the most common side effect for the age groups (30-44) is mild headaches.

Case (ii)
Again, to find the combined side effects of two shots of covid-19 vaccine given to HSR of same age group (45-60). Let $q_i, (i = 1,2)$, represent the experts who observe the HSRs, $R = (r_1, r_2, \dots, r_5)$ after each shots of covid -19 vaccine and present the results in PFSM. Let the criteria be $e_1 = mild\ \ fever, e_2 = muscle/joint\ \ aches, e_3 = mild\ \ headache, e_4 = nausea\ \ and\ \ e_5 = rashes\ \ over\ \ the\ \ body.$

The Method:
To find out the combined side effects of two shots of covid -19 vaccine, we use the following method.

step 1:
The experts observe HSRS and present the results of first shot in PFSMs as below,

$$q_1^1 = \begin{bmatrix} \langle 0.1,0.8,0.1\rangle & \langle 0.5,0.3,0.2\rangle & \langle 0.1,0.2,0.3\rangle & \langle 0.7,0.2,0.1\rangle & \langle 0.5,0.1,0.4\rangle \\ \langle 0.1,0.2,0.7\rangle & \langle 0.1,0.1,0.7\rangle & \langle 0.2,0.4,0.4\rangle & \langle 0.6,0.1,0.3\rangle & \langle 0.0,0.9,0.1\rangle \\ \langle 0.0,0.9,0.1\rangle & \langle 0.3,0.2,0.5\rangle & \langle 0.3,0.3,0.3\rangle & \langle 0.2,0.5,0.3\rangle & \langle 0.1,0.3,0.3\rangle \\ \langle 0.1,0.2,0.7\rangle & \langle 0.3,0.4,0.3\rangle & \langle 0.0,0.6,0.4\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.2,0.2,0.5\rangle \\ \langle 0.1,0.7,0.2\rangle & \langle 0.7,0.1,0.1\rangle & \langle 0.2,0.3,0.3\rangle & \langle 0.2,0.1,0.7\rangle & \langle 0.1,0.7,0.1\rangle \end{bmatrix}$$

and

$$q_1^2 = \begin{bmatrix} \langle 0.2,0.6,0.2\rangle & \langle 0.2,0.3,0.3\rangle & \langle 0.0,0.6,0.4\rangle & \langle 0.4,0.1,0.1\rangle & \langle 0.5,0.0,0.4\rangle \\ \langle 0.3,0.3,0.3\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.1,0.9,0.0\rangle & \langle 0.0,0.7,0.1\rangle & \langle 0.2,0.2,0.5\rangle \\ \langle 0.3,0.7,0.0\rangle & \langle 0.4,0.6,0.0\rangle & \langle 0.5,0.3,0.1\rangle & \langle 0.3,0.3,0.4\rangle & \langle 0.1,0.1,0.8\rangle \\ \langle 0.1,0.3,0.6\rangle & \langle 0.2,0.3,0.5\rangle & \langle 0.2,0.1,0.7\rangle & \langle 0.1,0.0,0.9\rangle & \langle 0.1,0.2,0.6\rangle \\ \langle 0.1,0.9,0.0\rangle & \langle 0.2,0.5,0.3\rangle & \langle 0.2,0.7,0.1\rangle & \langle 0.2,0.5,0.1\rangle & \langle 0.7,0.3,0.0\rangle \end{bmatrix}$$

q_1^1 and q_1^2 represent the observation of HSRs by two experts and present the result in PFSMs form for the first shot.

step 2:
The experts observe HSRs and present the results of second shot in PFSM as below

$$q_2^1 = \begin{bmatrix} \langle 0.3,0.7,0.0\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.1,0.7,0.2\rangle & \langle 0.3,0.2,0.1\rangle & \langle 0.6,0.3,0.1\rangle \\ \langle 0.1,0.1,0.5\rangle & \langle 0.7,0.0,0.2\rangle & \langle 0.3,0.4,0.3\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.0,0.1,0.9\rangle \\ \langle 0.1,0.8,0.1\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.2,0.1,0.1\rangle & \langle 0.4,0.4,0.2\rangle & \langle 0.2,0.1,0.7\rangle \\ \langle 0.1,0.1,0.1\rangle & \langle 0.3,0.3,0.4\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.1,0.2,0.7\rangle \\ \langle 0.1,0.6,0.3\rangle & \langle 0.2,0.6,0.2\rangle & \langle 0.1,0.7,0.2\rangle & \langle 0.0,0.7,0.2\rangle & \langle 0.1,0.1,0.8\rangle \end{bmatrix}$$

and

$$q_2^2 = \begin{bmatrix} \langle 0.2,0.5,0.3\rangle & \langle 0.1,0.1,0.3\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.4,0.4,0.2\rangle & \langle 0.3,0.1,0.6\rangle \\ \langle 0.1,0.8,0.1\rangle & \langle 0.0,0.9,0.0\rangle & \langle 0.1,0.9,0.0\rangle & \langle 0.4,0.6,0.0\rangle & \langle 0.6,0.2,0.1\rangle \\ \langle 0.0,0.9,0.1\rangle & \langle 0.3,0.4,0.3\rangle & \langle 0.1,0.1,0.1\rangle & \langle 0.2,0.3,0.5\rangle & \langle 0.2,0.1,0.7\rangle \\ \langle 0.8,0.1,0.1\rangle & \langle 0.1,0.6,0.2\rangle & \langle 0.3,0.3,0.1\rangle & \langle 0.2,0.3,0.3\rangle & \langle 0.7,0.1,0.1\rangle \\ \langle 0.1,0.8,0.1\rangle & \langle 0.1,0.7,0.2\rangle & \langle 0.2,0.6,0.1\rangle & \langle 0.4,0.4,0.1\rangle & \langle 0.5,0.5,0.0\rangle \end{bmatrix}$$

q_2^1 and q_2^2 represent the observation of HSRs by two experts and present the result in PFSMs form for the first shot.

step 3:
Combine PFSMs of $q_1^1 \cup q_1^2$ and $q_2^1 \cup q_2^2$ to form PFSBM as below $q_1 = q_1^1 \cup q_1^2$

$$\mathrm{q}_1 = \begin{bmatrix} \langle 0.1,0.8,0.1\rangle & \langle 0.5,0.3,0.2\rangle & \langle 0.1,0.2,0.3\rangle & \langle 0.7,0.2,0.1\rangle & \langle 0.5,0.1,0.4\rangle \\ \langle 0.1,0.2,0.7\rangle & \langle 0.1,0.1,0.7\rangle & \langle 0.2,0.4,0.4\rangle & \langle 0.6,0.1,0.3\rangle & \langle 0.0,0.9,0.1\rangle \\ \langle 0.0,0.9,0.1\rangle & \langle 0.3,0.2,0.5\rangle & \langle 0.3,0.3,0.3\rangle & \langle 0.2,0.5,0.3\rangle & \langle 0.1,0.3,0.3\rangle \\ \langle 0.1,0.2,0.7\rangle & \langle 0.3,0.4,0.3\rangle & \langle 0.0,0.6,0.4\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.2,0.2,0.5\rangle \\ \langle 0.1,0.7,0.2\rangle & \langle 0.7,0.1,0.1\rangle & \langle 0.2,0.3,0.3\rangle & \langle 0.2,0.1,0.7\rangle & \langle 0.1,0.7,0.1\rangle \end{bmatrix} \quad \cup$$

$$\begin{bmatrix} \langle 0.2, 0.6, 0.2\rangle & 0.2, 0.3, 0.3 & \langle 0.0, 0.6, 0.4\rangle & \langle\ 0.4, 0.1, 0.1\rangle & \langle 0.5, 0.0, 0.4\rangle \\ \langle 0.3, 0.3, 0.3\rangle & \langle 0.6, 0.1, 0.2\rangle & \langle 0.1, 0.9, 0.0\rangle & \langle 0.0, 0.7, 0.1\rangle & \langle 0.0,0.9,0.1\rangle \\ \langle 0.3, 0.7, 0.0\ \rangle & \langle 0.4, 0.6, 0.0\rangle & \langle\ 0.5, 0.3, 0.1\rangle & \langle 0.3, 0.3, 0.4\rangle & \langle 0.1, 0.1, 0.8\ \rangle \\ \langle 0.1, 0.3, 0.6\rangle & \langle 0.2, 0.3, 0.5\rangle & \langle\ 0.2, 0.1, 0.7\rangle & \langle 0.1, 0.0, 0.9\rangle & \langle 0.1, 0.2, 0.6\rangle \\ \langle 0.1, 0.9, 0.0\rangle & \langle 0.2, 0.5, 0.3\ \rangle & \langle 0.2, 0.7, 0.1\rangle & \langle 0.2, 0.5, 0.1\rangle & \langle 0.7, 0.3, 0.0\rangle \end{bmatrix}$$

Similarly $q_2 = q_2^1 \cup q_2^2$

$$q_2 = \begin{bmatrix} \langle 0.3, 0.7, 0.0\rangle & \langle 0.5, 0.1, 0.4\rangle & \langle 0.1, 0.7, 0.2\rangle & \langle 0.3, 0.2, 0.1\rangle & \langle 0.6, 0.3, 0.1\rangle \\ \langle 0.1, 0.1, 0.5\rangle & \langle 0.7, 0.0, 0.2\rangle & \langle 0.3, 0.4, 0.3\rangle & \langle 0.5, 0.2, 0.2\rangle & \langle 0.0, 0.1, 0.9\rangle \\ \langle\ 0.1, 0.8, 0.1\rangle & \langle\ 0.5, 0.2, 0.3\rangle & \langle 0.2, 0.1, 0.1\rangle & \langle 0.4, 0.4, 0.2\ \rangle & \langle 0.2, 0.1, 0.7\ \rangle \\ \langle 0.1, 0.1, 0.1\rangle & \langle 0.3, 0.3, 0.4\rangle & \langle 0.6, 0.1, 0.2\rangle & \langle\ 0.5, 0.1, 0.4\rangle & \langle 0.1, 0.2, 0.7\ \rangle \\ \langle 0.1, 0.6, 0.3\rangle & \langle 0.2, 0.6, 0.2\rangle & \langle 0.1, 0.7, 0.2\rangle & \langle 0.0, 0.7, 0.2\rangle & \langle 0.1, 0.1, 0.8\rangle \end{bmatrix}$$

$$\cup$$

$$\begin{bmatrix} \langle 0.2,0.5,0.3\rangle & \langle 0.1,0.1,0.3\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.4,0.4,0.2\rangle & \langle 0.3,0.1,0.6\rangle \\ \langle 0.1,0.8,0.1\rangle & \langle 0.0,0.9,0.0\rangle & \langle 0.1,0.9,0.0\rangle & \langle 0.4,0.6,0.0\rangle & \langle 0.6,0.2,0.1\rangle \\ \langle 0.0,0.9,0.1\rangle & \langle 0.3,0.4,0.3\rangle & \langle 0.1,0.1,0.1\rangle & \langle 0.2,0.3,0.5\rangle & \langle 0.2,0.1,0.7\rangle \\ \langle 0.8,0.1,0.1\rangle & \langle 0.1,0.6,0.2\rangle & \langle 0.3,0.3,0.1\rangle & \langle 0.2,0.3,0.3\rangle & \langle 0.7,0.1,0.1\rangle \\ \langle 0.1,0.8,0.1\rangle & \langle 0.1,0.7,0.2\rangle & \langle 0.2,0.6,0.1\rangle & \langle 0.4,0.4,0.1\rangle & \langle 0.5,0.5,0.0\rangle \end{bmatrix}$$

step 4: By using definition 1, we determine the score value of each element.

$$S(q_1) = \begin{bmatrix} 0.675 & 0.6 & 0.6 & 0.475 & 0.5 \\ 0.6 & 0.5 & 0.825 & 0.65 & 0.75 \\ 0.9 & 0.7 & 0.6 & 0.7 & 0.525 \\ 0.625 & 0.675 & 0.675 & 0.5 & 0.55 \\ 0.9 & 0.575 & 0.825 & 0.6 & 0.725 \end{bmatrix}$$

and

$$S(q_2) = \begin{bmatrix} 0.675 & 0.55 & 0.7 & 0.55 & 0.6 \\ 0.65 & 0.675 & 0.825 & 0.675 & 0.55 \\ 0.925 & 0.8 & 0.225 & 0.675 & 0.55 \\ 0.375 & 0.755 & 0.5 & 0.55 & 0.55 \\ 0.85 & 0.825 & 0.8 & 0.725 & 0.65 \end{bmatrix}$$

step 5:
Combine the corresponding elements of q_1 and q_2 and find the average of each HSR corresponding to the criteria e_j.

$$A = \begin{bmatrix} 0.675 & 0.575 & 0.65 & 0.512 & 0.55 \\ 0.625 & 0.587 & 0.825 & 0.662 & 0.65 \\ 0.912 & 0.75 & 0.412 & 0.712 & 0.537 \\ 0.5 & 0.712 & 0.587 & 0.525 & 0.55 \\ 0.875 & 0.7 & 0.812 & 0.662 & 0.687 \end{bmatrix}$$

step 6:
Determine the maximum value of each HSR and conclude the side effects of covid-19 vaccine, also

$$maximum\ \ value = \begin{bmatrix} 0.675 \\ 0.825 \\ 0.912 \\ 0.712 \\ 0.875 \end{bmatrix}$$

The maximum value for r_1, r_3 and r_5 HSR are $0.675, 0.912$ and 0.875 respectively. This shows that they had mild fever as a side effect towards covid-19 vaccine. similarly for r_2 and r_4 the maximum values are 0.875 and 0.712. This shows that r_2 has mild headaches as a side effect and r_4 had muscle/joint as a side effect towards covid-19 vaccine. From the above observation, we conclude that the most common side effect for the age groups (45-60) is mild fever.

6. Conclusion

We have presented the concept of $PFBM$ and have shown the result of Commutative, Associative, Distributive with algebraic operations. In the future, we will apply this concept with Picture Fuzzy Bi-vector Space.

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Chinnadurai.V received his Ph.D., in Mathematics from Annamalai University, India in 1998 and he is currently as Professor in the department of Mathematics, Annamalai University. His research interest include Fuzzy Sets and System, Picture Fuzzy Matrices and Picture Fuzzy Bimatrices.

Madhanraj.S received M.Sc., Mathematics from St.Joseph's College, Cuddalore in 2015 and M.phil., Mathematics from St.Joseph's College, Cuddalore in 2016. Since 2021, I am doing Ph.D. degree in the department of Mathematics, Annamalai University. My area of interest include Fuzzy Sets and System, Picture Fuzzy Matrices and Picture Fuzzy Bimatrices