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Measure of Slope Rotatability for Second Order Response Surface Designs under Tri-Diagonal Correlation Error Structure Using Pairwise Balanced Designs

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Abstract: In this paper, measure of slope rotatability for second order response surface designs using pairwise balanced designs under tri-diagonal correlation error structure is suggested and studied for $6 \leq v \leq 15$ (v number of factors).

Keywords: Response surface design, slope-rotatability, tri-diagonal correlation error structure, pairwise balanced designs, weak slope rotatability region.

2010 AMS Subject Classification: 62K05.

1. Introduction

Response surface methodology is a collection of mathematical and statistical techniques useful for analysing problems where several independent variables influence a dependent variable. The independent variables are often called the input or explanatory variables and the dependent variable is often the response variable. An important step in development of response surface designs was the introduction of rotatable designs by Box and Hunter (1957). Das and Narasimham (1962) constructed rotatable designs using balanced incomplete block designs (BIBD). Tyagi (1964) constructed second order rotatable designs using pairwise balanced designs (PBD).The study of rotatable designs mainly emphasized on the estimation of absolute response. Estimation of response at two different points in the factor space will often be a great importance. If differences at two points close together, estimation of local slope (rate of change) of the response is of interest. Hader and Park (1978) extended the notion of rotatability to cover the slope for the case of second order models. In view of slope rotatability of response surface methodology, a good estimation of derivatives of the response function is more important than estimation of mean response. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses, rate of disintegration of radioactive material in animal etc. (cf. Park 1987). Victorbabu and Narasimham (1991, 1993) studied second order slope rotatable designs (SOSRD) using BIBD and PBD respectively. Victorbabu (2002, 2007) suggested SOSRD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes and a review on SOSRD. To access the degree of slope rotatability Park and Kim (1992) introduced a measure for second order response surface designs. Park et.al (1993) introduced measure of rotatability for second order response surface designs. Victorbabu and Surekha (2011, 12a, 12b, 12c) studied measure of slope rotatability for second order response surface designs using central composite designs (CCD) and BIBD, PBD and SUBA with two unequal block sizes respectively.

Many authors have studied rotatable designs and slope rotatable designs assuming errors to be uncorrelated and homoscedastic. However, it is not uncommon to come across practical situations when the errors are correlated, violating the usual assumptions. Das (1997, 2003a) introduced and studied robust second order rotatable designs. Das (2003b) introduced slope rotatability with correlated errors and gave conditions for the different variance-covariance error structures. Das and Park (2007) introduced measure of robust rotatability for second order response surface designs. To access the degree of slope rotatability for correlated errors a new measure for second order response surface designs was introduced by Das and Park (2009). Rajyalakshmi and Victorbabu (2014, 15, 18, 19) studied SOSRD under tri-diagonal structure of errors using CCD, pairwise balanced designs, symmetrical unequal block arrangements (SUBA) with two unequal block sizes and BIBD respectively. Sulochana and Victorbabu (2020a, 2020b) studied SOSRD under tri-diagonal correlated structure of errors using a pair of SUBA with two unequal block sizes and a pair of BIBD respectively. Sulochana and Victorbabu (2020c, 2021a, 2021b, 2021c) studied measure of slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD, CCD, BIBD and SUBA with two unequal block sizes respectively. Sulochana and Victorbabu (2021d, 2021e) studied measure of slope rotatability for second order response surface designs under tri-diagonal correlated structure of errors using CCD and BIBD respectively.

In this paper, following the works of Park and Kim (1992), Das (2003a, 2003b, 2014), Das and Park (2009), Victorbabu and Surekha (2012b), Rajyalakshmi and Victorbabu (2015), the measure of slope-rotatability for second order response surface designs with tri-diagonal correlation error structure using PBD for $\rho(-0.9\leq\rho\leq 0.9)$ for $6\leq v\leq 15$ (v number of factors) is suggested.

2. Preliminaries

2.1. Tri-diagonal correlation structure

The tri-diagonal structure of errors arises when the variance is same (σ^2) and the correlation between any two errors having lag n is ρ, and 0 (zero) otherwise. The tri-diagonal error structure with 2n observations is given below. (cf. Das (2014) p.30).

$$W=\left\{D(e)=\sigma^2\left[\begin{pmatrix} I_n & I_n \\ I_n & I_n \end{pmatrix}\times\frac{1+\rho}{2}+\begin{pmatrix} I_n & -I_n \\ -I_n & I_n \end{pmatrix}\times\frac{1-\rho}{2}\right]=W_{2n\times 2n}(\rho),\text{ say}\right\}$$

$$W^{-1}_{2n\times 2n}(\rho)=\left(\sigma^2\right)^{-1}\left[\begin{bmatrix} I_n & I_n \\ I_n & I_n \end{bmatrix}\times\frac{1}{2(1+\rho)}+\begin{bmatrix} I_n & -I_n \\ -I_n & I_n \end{bmatrix}\times\frac{1}{2(1+\rho)}\right]$$

2.2. Conditions of slope rotatability for second order response surface designs with tri-diagonal correlation error structure (cf. Das (2003a, 2003b, 2014))

A second order response surface design $D=(X_{ui})$ for fitting,

$$Y_u(X)=b_0+\sum_{i=1}^{v}b_iX_{ui}+\sum_{i=1}^{v}b_{ii}X_{ui}^2+\sum\sum_{i\leq j=1}^{v}b_{ij}X_{ui}X_{uj}+e_u;1\leq u\leq 2n \qquad (2.1)$$

where X_{ui} denotes the level of the ith factor (i=1,2,…,v) in the uth run (u=1,2,…,2n) of the experiment, e_u's are correlated random errors, is said to be a SOSRD under tri-diagonal correlated structure of errors, if the variance of the estimate of first order partial derivative of $Y_u\left(X_{u1}, X_{u2}, X_{u3},...,X_{uv}\right)$ with respect to each independent variable $\left(X_i\right)$ is only a function of the distance $\left(d^2=\sum_{i=1}^{v} X_i^2\right)$ of the point $\left(X_{u1}, X_{u2}, X_{u3},...,X_{uv}\right)$ from the origin (centre of the design). i.e, $V\left(\frac{\partial \hat{Y}_u}{\partial X_i}\right)=h\left(d^2\right)$. Such a spherical variance function $h\left(d^2\right)$ for estimation of slopes in the second order response surface is achieved if the design points satisfy the following conditions (cf. Das 2003a, 2003b and 2014, Rajyalakshmi and Victorbabu 2014, 15, 18, 19).

Following Box and Hunter (1957), Hader and Park (1978), Victorbabu and Narasimham (1991a), Das (2003a, 2003b and 2014), Rajyalakshmi and Victorbabu (2014, 15, 18, 19) the general conditions for second order slope rotatability under the tri-diagonal correlated structure of errors can be obtained as follows. To simplify the fit of the second order polynomial from design points "D" through the method of least squares, we impose the following simple symmetry conditions on "D" to facilitate easy solutions of the normal equations.

$$\sum_{u=1}^{2n}\prod_{i=1}^{v} X_{ui}^{\alpha_i}=0, \text{ if any } \alpha_i \text{ is odd, for } \sum \alpha_i \le 4, \tag{2.2}$$

$$\sum_{u=1}^{2n} X_{ui}^2=\text{constant}=2n\lambda_2, \text{ for all i,} \tag{2.3}$$

$$\sum_{u=1}^{2n} X_{ui}^4=\text{constant}= c2n\lambda_4, \text{ for all i,} \tag{2.4}$$

$$\sum_{u=1}^{2n} X_{ui}^2 X_{uj}^2=\text{constant}=2n\lambda_4, \text{ for all values } i \ne j \tag{2.5}$$

$$\sum_{u=1}^{2n} X_{ui}^4 = c \sum_{u=1}^{2n} X_{ui}^2 X_{uj}^2 \tag{2.6}$$

where, c, N = 2n, λ_2 and λ_4 are constants. The summation is over the designs points,

The variances and covariances of the estimated parameters under the tri-diagonal correlated structure of errors are as follows:

$$V\left(\hat{b}_0\right) = \frac{\sigma^2 \lambda_4 (c+v-1)\ (1+\rho)}{2n\Delta} \tag{2.7}$$

$$V\left(\hat{b}_i\right) = \frac{\sigma^2 (1-\rho^2)}{2n\lambda_2} \tag{2.8}$$

$$V\left(\hat{b}_{ij}\right) = \frac{\sigma^2 (1-\rho^2)}{2n\lambda_4} \tag{2.9}$$

$$V\left(\hat{b}_{ii}\right) = \frac{\sigma^2 (1-\rho^2)[\lambda_4 \left(c+v-2\right) -(v-1)\lambda_2^2 (1-\rho)]}{(c-1)(2n)\lambda_4\Delta} \tag{2.10}$$

$$Cov\left(\hat{b}_0, \hat{b}_{ii}\right) = \frac{-\sigma^2 \lambda_2^2 (1-\rho^2)}{2n\Delta} \tag{2.11}$$

$$Cov\left(\hat{b}_{ii}, \hat{b}_{ij}\right) = \frac{\sigma^2 (1-\rho^2)[\lambda_2^2 \left(1-\rho\right) -\lambda_4]}{(c-1)(2n)\lambda_4\Delta} \tag{2.12}$$

where $\Delta = \left[\lambda_4 \left(c+v-1\right) - v\lambda_2^2 \left(1-\rho\right)\right]$ and the other covariances are zero.

An inspection of the variance of $\hat{b}_0$ shows that a necessary condition for the existence of a non-singular second order slope rotatable design with tri-diagonal correlated structure is

$$\left[\lambda_4 \left(c+v-1\right) - v\lambda_2^2 \left(1-\rho\right)\right] > 0 \tag{2.13}$$

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v(1\text{-}\rho)}{c+v\text{-}1} \text{(non-singularity condition)} \tag{2.14}$$

If the non-singularity condition (2.14) exists then only the design exists.

For the second order model

$$\frac{\partial \hat{Y}_u}{\partial X_i} = \hat{b}_i + 2\hat{b}_{ii} X_i + \sum_{i=1, j \neq i}^{v} \hat{b}_{ij} X_j,$$

$$V\left(\frac{\partial \hat{Y}_u}{\partial X_i}\right) = V\left(\hat{b}_i\right) + 4X_i^2 V\left(\hat{b}_{ii}\right) + \sum_{i=1, j \neq i}^{v} X_j^2 V\left(\hat{b}_{ij}\right) \tag{2.15}$$

The condition for right hand side of equation (2.15) to be a function of $(d^2 = \sum_{i=1}^{v} X_i^2)$ alone (for slope rotatability) is clearly,

$$V\left(\hat{b}_{ii}\right) = \frac{1}{4} V\left(\hat{b}_{ij}\right) \tag{2.16}$$

Equation (2.16) leads to condition,

$$\frac{cN\lambda_4}{\left(1\text{-}\rho^2\right)\left(1+\rho\right)}\left[4N\text{-}(c+v\text{-}2)N + v\left(\frac{N\lambda_2^2(1\text{-}\rho)}{\lambda_4}\right)\right] + \frac{N^2\lambda_4}{\left(1\text{-}\rho^2\right)\left(1+\rho\right)}\left[5v\text{-}9\right] \text{-} N^2\lambda_2^2\left[\frac{5v\text{-}4}{(1+\rho)^2}\right] = 0 \tag{2.17}$$

where N = 2n.

Simplifying (2.17) gives rise,

$$\lambda_4\left[v(5\text{-}c)\text{-}(c\text{-}3)^2\right] + \lambda_2^2\left[v(c\text{-}5)+4\right]\left(1-\rho\right) = 0 \tag{2.18}$$

For $\rho = 0$, equation (2.18) reduces to

$$\lambda_4\left[v(5\text{-}c)\text{-}(c\text{-}3)^2\right]+\lambda_2^2\left[v(c\text{-}5)+4\right]=0 \tag{2.19}$$

Equation (2.19) is similar to the SOSRD condition of Victorbabu and Narasimham (1991a).

Therefore, equations (2.2) to (2.12), (2.14) to (2.18) give a set of conditions for SOSRD under tri-diagonal correlated structure of errors for any general second order response surface design.

On simplification of (2.15) using (2.8) to (2.12) and (2.16), we have,

$$V\left(\frac{\partial \hat{Y}_u}{\partial X_i}\right)=V\left(\hat{b}_i\right)+4X_i^2\frac{V\left(\hat{b}_{ij}\right)}{4}+\sum_{i=1;j\neq i}^{v} X_j^2 V\left(\hat{b}_{ij}\right)$$

$$V\left(\frac{\partial \hat{Y}_u}{\partial X_i}\right)=V\left(\hat{b}_i\right)+\sum_{i=1}^{v} X_i^2 V\left(\hat{b}_{ij}\right)$$

$$V\left(\frac{\partial \hat{Y}_u}{\partial X_i}\right)=V\left(\hat{b}_i\right)+V\left(\hat{b}_{ij}\right)d^2$$

where $d^2=\sum_{i=1}^{v} X_i^2$ and $V\left(\hat{b}_i\right)$, $V\left(\hat{b}_{ij}\right)$ are stated in (2.8) and (2.9). Further, we have

$$V\left(\frac{\partial \hat{Y}_u}{\partial X_i}\right)=\frac{1\text{-}\rho^2}{N}\left(\frac{1}{\lambda_2}+\frac{d^2}{\lambda_4}\right)\sigma^2 \tag{2.20}$$

where N=2n.

2.4. Slope rotatability for second order response surface designs with tri-diagonal correlation error structure using PBD (cf. Rajyalakshmi and Victorbabu (2015))

Following the works of Hader and Park (1978), Victorbabu and Narasimham (1991, 93), Das (2003a, 2003b, 2014), Rajyalakshmi and Victorbabu (2015), the method construction of slope rotatability for second order response surface designs with tri-diagonal correlation error structure using PBD is given below.

Pairwise balanced designs: The arrangement of v treatments in b blocks will be called a PBD of index λ and type $(v, k_1, k_2, ..., k_m)$ if each block contains $k_1, k_2, ..., k_m$ treatments $(k_i \leq v, k_i \neq k_j)$ and each pair of distinct treatments occurs in exactly λ blocks of size $k_i (i=1,2,...,m)$ then $b=\sum_{i=1}^{m} b_i$ and $\lambda v(v-1)=b=\sum_{i=1}^{m} b_i k_i (k_i-1)$.

Let $(v, b, r, k_1, k_2, ..., k_m, \lambda)$, be an equi-replicated PBD, $k=\max(k_1, k_2, ..., k_m)$, Let $2^{t(k)}$ denote a fractional replicate of 2^k in ± 1 levels, in which no interaction with less than five factors is confounded. $[1-(v, b, r, k_1, k_2, ..., k_m, \lambda)]$ denote the design points generated from the transpose of incidence matrix of PBD. $[1-(v, b, r, k_1, k_2, ... k_m, \lambda)]2^{t(k)}$ are the $b\, 2^{t(k)}$ design points generated from PBD by 'multiplication' (Raghavarao, 1971). $(a, 0, 0, ...0)2^1$ denote the design points generated from $(a, 0, 0, ...0)$ point set, and $\cup$ denotes combination of the design points generated from different sets of points. n_0 denote the number of central points. The total number of factorial combinations in the design can be written as $N=bF+2v+n_0$. Here $F=2^{t(k)}$.

Here we consider SOSRD using PBD Victorbabu and Narasimham (1993) having 'n' (n=bF+2v) non-central design points. The set of 'n'- non central design points are extended to 2n design points by adding 'n' (n_0=n) central points just below or above the 'n' non-central design points. Hence 2n (= N) be the total number of design points of the slope rotatability for second order response surface designs with tri-diagonal correlation error structure using PBD.

Result (3.1): For the design points, $[1-(v, b, r, k_1, k_2,\dots k_m, \lambda)]F \cup (a,0,0,\dots.0)2^1 \cup (n_0)$ will give a v-dimensional SOSRD under tri-diagonal correlated structure of errors using PBD in $N=bF+2v+n_0$ design points, where a^2 is positive real root of the fourth degree polynomial equation,

$$\left[\left(8v(1-\rho)-4N\right)\right]a^8+\left[8vrF\left(1-\rho\right)\right]a^6+$$
$$\left[\left(2vr^2F^2(1-\rho)+\left\{\left((12-2v)\lambda-4r\right)N+\left(16\lambda-20v\lambda+4vr\right)(1-\rho)\right\}F\right)\right]a^4+$$
$$\left[\left(4vr+(16-20v)r\lambda\right)\right]F^2(1-\rho)a^2+\left[\left((5v-9)\lambda^2+(6-v)r\lambda-r^2\right)\right]NF^2+$$
$$\left[\left(vr+4\lambda-5v\lambda\right)\right](1-\rho)r^2F^3=0$$

Note: Values of SOSRD under tri-diagonal correlation error structure using PBD can be obtained by solving the above equation.

3. Measure of second order slope rotatability for correlated structure of errors (cf. Das and Park (2009))

Following Das and Park (2009), equations (2.2) to (2.18) give necessary and sufficient conditions for a measure for any second order response surface designs with correlated errors. Further we have,

$V(b_i)$ eual for all i,

$V(b_{ii})$ eual for all i,

$V(b_{ij})$ eual for all i, j, where $i \neq j$

$Cov(b_i, b_{ii})=Cov(b_i, b_{ij})=Cov(b_{ii},b_{ij})=Cov(b_{ij},b_{il})=0$ for all $i \neq j \neq l$, and for all ρ (3.1)

Das and Park (2009) proposed that, if the conditions in (2.2) to (2.18) and (3.1) are met, $M_V(D)$ is the proposed measure of slope rotatability for second order response surface designs for any correlated error structure.

$$M_v(D)=\frac{1}{1+Q_v(D)}$$

$$\text{where } Q_v(D)=\frac{1}{2(v-1)\sigma^4}\left\{(v+2)(v+4)\sum_{i=1}^{v}\left[(V(b_i)-\overline{V})+\frac{a_i-\overline{a}}{v+2}\right]^2\right.$$

$$+\frac{4}{v(v+2)}\sum_{i=1}^{v}(a_i-\overline{a})^2+2\sum_{i=1}^{v}\left[\left(4V(b_{ii})-\frac{a_i}{v}\right)^2+\sum_{i=1;\, j^1 i}^{v}\left(V(b_{ij})-\frac{a_i}{v}\right)^2\right]$$

$$+4(v+4)\left(4Cov(b_i,b_{ii})^2+\sum_{j=1;j\neq i}^{v}Cov(b_i,b_{ij})^2\right)$$

$$\left.+4\sum_{i=1}^{v}\left(4\sum_{j=1;\,j\neq i}^{v}Cov(b_{ii},b_{ij})^2\right)+\sum_{j<l;\,j,l\neq i}\sum^{v}Cov(b_{ij},b_{ij})^2\right\} \tag{3.2}$$

here $\overline{V}=\frac{1}{v}\sum_{i=1}^{v}V(b_i)$, $a_i=4V(b_{ii})+\sum_{j=1;j\neq i}^{v}V(b_{ij})\ (1\leq i\leq v)$ and $\overline{a}=\frac{1}{v}\sum_{i=1}^{v}a_i$.

It can be easily shown that $Q_v(D)$ in equation (3.2) becomes zero for all values ρ, if and only if the conditions in equations (3.1) hold.

Further, it is simplified to

$$Q_v(D)=\frac{1}{\sigma^4}\left[4V(b_{ii})-V(b_{ij})\right]^2. \tag{3.3}$$

Note that $0\leq M_v(D)\leq 1$, and it can be easily shown that $M_v(D)$ is one if and only if the design is slope rotatable with any correlated error structure for all values of ρ, and $M_v(D)$ approaches to zero as the design ‘D’ deviates from the slope-rotatability under specified correlated error structure.

4. Measure of slope rotatability for second order response surface designs with tri-diagonal correlation error structure using pairwise balanced designs

In this paper, the degree of slope rotatability for second order response surface designs under tri-diagonal correlated structure of errors $\left(\rho\left(-0.9\leq\rho\leq0.9\right)\right)$ using pairwise balanced designs for $6\leq v\leq15$ (v number of factors) is suggested.

Following Park and Kim (1992), Das and Park (2009), Victorbabu and Surekha (2012b), Rajyalakshmi and Victorbabu (2015), the proposed measure of slope-rotatability for second order response surface designs under tri-diagonal correlated structure of errors using PBD is given below.

Let $(v, b, r, k_1, k_2, ..., k_m, \lambda)$, be an equi-replicated PBD, $k=\max(k_1, k_2, ..., k_m)$, Let $2^{t(k)}$ denote a fractional replicate of 2^k in ±1 levels, in which no interaction with less than five factors is confounded, then the design points, $[1-(v, b, r, k_1, k_2, ... k_m, \lambda)]F \cup (a,0,0,....0)2^1 \cup (n_0)$ will give slope rotatability for second order response surface designs under tri-diagonal correlated structure of errors using PBD in $N=2n=bF+2v+n_0$ design points. Here $F=2^{t(k)}$. For the design points generated from PBD, equations in (2.2) are true. Further, from (2.3), (2.4), and (2.5), we have,

(I) $\sum_{u=1}^{2n} x_{iu}^2=rF+2a^2=N\lambda_2$

(II) $\sum_{u=1}^{2n} x_{ui}^4=rF+2a^4=cN\lambda_4$

(III) $\sum_{u=1}^{2n} x_{ui}^2 x_{ui}^2=\lambda F=N\lambda_4$ (4.1)

Measure of slope rotatability of second order response surface designs under tri-diagonal correlated structure of errors using PBD can be obtained by

$$M_V(D)=\frac{1}{1+Q_V(D)}$$

$$Q_V(D)=\frac{1}{\sigma^4}\left[4V(b_{ii})-V(b_{ij})\right]^2$$
$$=\frac{1}{\sigma^4}\left[4G-\frac{(1-\rho^2)\,\sigma^2}{\lambda F}\right]^2$$

(4.2)

where G= $V(b_{ii})$

$$=\frac{\left(1-\rho^2\right)\sigma^2}{\left(F(r-\lambda)+2a^4\right)}\left[\frac{N\left((r-\lambda)F+2a^4\right)+(v-1)\left(N\lambda F-(r^2\lambda^2+4rFa^2+4a^4)(1-\rho)\right)}{N\left((r-\lambda)F+2a^4\right)+(v)\left(N\lambda F-(r^2F^2+4rFa^2+4a^4)(1-\rho)\right)}\right]$$

by substituting (3.3) and (4.1) in $V(b_{ii})$ of (2.10) we get above G value.

If $M_V(D)$ is one if and only if the design 'D' is slope rotatable under tri-diagonal correlated structure of errors using PBD for all values of ρ, and $M_V(D)$ approaches to zero as the design 'D' deviates from the slope-rotatability under tri-diagonal correlated structure of errors using PBD.

Example: We illustrate the method of measure of slope-rotatability for second order response surface designs under tri-diagonal correlated structure of errors with the help of a PBD $(v=6,\, b=7,\, r=3,\, k_1=3,\, k_2=2, \lambda=1)$.

The design points, $[1\text{-}(v=6, b=7, r=3, k_1=3, k_2=2,\lambda=1)]2^3$ U $(a,0,0,....0)2^1$ U $(n_0=68)$ will give slope rotatability for second order response surface designs under tri-diagonal correlated structure of errors in

N = 2n = 136 design points for 6 factors. From equations (4.1), we have,

(I) $\sum_{u=1}^{2n} x_{ui}^{2} = 24+2a^{2} = N\lambda_{2}$

(II) $\sum_{u=1}^{2n} x_{ui}^{4} = 24+2a^{4} = cN\lambda_{4}$

(III) $\sum_{u=1}^{2n} x_{ui}^{2} x_{ui}^{2} = 8 = N\lambda_{4}$ (4.3)

From (I), (II) and (III) of (4.3), we get $\lambda_2 = \frac{24+2a^2}{136}$, $\lambda_4 = \frac{8}{136}$ and $c = \frac{24+2a^4}{8}$. Substituting λ_2, λ_4 and c in (2.17) and on simplification, we get the following biquadratic equation in a^2.

$$\left[48(1-\rho)-544\right]a^{8}+1152(1-\rho)a^{6} + \left[6912(1-\rho)-\{1632+32(1-\rho)\}8\right]a^{4}-6144(1-\rho)a^{2}+\left[104448(1-\rho)-36864(1-\rho)\right]=0. \quad (4.4)$$

Equation (4.4) has only one positive real root for each value of ρ, $a^2=2.6772$(by taking $\rho=0.1$). This can be alternatively written directly from result (2.1). Solving (4.4), we get $a=1.6362$(by taking $\rho=0.1$). From (4.2) we get $Q_V(D)=0$, $M_V(D)=1$ for each value of ρ.

Suppose if we take a=1 instead of taking a=1.6362 for the above PBD we get $Q_V(D)=0.005813$, then $M_V(D)=0.9941$ (taking $\rho=0.1$). Here $M_V(D)$ deviates from slope rotatability for second order response surface designs under tri-diagonal correlated structure of errors using PBD.

Table 1, gives the values of $M_V(D)$ for second order rotatable designs under tri-diagonal correlated structure of errors using PBD for $\rho(-0.9 \le \rho \le 0.9)$ and $6 \le v \le 15$ (v number of factors).

4.1.Weak slope rotatability region for correlated errors (cf. Das and Park (2009)

Following Das and Park (2009), we also find weak slope rotatability region (WSRR) for second order response surface designs with tri-diagonal correlation error structure using PBD.

$$M_v(D) \geq s$$

$M_v(D)$ involves the correlation parameter $\rho \in W$ and as such, $M_v(D) \geq s$ for each value of ρ is too strong to be met. On the other hand, for a given v, we can find range of values of ρ for which $M_v(D) \geq s$. Das and Park (2009) call this range as the weak slope rotatability region $(WSRR(R_{D(s)}(\rho))$ of the design ‘ D ’. Naturally, the desirability of using ‘ D ’ will rest on the wide nature of $(WSRR(R_{D(s)}(\rho))$ along with its strength s. Generally, we would require ‘s’ to be very high say, around 0.95 (cf. Das and Park (2009)).

Table 2, gives the values of weak slope rotatability region $(WSRR(R_{D(s)}(\rho))$ for second order slope rotatable designs with tri-diagonal correlation error structure using PBD for $\rho(-0.9 \leq \rho \leq 0.9)$ and $6 \leq v \leq 15$ (v number of factors) respectively.

Table 1: Measure of slope rotatability for second order response surface designs $(M_v(D))$ with tri-diagonal correlation error structure using PBD for $\rho(-0.9 \leq \rho \leq 0.9)$ and $6 \leq v \leq 15$

(v=6, b=7, r=3, k_1=3,k_2=2, λ=1), n=68, n_0 =68, 2n=136												
a \ ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	1	0.9994	0.9996	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9995	0.9995	0.9994	2.0451
-0.8	0.9983	0.9988	0.9995	0.9999	0.9997	0.9993	0.9989	0.9986	0.9984	0.9983	0.9982	0.9981	1.9398
-0.7	0.9972	0.9984	0.9996	0.9999	0.9994	0.9985	0.9978	0.9984	0.9969	0.9966	0.9964	0.9963	1.8568
-0.6	0.9961	0.9980	0.9996	0.9999	0.9989	0.9976	0.9965	0.9956	0.9951	0.9947	0.9944	0.9942	1.7960
-0.5	0.9953	0.9978	0.9997	0.9997	0.9984	0.9966	0.9951	0.9940	0.9932	0.9927	0.9923	0.9921	1.7519
-0.4	0.9946	0.9977	0.9998	0.9996	0.9978	0.9956	0.9938	0.9925	0.9915	0.9909	0.9904	0.9901	1.7194
-0.3	0.9940	0.9976	0.9998	0.9994	0.9972	0.9948	0.9927	0.9911	0.9900	0.9893	0.9887	0.9884	1.6945

-0.2	0.9938	0.9976	0.9999	0.9992	0.9968	0.9940	0.9918	0.9901	0.9889	0.9881	0.9875	0.9871	1.6751
-0.1	0.9937	0.9976	0.9999	0.9991	0.9964	0.9936	0.9912	0.9894	0.9882	0.9873	0.9867	0.9862	1.6595
0	0.9938	0.9978	0.9999	0.9990	0.9962	0.9933	0.9909	0.9892	0.9879	0.9870	0.9864	0.9860	1.6468
0.1	0.9941	0.9980	0.9999	0.9989	0.9962	0.9934	0.9911	0.9893	0.9881	0.9873	0.9867	0.9862	1.6362
0.2	0.9947	0.9982	0.9999	0.9989	0.9964	0.9937	0.9915	0.9899	0.9888	0.9880	0.9874	0.9870	1.6272
0.3	0.9953	0.9984	0.9999	0.9990	0.9967	0.9943	0.9923	0.9909	0.9899	0.9892	0.9887	0.9883	1.6195
0.4	0.9961	0.9987	0.9999	0.9991	0.9971	0.9951	0.9934	0.9922	0.9914	0.9908	0.9903	0.9900	1.6128
0.5	0.9969	0.9990	0.9999	0.9992	0.9976	0.9960	0.9947	0.9938	0.9931	0.9926	0.9923	0.9920	1.6070
0.6	0.9978	0.9993	1	0.9994	0.9982	0.9971	0.9961	0.9954	0.9949	0.9946	0.9943	0.9942	1.6019
0.7	0.9986	0.9995	1	0.9996	0.9989	0.9981	0.9975	0.9971	0.9968	0.9965	0.9964	0.9963	1.5973
0.8	0.9993	0.9998	0.9999	0.9998	0.9994	0.9990	0.9987	0.9985	0.9984	0.9982	0.9982	0.9981	1.5932
0.9	0.9998	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9995	0.9995	0.9995	0.9994	1.5895

Note : Here a* indicates that the values of slope rotatability for second order response surface designs with tri-diagonal correlation error structure using PBD. For each value of a*, $M_V(D)$ is equal to 1.

(v=8, b=15, r=6, k_1=4,k_2=3, λ=2,), n=256, n_0 =256, 2n=512													
a / ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.6098
-0.8	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.5292
-0.7	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	2.4708
-0.6	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9997	0.9997	0.9997	2.4286
-0.5	0.9995	0.9996	0.9997	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	2.3973
-0.4	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9998	0.9998	0.9996	0.9995	0.9995	2.3735
-0.3	0.9994	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9994	2.3550
-0.2	0.9994	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	0.9997	0.9996	0.9995	0.9994	0.9993	2.3402
-0.1	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3281
0	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3181
0.1	0.9994	0.9995	0.9997	0.9998	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3097
0.2	0.9994	0.9995	0.9997	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3025
0.3	0.9995	0.9996	0.9997	0.9999	0.9999	0.9999	0.9999	0.9997	0.9996	0.9995	0.9994	0.9994	2.2963
0.4	0.9995	0.9996	0.9998	0.9999	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9995	2.2909

0.5	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9998	0.9997	0.9997	0.9996	0.9996	2.2862
0.6	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9997	2.2820
0.7	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	2.2782
0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.2749
0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.2718

(v=9, b=15, r=6, k_1=4,k_2=3, λ=2), n=258, n_0 =258, 2n=516													
a / ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.6387
-0.8	0.9999	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.5387
-0.7	0.9999	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	2.4714
-0.6	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	2.4261
-0.5	0.9995	0.9996	0.9997	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9996	2.3944
-0.4	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9995	2.3714
-0.3	0.9994	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	2.3539
-0.2	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	0.9997	0.9996	0.9995	0.9994	0.9993	2.3403
-0.1	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3294
0	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3205
0.1	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3131
0.2	0.9994	0.9995	0.9997	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	2.3069
0.3	0.9995	0.9996	0.9997	0.9999	0.9999	0.9999	0.9999	0.9997	0.9996	0.9995	0.9995	0.9994	2.3016
0.4	0.9995	0.9996	0.9998	0.9999	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9995	2.2969
0.5	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9998	0.9997	0.9997	0.9996	0.9996	2.2929
0.6	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9997	2.2893
0.7	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	0.9998	2.2862
0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.2834
0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.2808

(v=10, b=11, r=5, k_1=5, k_2=4, λ=2), n=196, n_0 =196, 2n=392													
a / ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7515
-0.8	0.9997	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9999	0.9999	2.6472
-0.7	0.9994	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	2.5855
-0.6	0.9991	0.9993	0.9995	0.9997	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	2.5475
-0.5	0.9988	0.9991	0.9994	0.9997	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9996	2.5224
-0.4	0.9986	0.9989	0.9993	0.9996	0.9999	1	0.9999	0.9998	0.9997	0.9996	0.9995	0.9995	2.5047
-0.3	0.9984	0.9987	0.9992	0.9996	0.9999	1	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	2.4917
-0.2	0.9982	0.9986	0.9991	0.9996	0.9999	0.9999	0.9999	0.9998	0.9996	0.9995	0.9994	0.9993	2.4817
-0.1	0.9981	0.9985	0.9991	0.9996	0.9999	0.9999	0.9999	0.9998	0.9996	0.9995	0.9994	0.9993	2.4738
0	0.9981	0.9985	0.9991	0.9996	0.9999	0.9999	0.9999	0.9997	0.9996	0.9995	0.9994	0.9993	2.4675
0.1	0.9982	0.9986	0.9991	0.9996	0.9999	0.9999	0.9999	0.9997	0.9996	0.9995	0.9994	0.9993	2.4622
0.2	0.9983	0.9987	0.9992	0.9996	0.9999	0.9999	0.9999	0.9998	0.9996	0.9995	0.9994	0.9993	2.4578
0.3	0.9985	0.9988	0.9992	0.9997	0.9999	0.9999	0.9999	0.9998	0.9997	0.9995	0.9995	0.9994	2.4540
0.4	0.9987	0.9990	0.9994	0.9997	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9995	2.4508
0.5	0.9990	0.9992	0.9995	0.9998	0.9999	0.9999	0.9999	0.9998	0.9997	0.9997	0.9996	0.9996	2.4480
0.6	0.9992	0.9994	0.9996	0.9998	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	2.4455
0.7	0.9995	0.9996	0.9997	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9998	2.4433
0.8	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.4414
0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.4396

(v=12, b=16, r=6, k_1=6, k_2=5, k_3=4, k_4=3, λ=2), n=536, n_0 =536, 2n=1072													
a / ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.9829
-0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.9073
-0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.8615
-0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8318
-0.5	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8112
-0.4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7961
-0.3	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7847
-0.2	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7757

-0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7684
0	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7625
0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7575
0.2	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7533
0.3	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7497
0.4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7466
0.5	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7438
0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7414
0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7392
0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7373
0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7356

(v=13, b=16, r=6, k_1=6, k_2=5, k_3=4, k_4=3, λ=2), n=538, n_0 =538, 2n= 1076													
a / ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.9928
-0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.9078
-0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.8600
-0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8304
-0.5	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8105
-0.4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7962
-0.3	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7856
-0.2	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7773
-0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7707
0	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7653
0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7609
0.2	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7571
0.3	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7539
0.4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7511
0.5	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7487
0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7465
0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7446
0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7429
0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7414

(v=14, b=16, r=6, k_1=6, k_2=5, k_3=4, λ=2), n=540, n_0 =540, 2n= 1080													
a / ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	3.0037
-0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.9082
-0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.8584
-0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8290
-0.5	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8098
-0.4	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7964
-0.3	0.9998	0.9999	0.9998	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7864
-0.2	0.9998	0.9999	0.9998	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9998	2.7788
-0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7679
0	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7639
0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7605
0.2	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7576
0.3	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7576
0.4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7551
0.5	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7529
0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7510
0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7493
0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7478
0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7465

(v=15, b=16, r=6, k_1=6, k_2=5, λ=2), n=542, n_0 =542, 2n=1084													
a / ρ	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3	a^*
-0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	3.0158
-0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.9082
-0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.8567
-0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8276
-0.5	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.8092
-0.4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7965
-0.3	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7873
-0.2	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9998	2.7748

-0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7703
0	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7667
0.1	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7636
0.2	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	2.7610
0.3	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7587
0.4	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7568
0.5	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7551
0.6	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7536
0.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	2.7522
0.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7522
0.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	0.9999	0.9999	0.9999	0.9999	0.9999	2.7510

Table 2: Values of WSRRs $R_{D(0.95)}(\rho)$ for slope rotatability for second order response surface designs with tri-diagonal correlation error structure using PBD for $\rho(-0.9 \le \rho \le 0.9)$ and for $6 \le v \le 15$

a / (v, b ,r, k_1, k_2,..λ)	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4	4.3
(6,7,3,3,2,1)	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9
(8,15,6,4,3,2)	-0.9 to 0.9	-0.9 to 0.9	-0.9 - 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9
(9,15,6,4,3,2)	-0.9- 0.9	-0.9 to 0.9	-0.9 – 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9
(10,11,5,5,4,2)	-0.9 to 0.9	-0.9 to 0.9	-0.9 – 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9
(12,16,6,6,5,4,3,2)	-0.9 to 0.9	-0.9 to 0.9	-0.9 – 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9
(13,16,6,6,5,4,3,2)	-0.9 to 0.9	-0.9 to 0.9	-0.9 – 0.9	0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9
(14,16,6,6,5,4,2)	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9
(15,16,6,6,5,2)	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9	-0.9 to 0.9

Conclusions:

In this paper, the measure of slope rotatability for second order response surface designs with tri-diagonal correlation error structure using PBD is studied. The degree of slope rotatability of the given design can be calculated for different values of $\rho(-0.9 \le \rho \le 0.9)$ and for $6 \le v \le 15$ (v number of factors). This enables us to assess the degree of slope rotatability for second order response surface designs with tri-diagonal correlation error structures using PBD.

In this method, we obtain designs with fewer number of design points. The implications of fewer number of design points leads to effective and reduced cost of experimentation. Here, we may point out this measure of slope rotatability for second order response designs under tri-diagonal correlated structure of errors using PBD has only 136 design points for v = 6 $(v=6, b=6, r=3, k_1=3, k_2=2, \lambda=1)$ factors, whereas the corresponding measure of slope rotatability for second order response designs under tri-diagonal correlated structure of errors using CCD (v =6) and BIBD $(v=6, b=15, r=5, k=2, \lambda=1)$ obtained by Sulochana and Victorbabu (2021d, 2021e) need 88 and 142 design points respectively.

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