“Solutions of Polynomial Equation of 5th degree using Vedic Method”

Prof. C. Santhamma, B.Sc. (Hons), M.Sc., D.Sc., FAPASc.

Emeritus Professor, Centurion University of Technology and Management AP, Vizianagaram

Mob : +91 7799384448, email_id: santhammachilukuri2018@gmail.com / csanthamma@cutmap.ac.in

Introduction

Under a series of investigations by the author on evaluation of different roots of numbers, polynomials and solutions of Polynomial Equations (one variable) by using Vedic Sutrams and applying Straight Division Method and Purana Apuranabhyam principle, this paper is also towards a part-of determination of solutions of a 5^{th} order polynomial equation. The principle of working is similar to the details shown in the working of 3^{rd} and 4^{th} degree polynomial equations. In this paper, the purana apuranabhyam principle is included along with the Straight Division method. The working details are clearly presented and finally, the five solutions of the equation are evaluated by the method(s) envisaged by Jagadguru Shankaracharya, Shri Bharati Krishna Tirtha Swamiji.

Abstract

Swamiji’s Straight Division method is adopted. The 5^{th} order polynomial equation is $x^5+4x^4-2x^3+10x^2-2x-962=0$. Started for a location of a single and most probable solution through the method of Vilokanam, exemplifying by giving different values for “x” insearch of the first solution. It has to lead to solving four other solutions of the given polynomial equation. Hence, one has to follow the necessary tools such as

(1) The General Terms, in the expansion of $(a+b+c+d+e+\ldots)^5$, where “a” as the integer value of the solution and b,c,d,e.. , the successive decimals, the terms as the deductions, are to be sorted out. Following the respective expressions of such deductions, the values, which are used from the derivative principle, as obtained from the representations of the polynomial equation, at the probable solutions are worked out. After the respective deductions from the Dividends, the results are to be divided by the Common Divisor (CD) of the problem to yield the decimal values together with the remainders, which are to be considered with the existing dividends in succession, thus forming the new dividends. The process is continued till all the considered decimals are evaluated. Thus by obtaining the first solution of the given polynomial, the given equation is converted to a 4^{th} degree polynomial with the exit of the first solution.

(2) The evaluation of the Common Divisor is through the first derivative of the equation at x=a, the first selected value by Vilokanam method (from RHS – LHS values at different “x” values). As the value did not result in fitting the solution, finally a different variable as $x=\frac{z}{100}$ is considered and applying the Straight Division method, the value for x, 3.3548488 is acceptable.

The resulting 4^{th} degree equation after the exit of the 1^{st} solution, is subjected to Purana Apuranabhyam principle a using relation between “x” and “y”. This is expanded in terms of two quadratics, $(y^2+by+c_1)(y^2-by+c_2)$, which are converted to a 3^{rd} degree equation in “p” also subjected to Straight Division method. This finally, lead to solving all the 4 solutions in relation to “y” and “x”.

Finally all the five solutions could be solved. Out of the 5 solutions of the equation, one is real and the other 4 are complex.

Let us consider the 5th order Polynomial Equation $x^5+4x^4-2x^3+10x^2-2x-962=0$

This work deals with the determination of all the five solutions of the 5th order polynomial equation $E=F(x)= x^5+4x^4-2x^3+10x^2-2x=962$

One has to arrive at atleast one most probable solution, which will lead to the other four. At first, let us consider the equation to have RHS and LHS divisions.

Such as LHS = $x^5+4x^4-2x^3+10x^2-2x$ and the RHS as 962.

The difference between RHS and LHS will enable to locate the approximate position of one value of the solution as "a", Table – 1 gives a few such differences worked out of the given values for x.

Table – 1

Value of x	**L.H.S.** F(x)	**R.H.S.**	**R.H. S – L.H.S.**
1	11	962	951
2	116	"	846
3	597	"	365
4	2072	"	– 1110
5	5615	"	– 4653
– 1	17	"	+ 945
– 2	92	"	+ 870
– 3	231	"	+ 731
– 4	296	"	+ 666
– 5	– 115	"	+ 1077
– 6	– 1788	"	2750
– 7	– 6013	"	+ 6975

The solution is of the form x=a.bcdef, Out of the values, from the Table – 1 one can pick out a value between x=3 to 4.

Let us consider that as x=a=3. The work is carried out digit by digit by using Swamiji's Straight Division method, which requires a Common Divisor, which is the first derivative at x=3 of the given polynomial. The work, is carried out digit by digit with the proper deduction of the corresponding terms, the results being from the general expansion of $(a+b+c+d+e+f)^5$, which can give rise to the decimals as the quotients when the new dividends are obtained after the corresponding deductions of the expansion terms, followed by the division by the Common Divisor (CD). So, one has to consider the general expansion of $(a+b+c+d+e+f+g)^5$ with the terms being sorted out for the deductions at the requisite positions of the decimals, the result of which are divided by the CD to arrive at the decimals and the remainders are carried out to form the next dividends. Thus the first solution is arrived at. It has to be tested. If it fits well, the solution can be continued. Otherwise a different relation between an unknown "z" and its relation to "x" is to be considered until one fitting solution is obtained (as $x=\frac{z}{10}, \frac{z}{100}, etc)$

The Straight Division is carried out with the 1st dividend as the RHS-LHS value and "a" is equal to its corresponding variable x=3, the integer value of the solution as x=a.bcdef… Table – 2 shows the details of the results obtained by the application of the straight division method. The starting dividend is 365-0 and the Common divisor is the first derivative of the polynomial equation at x=3.

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Table – 2

	0	0	0	0	0	
CD= 841	365	286	$\overline{583}$	381	516	0 $\overline{59}$
		$\overline{7648}$	$\overline{8704}+19120$ $=10416$	$\overline{4864}+\overline{11950}+$ $\overline{19120}+32640=$ $\overline{3294}$	$\overline{1024}+0+23900$ $+24320+\overline{32640}+$ $\overline{40800}=\overline{26244}$	
		$10a^3b^2$	$10a^2.b^3+10a^3.2bc$	$5a.b^4+10a^3c^2$ $+10a^3.2bd+10a^2.3b^2c$	$b^5+10a^3.2be$ $+10a^3.2cd+5a.4b^3c$ $+10a^2.3b^2d+10a^2.3bc^2$	
3	4	$\overline{5}$	5	0	$\overline{25}$	
a	b	c	d	e	f	

One has to sort out the terms, from the expansion $(a+b+c+d+e+f)^5$ (Refer Table – 2) which need to be deducted from the respective positions of the dividends.

The expressions corresponding to the contributions of different decimals are

For b : $5a^4b$
For c : $5a^4c+10a^3b^2$
For d : $5a^4d+10a^2b^3+20a^3bc$
Etc…

These can be seen in Table - 6

With a=3, RHS-LHS (1st dividend) is 365-0. The following details are the Derivatives of the Polynomial used as representatives in the evaluation.

F'(x), first derivative of the given equation : $5a^4$ Representation at x = 3 $\Rightarrow$ $[5x^4 + 16x^3 - 6x^2 + 20x - 2]$ at x = 3 $\Rightarrow$ 841 (CD)
F"(x), second derivative of the given equation : $10a^3$ Representation at x = 3
$\Rightarrow \frac{1}{2}[20x^3 + 48x^2 - 12x + 20]$ at x = 3 $\Rightarrow$ 478
F'"(x), third derivative of the given equation : $10a^2$ Representation at x = 3
$\Rightarrow \frac{1}{6}[60x^2 + 96x - 12]$ at x = 3 $\Rightarrow$ 136
$F^{iv}(x)$, fourth derivative of the given equation : 5a Representation at x = 3
$\Rightarrow \frac{1}{24}[120x + 96]$ at x = 3 $\Rightarrow$ 19
$F^{v}(x)=\frac{1}{120}(120)=1$

These values of the representations are used in the expansion terms together with the individual values of decimals, as the case may be.

The first decimal is to be obtained as the quotient, when the first dividend is divided by the CD 841. The remainder is 286-0. One has to deduce $10a^3b^2$ from the dividend (2860-7648) = -5 with and -583 as remainder. The result is divided by 841. For example, $20a^3bc$ = $10a^3.2bc$. ($10a^3$=478 2bc=40)=19120

The deduction terms from the expansion $(a+b+c+d+e+f)^5$ to be subtracted from the dividends and after undergoing division by the CD, the resulting quotients as the decimal together with the remainders are shown clearly in the Table – 2.

b=4, c=-5, d=5, e=0, f=-25, giving the value as 3.35475.

The first solution is 3.35475, E=-0.121940667 as such, it needed refinement.

(When the value of x is substituted in the polynomial equation, E=0.121940667. instead of zero)

$\therefore$ one has to refine the result by considering a variable such as x= $\frac{z}{10}, \frac{z}{100}, etc.$

Hence x=$\frac{z}{100}$ is considered and the entire equation is written in "z" as

$$\frac{z^5}{10^{10}} + \frac{4z^4}{10^8} - \frac{2z^3}{10^6} + \frac{10z^2}{10^4} - \frac{2z}{10^2} = 962$$

E = $z^5+400z^4-20000z^3+10000000z^2-200000000z=962\times10^{10}$

F'(z), F"(z), F'"(z), $F^{iv}(z)$, $F^{v}(z)$ derivatives are at x=336. To deterimine the 1st probable solution in z, the RHS-LHS is worked out for the different values of "z", RHS=962×10^{10}

It is noticed that the possible value is between 335 and 336 by vilokanam. Z is finally considered as 336

With the value of z=336, the various representations (derivatives) of the equation,$5a^4$, $10a^3$, $10a^2$, 5a and 1.
CD= $F^{/}(z)$ = 12416666370; $\frac{1}{2}F^{//}(z)$ = 640120960, $\frac{1}{6}F^{///}(z)$ = 164560, $\frac{1}{24}F^{iv}(z)$ = 2080, $\frac{1}{120}$ $F^{v}(z)$ = 1

With these being incorporated as the corresponding deductions along with new dividends, the decimals are shown in the Table – 3

Table – 3

CD=$5a^4$ representation =124166663700	0 63791897000	0 17085651500	0 62692875300	0 12290824100	0 31265554260
$10a^3$ representation = 640120960		$\overline{16003024000}$ (b^2)	$\overline{6401209600}$ (2bc)	$\overline{32646168960}$ ($2bd + c^2$)	$\overline{12802419200}$ (2be+2cd)
$10a^2$ representention = 1646560			205850000 (b^3)	123492000 ($3b^2c$)	642158400 ($3b^2d+3bc^2$)
5a representation =2080				$\overline{1300000}$ (b^4)	$\overline{1040000}$ ($4b^3c$)
Representation of 1					3125 (b^5)
336 a.	$\bar{5}$ b	$\bar{1}$ c	$\bar{5}$ d	$\bar{1}$ e	$\bar{2}$ f

The value of z = 336. $\bar{5}\,\bar{1}\,\bar{5}\,\bar{1}\ \bar{2}$ = 335.48488; X=$\frac{z}{100} = \frac{335.48488}{100}$ = 3.3548488, E=0.00007950

With this value of x, the remaining 4 values are worked out, as follows :

E=(x-3.3548488)A

A should have x^4, x^3, x^2, x and constant terms.

(Applying Adyamadyena, Antyamantyena)

$E=F(x) = x^5+4x^4-2x^3+10x^2-2x-962 = (x-3.3548488)(x^4+\alpha x^3+\beta x^2+\gamma x+286.7491345)$ and

Comparing coefficients of like terms,

x coeff: $286.7491375 - 3.3548488\,\gamma = -2$

$\gamma = 86.06919558$

x^2 coeff: $86.06919558 - 3.3548488\beta = 10$

$\beta = 22.67440356$

x^3 coeff: $22.67440356 - 3.3548488\alpha = -2$

$\alpha = 7.354848171$

$\therefore\ E = (x - 3.3548488)\ (x^4 + 7.354848171x^3 + 22.67440356x^2 + 86.06919558x + 286.7491375)$

The solutions of the equation $E_1 = x^4 + 7.354848171x^3 + 22.67440356x^2 + 86.06919558x + 286.7491375=0$ are to be determined

At this stage Purana, Apuranabhyam is applied

$E_1 = x^4 + 7.354848171x^3 = - 22.67440356x^2 - 86.06919558x - 286.7491375$

$(x + \frac{7.35484817}{4})^4 = (x + 1.838712043)^4$

$= x^4 + 4(1.838712043)x^3 + 6(1.838712043)^2x^2 + 4(1.838712043)^3x + (1.838712043)^4$

$= x^4 + 7.354848172x^3 + 20.28517186x^2 + 24.86572653x + 11.43022771$

Substitutes the values of first two terms from E_1

$(x + 1.838712043)^4 = - 22.67440356x^2 - 86.06919558x - 286.7491375 + 20.28517186x^2 + 24.86572653x + 11.43022771$

$(x + 1.838712043)^4 = - 2.389231710x^2 - 61.20346905x - 275.3189098$

Let $(x + 1.838712043) = y \Rightarrow x = y - 1.838712043$

$y^4 = - 2.38923170\ (y - 1.8387120430)^2 - 61.20346906\ (y - 1.838712043) - 275.3189098$

$= - 2.38923170y^2 - 52.41725082y - 170.8610168$

$E_2 = y^4 + 2.38923170y^2 + 52.41725082y + 170.8610168 = 0$

$= (y^2 +by + c_1)\ (y^2 - by + c_2)$

$= y^4 + y^2\ (c_2 + c_1 - b^2) + yb(c_2 - c_1) + c_1\ c_2 = 0$

$c_2 + c_1 - b^2 = 2.38923170$

$c_2 + c_1 = 2.38923170 + b^2$ ———— (A)

$c_2 - c_1 = \dfrac{52.41725082}{b}$ ———— (B)

$(c_2 + c_1)^2 - (c_2 - c_1)^2 = 4c_1\ c_2$

$(b^2 + 2.38923170)^2 - \left(\dfrac{52.41725082}{b}\right)^2 = 4(170.8610168)$

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Let $b^2 = p$

$$(p + 2.38923170)^2 - \frac{(52.41725082)^2}{p} = 4(170.8610168)$$

$$(p^2 + 5.708428116 + 4.77846342p) - \frac{2747.568184}{p} = 683.4440672$$

$E_2 = p^3 + 4.77846342p^2 - 677.735639p = 2747.568184$ is subjected to Straight Division

$F(p) = p^3 + 4.77846342p^2 - 677.735639p = 2747.568184$

Table – 4

z value	f(p)	RHS	RHS – LHS
– 1	681.5141024	2747.568184	2066.054082
– 2	1366.585132		1380.983052
– 3	2049.213088		698.355096
– 4	2723.397971		24.170213
– 5	3383.139781		– 635.571597

$F^{/}(z) = 3p^2 + 9.55692684p - 677.735639 \Rightarrow$ when $p = -4$, $F^{/}(-4) = -667.9633464$ (CD)

$F^{//}(p) = 6p + 9.55692684 \Rightarrow \frac{1}{2}F^{//}(-4) = -7.22153658$

$F^{///}(p) = 6 \Rightarrow \frac{1}{6}F^{///}(3) = 1$

Table – 5

CD=$3a^2$ representation= 667.9633464	241.70213	2417.0213	4131.312608	1235.325296	6323.55778	5718.62973	372.641484
3a representation = 7.22153658		0 (b^2)	0 (2bc)	64.99382922 ($2bd + c^2$)	259.9753169 (2be + 2cd)	303.304536 ($2bf+2ce+d^2$)	476.6214143 (2bg+2cf+2de)
Representation of 1			0 (b^3)	0 ($3b^2c$)	0 ($3b^2d+3bc^2$)	27 ($3b^2e+c^3+6bcd$)	162 ($3b^2f+3bd^2+3c^2d+6bce$)
– 4 a	0 b	$\bar{3}$ c	$\bar{6}$ d	$\bar{1}$ e	$\bar{9}$ f	$\bar{9}$ g	$\bar{1}$ i

$p = -4.0361991$; $E = -0.00005166$

$\therefore$ $(p + 4.0361991)$ is a factor of E_2

$\therefore$ $E_2 = (p + 4.0361991)A$. A is quadratic, should have p^2, p and constant terms

By applying Adyamadyena Antyamantyena

$E_2 = (p + 4.0361991)(p^2 + \alpha p - 680.7315784)$ comparing p coefficient

$-680.7315784 + 4.0361991\alpha = -677.735639$

$\alpha = 0.742267496$

$\therefore\ E_2 = (p + 4.0361991)(p^2 + 0.742267496p - 680.7315784)$

The Quadratic is factorized using differential relation

$2p + 0.742267496 = \pm\sqrt{0.550961035 + 2722.926314} = \pm 52.18694544$

$p = -0.371133748 \pm 26.09347272$

$= 25.72233897, -26.4640647$

$p = b^2 = 25.72233897$

$b = \pm 5.071719528$

from (A) $c_2 + c_1 = 2.3892317 + 25.72233897 = 28.11157068$

from (B) $c_2 - c_1 = \dfrac{52.41725082}{5.071719528} = 10.33520299$

$c_2 = 19.22338683$

$c_1 = 8.888183846$

$\therefore\ E = (y^2 + 5.071719528y + 8.888183846)(y^2 - 5.071719528y + 19.22338683)$

$(y^2 + 5.0717195284y + 8.888183846)$ is factorized by using differential relation

$2y + 5.071719528 = \pm\sqrt{25.72233897 - 35.55273538} = \pm 3.135346299i$

$y_1, y_2 = -2.535859764 \pm 1.567673149\ i$

$(y^2 - 5.071719528y + 19.22338683)$ is factorized using differential relation

$2y - 5.071719528 = \pm\sqrt{25.72233897 - 76.89354136} = \pm 7.153405367i$

$y_3, y_4 = 2.535859764 \pm 3.576702684i$

but $x = y - 1.838712043$

$x_1 = -4.374571807 + 1.567673149i$

$x_2 = -4.374571807 - 1.567673149i$

$x_3 = 0.697147721 + 3.576702684i$

$x_4 = 0.697147721 - 3.576702684i$

After substituting all the required values for b, c_1 and c_2, the product of the two quadratic equations in “y” are solved for the remaining values in “y” and consequently in “x”

$E_1 = (x - 3.3548488)(x + 4.374571807 - 1.567673149i)(x + 4.374571807 + 1.567673149i)(x - 0.697147721 + 3.576702684i)(x - 0.697147721 - 3.576702684i)$

Applying Gunita Samuccaya Sutram

$S_c = -951 = (-2.3548488)(5.374571807 - 1.567673149i)(5.374571807 + 1.567673149i)(0.302852279 + 3.576702684i)(0.302852279 - 3.576702684i) = -950.9999521 \sim -951$

This 5^{th} degree equation has one real and four imaginary solutions.

Table – 6
Expansion Table

$(a+b+c+d+e+f+g)^5$

Coeff:	1	5	10	20	30	60
10^0	a^5					
10^1		a^4b				
10^2		a^4c	a^3b^2			
10^3		a^4d	a^2b^3	a^3bc		
10^4		a^4e, ab^4	a^3c^2	a^3bd	a^2b^2c	
10^5	b^5	a^4f		a^3be, b^3ca, a^3cd	a^2b^2d, a^2c^2b	
10^6		a^4g, b^4c	a^3d^2, c^3a^2	a^3bf, a^3ce, ab^3d	a^2b^2e, b^2c^2a	$bcda^2$
10^7		a^4h, b^4d	b^3c^2	a^3bg, c^3ab, a^3cf, a^3de, ab^3e	a^2b^2f, a^2d^2b, a^2c^2d,	$bcea^2$, ab^2cd
10^8		a^4i, b^4e, ac^4	a^3e^2, c^3b^2	a^3bh, b^3cd, a^3cg, a^3df, ab^3f	a^2b^2g, a^2c^2e, a^2d^2c, ab^2d^2	a^2bcf, a^2bde, ab^2ce abc^2d

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