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Vedic Method of Differentiation of Ratios of Polynomials

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Introduction

The simplicity and elegance in working out the differentiation of ratios of polynomials is exemplified by demonstrating a few problems using the formulae similar to the UrdhvaTiryagbhyam multiplication and principle of digression. The working details are also compared with the method in vogue using the formula $\frac{vdu-udv}{v^2}$ for the digression in the form of $\frac{u}{v}$ functions of $'x'$.

Abstract

The Vedic Method involves in writing down the problem in a specific manner leading to the result and is as follows :

For example, working process of the problem $\frac{3+6x+9x^2+18x^3}{1+2x+4x^2+8x^3} = \frac{u}{v}$ is explained.

First Step		x	x^2	x^3
Numerator (u)	3	6	9	18
Denominator (v)	1	2	4	8

The numbers (1.6 – 3.2) (1-0) Powers

The Multiplication deals with differences in powers. The first step is (1.6 – 3.2)(1 – 0)

The second step is : (1.9 – 3.4)(2-0)x = -6x

Third step : (1.18 – 3.8)(3-0)x^2
Fourth step : (2.9 – 6.4)(2-1)x^2 } = $-24x^2$

5th step : (2.18 – 6.8)(3-1)x^3=$-24x^3$

6th step : (4.18 – 9.8)(3-2)x^4=0 and the final result is equal to $\frac{-6x-24x^2-24x^3}{v^2}$

Differentiate (w.r.t. x) the following :

1) $z = \frac{4x+2}{2x^2+2x}$

Western Method :

$z = \frac{4x+2}{2x^2+2x} = \frac{U}{V}$ $\quad dz = \frac{\overset{A}{vdu} - \overset{B}{udv}}{v^2}$

$z=\frac{2+4x+0x^2}{0+2x+2x^2} = \frac{u}{v}$

A=vdu ; B=udv

A→$(2x+2x^2)4 - (2+4x)(2+4x)$←B

A – B = $8x+8x^2-4-8x-8x-16x^2$

$$\frac{-4-8x-8x^2}{v^2} = \frac{-1-2x-2x2}{x^2+2x^3+x^4}$$

Vedic Method :

A - B

$Z = \frac{2+4x+0x^2}{0+2x+2x^2} = \frac{u}{v}$ $dz = \frac{vdu-udv}{v^2}$

$(0.4 - 2.2)(1\text{-}0) + (0.0 - 2.2)(2 - 0)x + (2.0 - 4.2)(2 - 1)x^2$

$$\frac{dz}{dx} = \frac{-1 - 2x - 2x^2}{x^2 + 2x^3 + x^4}$$

2) $z = \frac{3+6x+9x^2+18x^3}{1+2x+4x^2+8x^3} = \frac{u}{v}$

Western Method :

$$\frac{dz}{dx} = \frac{vdu - udv}{v^2}$$

$$\frac{dz}{dx} = \frac{(1 + 2x + 4x^2 + 8x^3)(6 + 18x + 54x^2) - (3 + 6x + 9x^2 + 18x^3)(2 + 8x + 24x^2)}{v^2}$$

On simplifying

A→vdu : $(1 + 2x + 4x^2 + 8x^3)(6 + 18x + 54x^2)$

6	18x	$54x^2$			
	12x	$36x^2$	$108x^3$		
		$24x^2$	$72x^3$	$216x^4$	
			$48x^3$	$144x^4$	$432x^5$
6	30x	$114x^2$	$228x^3$	$360x^4$	$432x^5$

-B→ -udv : $-(3+6x+9x^2+18x^3)(2+8x+24x^2)$

-6	-24x	$-72x^2$			
	-12x	$-48x^2$	$-144x^3$		
		$-18x^2$	$-72x^3$	$-216x^4$	
			$-36x^3$	$-144x^4$	$-432x^5$
-6	-36x	$-138x^2$	$-252x^3$	$-360x^4$	$-432x^5$

$(A - B) = \frac{-6x-24x^2-24x^3}{v^2}$

Vedic Method :

$(1.6 - 3.2)(1\text{-}0) + (1.9 - 3.4)(2\text{-}0)x + [(1.18 - 3.8)(3 - 0)x^2 + (2.9 - 6.4)(2 - 1)x2] + (2.18 - 6.8)(3\text{-}1)x^3 + (4.18 - 9.8)(3 - 2)x^3 + (4.18 - 9.8)(3\text{-}2)x^4$

On simplifying ; the result $dz = \frac{-6x-24x^2-24x^3}{v^2}$

A - B

3) $z = \frac{2+7x+3x^2}{4+9x+2x^2} = \frac{u}{v}$ $\quad dz = \frac{vdu-udv}{v^2}$

Western Method :

(4+9x+2x2)(7+6x)-(2+7x+3x2)(9+4x)

A→	28	24x		
		63x	$54x^2$	
			$14x^2$	$12x^3$
	28	87x	$68x^2$	$12x^3$
-B→	-18	-8x		
		-63x	$-28x^2$	
			$-27x^2$	$-12x^3$
	-18	-71x	$-55x^2$	$-12x^3$

$(A – B) = 10+ 16x + 13x^2$

Vedic Method :

$(4.7 – 2.9)(1-0)+(4.3 – 2.2)(2-0)x + (9.3 – 7.2)(2-1)x^2$

On simplifying : $dz = \frac{10+16x+13x2}{v^2}$

$$4)\ \ z = \frac{1+4x+9x2+3x3}{2+3x+4x2+9x3} = \frac{u}{v} \qquad dz = \frac{\overset{A\ -\ B}{vdu-udv}}{v^2}$$

Western Method :

$(2+3x+4x^2+9x^3)(4+18x+9x^2) – (1+4x+9x^2+3x^3)(3+8x+27x^2)$

A→	8	36x	$18x^2$			
		12x	$54x^2$	$27x^3$		
			$16x^2$	$72x^3$	$36x^4$	
				$36x^3$	$162x^4$	$81x^5$
	8	48x	$88x^2$	$135x^3$	$198x^4$	$81x^5$

B→	3	8x	$27x^2$			
		12x	$32x^2$	$108x^3$		
			$27x^2$	$72x^3$	$243x^4$	
				$9x^3$	$24x^4$	$81x^5$
	3	20x	$86x^2$	$189x^3$	$270x^4$	$81x^5$

$(A-B)= 5 + 28x + 2x^2 - 54x^3 - 69x^4$

$dz = \frac{5+28x+2x^2-54x^3-69x^4}{v^2}$

Vedic Method :

$(2.4 – 1.3) + (2.9 – 1.4)(2-0)x + [(2.3 – 1.9)(3 – 0)x^2 –(3.9 – 4.4)(2 – 1)x^2] + (3.3 – 4.9)(3 – 1)x^3 + (4.3 – 9.9)(3 – 2)x^4 = 5 + 28x + 2x^2 - 54x^3 - 69x^4$

$dz = \frac{5+28x+2x^2-54x^3-69x^4}{v^2}$

$$5)\ \ z = \frac{2+3x+4x^2+x^3}{2+4x+9x^2+2x^3} = \frac{u}{v} \qquad dz = \frac{\overset{A\ -\ B}{vdu-udv}}{v^2}$$

Western Method :

$(2+4x+9x^2+2x^3)(3+8x+3x^2)-(2+3x+4x^2+x^3)(4+18x+6x^2)$

A→	6	16x	$6x^2$			
		12x	$32x^2$	$12x^3$		
			$27x^2$	$6x^3$	$16x^4$	
				$72x^3$	$27x^4$	$6x^5$
	6	28x	$65x^2$	$90x^3$	$43x^4$	$6x^5$

B→	8	36x	$12x^2$			
		12x	$54x^2$	$18x^3$		
			$16x^2$	$72x^3$	$24x^4$	
				$4x^3$	$18x^4$	$6x^5$
	8	48x	$82x^2$	$94x^3$	$42x^4$	$6x^5$

$(A-B) = -2-20x-17x^2-4x^3+x^4$

Vedic Method :

(2.3 – 2.4)(1 – 0) + (2.4 – 2.9)(2 – 0)x [(2.1 – 2.2)(3-0)x^2+(4.4 – 3.9)(2-1)x^2] + (4.1 – 3.2)(3 – 1)x^3 + (9.1 – 4.2)(3 – 2)x^4

$$dz = \frac{-2 - 20x - 17x^2 - 4x^3 + x^4}{v^2}$$

Conclusions

The " . " between two numbers indicates multiplication. The method suggested from Vedic principles has an advantage in preparing the answer in quick succession using digression principle and the method similar to 'UrdhwaTiryagbhyam' with the Vertical multiplication, being absent and finally the simplification can be carried out with extreme simplicity thus saving time and introducing elegance in working steps. Author has given the (Western) existing method (method in vogue) by the ordinary multiplication followed by simplification, which requires more time. As such, the author has worked out the problem by both the methods. The problem deals with differentiation of the ratio of polynomials valid for any number of terms and of any order. The author opines such simple methods need to be introduced in the text books – as an alternative method and of Indian Origin.

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