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Mathematical Analysis of Unsteady Magneto Pulsatile Blood Flow Through Porous Medium Between Two Concentric Cylinders Using Giesekus Model

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Abstract

In this paper, the problem of magnetohydrodynamic (MHD) human blood flow between two concentric cylinders filled with porous medium is described using Giesekus model. A uniform circular magnetic field is applied normal to the blood flow which is driven by an unsteady pulsating pressure gradient. The momentum equation and the Giesekus equation of state that characterize the fluid are solved considering the modified Darcy's law to account the resistance offered by the porous medium. Exact solutions for the axial velocity and volume flow rate are presented. The effects of various parameters on the blood pulsatile flow characteristics such as geometry, viscoelasticity, magnetic and permeability parameters are presented through graphical illustrations. The main results of the present study is that, the blood velocity distribution increases with an increase of permeability parameter of the porous medium, while it decreases as the magnetic and frequency parameters are increase. Also it is found that the effect of the magnetic field is to decrease the blood flow rate. The decrease in the flow rate is due to an increase in the apparent viscosity of blood due to the magnetic field. The results are presented graphically and the effects of the various parameters on the human blood flow rate are discussed and compared with reported literature values.

Keywords: *MHD blood flow, Giesekus model, Flow rate, Concentric cylinders, Pulsating flow, Porous medium.*

1. Introduction

A substantial number of studies have evaluated the relationship between blood viscosity and cardiovascular disease. The risk of a major cardiovascular event (death, acute myocardial infarction etc.) increases with elevated blood viscosity. Elevated blood viscosity has been found in a number of conditions associated with increase risk of cardiovascular disease, such as hypertension [1], smoking [2] and diabetes [3]. Factors influencing blood viscosity are the concentration of red blood cells (RBCs), Plasma viscosity, the flexibility of RBCs, the tendency of RBCs to form larger groups, and the concentration of white blood cells [4-6]. To understand the complete flow properties of blood, it is necessary to determine the viscosity under a range of shear rates. Recently new laser-diffraction slit rheometer was introduced to measure the RBCs deformability Shin et al. [7]. He achieved experimentally that the whole blood viscosity is strongly dependent on the RBCs deformability. It has been well established that blood is both a viscoelastic and a shear-thinning liquid [8]. Therefore, many mathematical models, for describing rheological behaviour of blood have been extensively developed, [9-13].

Viscoelasticity is a property of human blood that is due to elastic energy that is stored in the deformation of RBCs as the heart pumps the blood through the body. The energy transferred to the blood by the heart is partially stored in the elastic structure, another part is dissipated by viscosity, and the remaining energy is stored in the kinetic motion of the blood. When the pulsation of heart is taken into consideration, an elastic regime becomes clearly evident. The RBCs occupy about half of the volume of blood and possess elastic properties. Therefore, the elastic feature is the largest contributing factor to the viscoelastic behaviour of blood. Due to the limited space between RBCs, it is clear that, significant cell to cell interaction will play an important role. This interaction and tendency for cells to aggregate is a major contributor to the viscoelastic behaviour of blood. Red blood cell deformation and aggregation is also coupled with flow induced charges in the arrangement and orientation as the third major factor in its viscoelastic behaviour as shown by Thurston and Henderson [14] and Thurston [15]. An overview of the developments in modelling of blood rheology and clot formation is given by Anand and Rajagopal [16].

In addition to a complicated structure of blood as a viscoelastic fluid, the vascular system is characterized by complicated geometry. Thus, whether the blood is arterial or venous can be recognized by physical characteristics. The blood incomes from the heart under a large pressure, whereas, the pressure difference is insignificant and the process of returning the blood to the heart is subject to another principle. Because of such a variety structures of the cardiovascular system, we need an adequate description of the rheological viscoelastic characteristics of blood. On the other hand, the resulting problem should be simple in mathematical handling. Therefore, in this paper we focus on the selection of an adequate model that, with an acceptable accuracy, govern an interaction between a pulsating blood flow and its viscoelasticity nature. In the present study, Giesekus model is used as the constitutive equation for dealing with a viscoelastic blood possessing shear-thinning characteristics. This model is defined for concentrated and dilute viscoelastic solutions. Giesekus model predicts a shear-thinning viscosity and non-vanishing first and second normal stress differences in viscometric flows [17-19]. It gives material functions that are much more realistic than any other model.

The movement of blood in an externally applied magnetic field is governed by the laws of MHD. When the body is subjected to a magnetic field the charged particles of the blood flowing transversally to the field get deflected by the Lorentz force, thus including electrical currents and voltages across the vessel walls and in the surrounding tissues strong enough to be detected at the surface of the thorax in the electrocardiogram. Therefore, the interactions between these induced currents and applied magnetic field can cause a reduction of blood flow rate [20]. Magnetic field interactions with blood flow have been demonstrated by many authors. Shit and Magee [21] investigated the unsteady flow of blood and heat transfer characteristics in the neighborhood of an overlapping constricted artery in the presence of magnetic field and whole body vibration. The unsteady flow mechanism is subjected to a pulsatile pressure gradient arising from systematic functioning of the heart and from the periodic body acceleration.

In biophysics, blood flow is considered to be a two-phase flow where cells form a suspension in the blood plasma [22]. In larger vessels the nature of blood is almost homogenous and we may readily compute the MHD effects using parabolic velocity distribution [23]. But for flow in narrow vessels, the blood cells tend to migrate along the centre of the vessel which give rise to a core region in which the concentration of blood cells is high, and a slower moving peripheral plasma region which is almost cell free [24, 25]. So the flow of blood in a narrow vessel may be explained by a two-layered model [26]. A good number of models have been developed regarding this phenomenon [27, 28]. Paries et al. derived empirical relationships of the relative apparent viscosity and mean tube hematocrit from in vitro [29, 30] and in vivo [31]. Nair et al. [32] used a two-phase model for the blood in modeling transport of oxygen in the arterioles. Including some recent studies, a number of investigations have been conducted in the literature using particulate suspension theory to describe the flow of blood in small vessels. Srivastava and Srivastava [33] proposed a two-phase model to address pulsatile blood flow in the entrance region of an artery. Srivastava et al. [34] applied the theory to study the effects of an external body acceleration on blood flow through small diameter tubes.

Initial theoretical works had evaluated blood during steady flow and later, using oscillating flow as shown by Womersley [35]. The early studies used the properties found in steady flow to derive properties for unsteady flow situations as given by Thurston [36, 37]. Advancements in medical procedures and devices required a better understanding of the mechanical properties of blood. With the development of cardiovascular prosthetic devices such as heart valves and blood pumps, the understanding of pulsating blood flow in complex geometries is required. A few specific examples are the effects of viscoelasticity of blood and its implications for the testing of a pulsatile blood pumps. In reference [38], Pontrelli analyzed the flow of an impulsive pressure gradient. The used model is the Oldroyd-B fluid modified with the use of a nonconstant viscosity, which includes the effect of the shear-thinning of many fluids. Labbio et al. [39] studied numerically the simulation process of flows in a circular pipe transversely subjected to a localized impulsive body force with applications to blunt traumatic aortic rupture. The pulsatile flow in a straight circular pipe is subjected to a transverse body force on a localized volume of fluid. Oscillating flow of a simple fluid in a circular pipe under a pulsating pressure gradient was investigated by Siginer [40] and Tanga and Kalyon [41]. Periodic pressure gradient for second-order fluid has been studied by Hayat et al. [42]. An excellent computational study of pulsatile flow dwelling on nonlinear flow aspects was presented by Hung [43]. Rahaman and Ramkissoon [44] have studied the unsteady axial viscoelastic pipe flows under the influence of periodic pressure gradient. Sharp [45] studied the effect of blood viscoelasticity on pulsatile flow in stationary and axially moving tubes. An analytical solution for pulsatile flow of a generalized Maxwell fluid in straight rigid tubes has been used to calculate the effect of blood viscoelasticity on velocity profiles and shear stress in flows representative of those in the large arteries.

The porous medium is a solid body that contains pores. It has very fine opening through which fluid flows. Porous medium is more related to Darcy's law, it is depicted that proportionality between the flow velocity and the pressure difference for low speed in an unbounded porous medium. It is characterized by its porosity and its permeability which is a measure of the flow conductivity in the porous medium. Ebaid et al. [46] have studied the peristaltic transport in an asymmetric channel through a porous medium. Jyothi et al. [47]` have considered the pulsatile flow of a Jeffrey fluid in a circular tube lined internally with porous material. Mortimer and Eyring [48], Mustapha et al. [49] and Mekheimer [50] are investigated the MHD effects on blood flow takes place through a porous medium. A porous structure is formed by fatty substances, blood clots, and cholesterol. The MHD peristaltic flow of the fractional Jeffrey fluid through porous medium in a nonuniform channel is presented by Xiaoyi et al. [51]. The fractional calculus is considered in Darcy's law and the constitutive relationship which included the relaxation and retardation behavior. The effects of fractional viscoelastic parameters of the generalized Jeffrey fluid on the peristaltic flow and the influence of magnetic field, porous medium, and geometric parameter of the nonuniform channel are presented through graphical illustration. Sharma et al. [52] studied a mathematical model for the hydro-magnetic non-Newtonian blood flow in the non-Darcy porous medium with a heat source and Joule effect is proposed. A uniform magnetic field acts perpendicular to the porous surface. The governing non-linear partial differential equations have been solved numerically by applying the explicit finite difference Method.

From this survey it is clear that, this problem being more significant for applications in biotechnology. A pulsating blood flow in the presence of magnetic field has been of great interest over recent years. Many researchers have examined the effects of magnetic fields on velocity profiles and major studies have focused on the effects of magnetic field on the volumetric rate of flow. Up to date, no one has obtained the analytical form of the magnetic field and the medium porosity effects on thehuman blood flow. Therefore, the purpose of the present study is to analytically explain the unsteady MHD flow of human blood as viscoelastic fluid between two concentric cylinders filled with porous medium. The fluid is driven by pulsating pressure gradient. The solutions of the governing equations have been obtained in an exact manner. The velocity and flow rate are determined. The effects of the magnetic field parameter, the permeability parameter and frequency parameter on the velocity field and on the flow rate are obtained analytically.

2. Formulation of the Problem

Consider the unsteady flow of human blood as an incompressible electrically conducting fluid in the gap between two concentric cylinders of radii R_1 and R_2 ($R_2 > R_1$) filled with porous medium, see Fig. (1). The blood flow through the porous gap is started due to pulsating pressure gradient and can be described by means of the velocity vector $\underline{V}$, pressure $p(z,t)$, and tangential stress tensor $\underline{\underline{S}}$. In large and medium sized vessels human blood can be modelled as an incompressible viscoelastic fluid. In porous medium, the momentum and continuity equations for unsteady MHD blood flow are:

$$\rho\left(\frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V}\right) = -\nabla p + \nabla\cdot\underline{\underline{S}} + \underline{f}, \qquad (1)$$

$$\nabla\cdot\underline{V} = 0. \qquad (2)$$

where ρ is the density and $\underline{f}$ is the body force per unit volume. The MHD blood flow through a porous medium requires that an additional force be included in the equations of fluid motion aside from the usual pressure and shear forces. The added force takes the form:

$$\underline{f} = \underline{f}_m + \underline{\Re}, \qquad (3)$$

where $\underline{f}_m$ is the magnetic body force due to magnetic field, and $\underline{\Re}$ is Darcy's resistance in porous medium.

A cylindrical polar coordinate system (r, θ, z) is used with z-axis parallel to the flow direction and there is no flow in the azimuthal direction. By assuming that the flow is unidirectional and axisymmetric ($\partial/\partial\theta = 0$) with no swirl in a cylindrical rigid gap, the blood dynamical variables do depend on r and t only except for the pressure which depends on z and t. Therefore, the velocity field can be written as:

$$\underline{V} = [0,\ 0,\ W(r,t)], \qquad (4)$$

which satisfies the equation of continuity, Eq. (2), identically. In such hypothesis the convection term, ($\underline{V}\cdot\nabla\underline{V}$), in Eq. (1) vanishes.

2.1. Magnetic Force

In the presence of magnetic field, the Lorentz force takes the form:

$$\underline{f}_m = \underline{J}\times\underline{B}, \qquad (5)$$

where $\underline{J}$ is the current density, $\underline{B}$ is the magnetic induction vector. The current density may be expressed by the generalized Ohm's law as:

$$\underline{J} = \sigma(\underline{E} + \underline{V}\times\underline{B}), \qquad (6)$$

where the terms $\sigma\underline{E}$ and $\sigma(\underline{V}\times\underline{B})$, respectively, represent the conduction and induction currents and σ is the electrical conductivity. In the low magnetic Reynolds number approximation, in which the induced magnetic field can be ignored, the magnetic body force becomes:

$$\underline{f}_m = \sigma(\underline{V}\times\underline{B})\times\underline{B}, \qquad (7)$$

a uniform circular magnetic field $\underline{B}$ of strength B_o is acting on the azimuthal direction, Fig. (1), such as:

$$\underline{B} = [0,\ B_o,\ 0], \qquad (8)$$

with the help of Eqs. (4) and (8), the magnetic body force becomes:

$$\underline{f}_m = [0,\ 0,\ -\sigma B_o^2 W], \qquad (9)$$

2.2. Darcy's Resistance

The resistance for blood flow through porous medium is given by Darcy's law,

$$\underline{\Re} = -\frac{\mu\phi}{k}\underline{V}, \qquad (10)$$

where $\underline{\Re}$ is a measure of the flow resistance offered by the porous medium, ϕ and k are the porousity and the permeability of the porous medium. The Darcy's resistance for Oldroyd-B fluid satisfies the following expression [53]:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\underline{\Re} = -\frac{\mu\phi}{k}\left(1+\lambda_2\frac{\partial}{\partial t}\right)\underline{V}, \qquad (11)$$

we assumed that:

$$\underline{\Re} = [0,\ 0,\ \Re_z], \qquad (12)$$

with the help of Eqs. (4) and (12) we can simplify Eq. (11) to final form:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\Re_z = -\frac{\mu\phi}{k}\left(1+\lambda_2\frac{\partial}{\partial t}\right)W. \qquad (13)$$

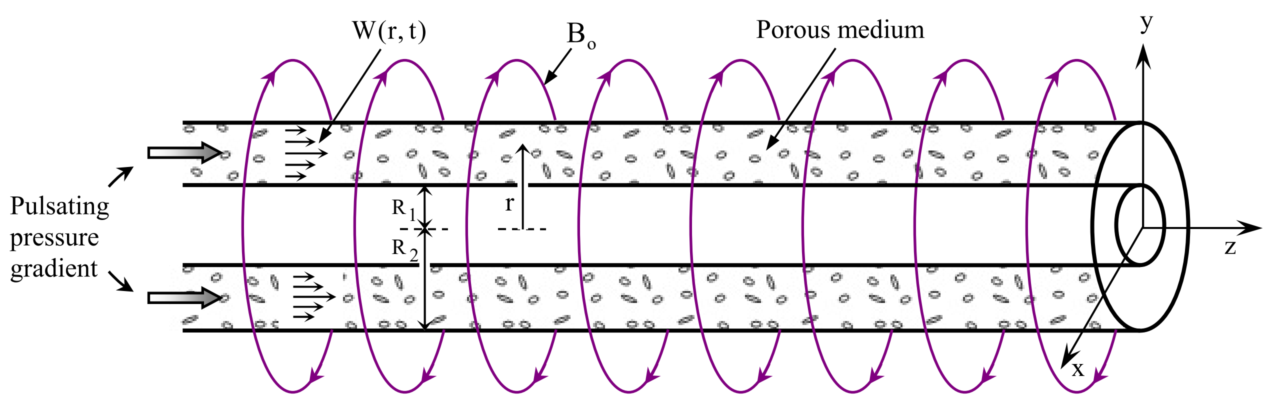


Fig 2. Schematic diagram of rheological blood flow through a porous medium filled the gap between two concentric cylinders.

2.3. Blood Constitutive Equation

As already pointed out, human blood is a viscoelastic and shear thinning fluid. Therefore, the constitutive equations for blood must incorporate its non-linear viscometric properties, especially the shear-thinning behaviour. A simple way to account for the elastic effects in blood is to consider the constitutive equation for the Giesekus model. The differential form of this model can be written as [54]:

$$\underline{\underline{S}} = \underline{\underline{S}}_s + \underline{\underline{S}}_p \ , \tag{14}$$

$$\underline{\underline{S}}_s = 2\mu_s \underline{\underline{d}} \ , \tag{15}$$

$$\underline{\underline{S}}_p + \lambda_1 \overset{\nabla}{\underline{\underline{S}}}_p + \alpha \frac{\lambda_1}{\mu_p}\left(\underline{\underline{S}}_p \cdot \underline{\underline{S}}_p\right) = 2\mu_p \underline{\underline{d}} \ . \tag{16}$$

The parameters μ_s and μ_p are the solvent and viscoelastic contributions to the dynamic viscosity μ,

$$\mu = \mu_s + \mu_p \ , \tag{17}$$

λ_1 is a relaxation time and α denotes Giesekus's mobility factor ($0 \le \alpha \le 1$). The stress tensor $\underline{\underline{S}}$ is decomposed into a viscoelastic contribution $\underline{\underline{S}}_p$ and a Newtonian solvent contribution $\underline{\underline{S}}_s$. The tensor $\overset{\nabla}{\underline{\underline{S}}}_p$ is the upper convected derivative of $\underline{\underline{S}}_p$:

$$\overset{\nabla}{\underline{\underline{S}}}_p = \frac{\partial \underline{\underline{S}}_p}{\partial t} + \underline{V} \cdot \nabla \underline{\underline{S}}_p - \underline{\underline{S}}_p \cdot \underline{\underline{L}} - \left(\underline{\underline{S}}_p \cdot \underline{\underline{L}}\right)^T \ , \tag{18}$$

The relation between the stretching tensors $\underline{\underline{L}}$ and the deformation tensor $\underline{\underline{d}}$ is given by:

$$\underline{\underline{L}} = \nabla \underline{V} \ , \quad \underline{\underline{d}} = \tfrac{1}{2}(\underline{\underline{L}} + \underline{\underline{L}}^T) \ . \tag{19}$$

The symbol "T" over any tensor denotes the matrix transpose. Equations (14-16) can be rewritten as a single constitutive equation by replacing $\underline{\underline{S}}_p$ in the last equation with $\underline{\underline{S}} - \underline{\underline{S}}_s = \underline{\underline{S}} - 2\mu_s \underline{\underline{d}}$. This leads to:

$$\begin{aligned} &\underline{\underline{S}} + \lambda_1 \overset{\nabla}{\underline{\underline{S}}} + a\frac{\lambda_1}{\mu}\left(\underline{\underline{S}} \cdot \underline{\underline{S}}\right) - 2a\lambda_2\left(\underline{\underline{S}} \cdot \underline{\underline{d}} + \underline{\underline{d}} \cdot \underline{\underline{S}}\right) \\ &\qquad = 2\mu\left[\underline{\underline{d}} + \lambda_2 \overset{\nabla}{\underline{\underline{d}}} - 2a\frac{\lambda_2^2}{\lambda_1}\left(\underline{\underline{d}} \cdot \underline{\underline{d}}\right)\right], \end{aligned} \tag{20}$$

where the retardation time λ_2 and the modified mobility parameter “a”, are given in terms of μ_s , μ_p and α as:

$$\lambda_2 = \lambda_1 \frac{\mu_s}{\mu} ; \quad a = \frac{\alpha}{1 - (\lambda_2 / \lambda_1)} \ . \tag{21}$$

In this work, we will study a subclass of Giesekus model known as Oldroyd-B fluid model. It has a measurable relaxation and retardation time and can relate the viscoelastic manner of blood solution under general flow conditions. The stress tensor $\underline{\underline{S}}$ for

Oldroyd-B model can be obtained as a limiting case of Giesekus model for $a \to 0$ in Eq. (20). So,

$$\underline{\underline{S}} + \lambda_1 \overset{\nabla}{\underline{\underline{S}}} = 2\mu\left(\underline{\underline{d}} + \lambda_2 \overset{\nabla}{\underline{\underline{d}}}\right), \tag{22}$$

taking Eq. (18) into account, Eq. (22) takes the form:

$$\begin{aligned}\underline{\underline{S}} + \lambda_1\left[\frac{\partial \underline{\underline{S}}}{\partial t} + \underline{V}\cdot\nabla\underline{\underline{S}} - \underline{\underline{L}}\cdot\underline{\underline{S}} - (\underline{\underline{L}}\cdot\underline{\underline{S}})^T\right] = \\ 2\mu\left[\underline{\underline{d}} + \lambda_2\left(\frac{\partial \underline{\underline{d}}}{\partial t} + \underline{V}\cdot\nabla\underline{\underline{d}} - \underline{\underline{L}}\cdot\underline{\underline{d}} - (\underline{\underline{L}}\cdot\underline{\underline{d}})^T\right)\right].\end{aligned} \tag{23}$$

It is clear that, the viscoelastic properties of blood through this model is specified by three parameters μ, λ_1 and λ_2. The used model reduces to the viscous (Newtonian) fluid when $\lambda_1 = \lambda_2 = 0$, the Maxwell fluid when $\lambda_2 = 0$ and an Oldroyd-B fluid when $0 < \lambda_2 < \lambda_1 < 1$. Since, the stress tensor in its components form is:

$$\underline{\underline{S}} = \begin{bmatrix} S_{rr} & S_{r\theta} & S_{rz} \\ S_{\theta r} & S_{\theta\theta} & S_{\theta z} \\ S_{zr} & S_{z\theta} & S_{zz}\end{bmatrix}, \tag{24}$$

so, with the choice of the velocity given in Eq. (4), the stress tensor components are:

$$S_{rr} + \lambda_1\left(\frac{\partial S_{rr}}{\partial t} - 2\frac{\partial W}{\partial r}S_{rz}\right) = -2\mu\lambda_2\left(\frac{\partial W}{\partial r}\right)^2, \tag{25}$$

$$S_{r\theta} + \lambda_1\left(\frac{\partial S_{r\theta}}{\partial t} - \frac{\partial W}{\partial r}S_{\theta z}\right) = 0, \tag{26}$$

$$S_{rz} + \lambda_1\left(\frac{\partial S_{rz}}{\partial t} - \frac{\partial W}{\partial r}S_{zz}\right) = \mu\left(\frac{\partial W}{\partial r} + \lambda_2\frac{\partial^2 W}{\partial r\partial t}\right), \tag{27}$$

$$S_{\theta\theta} + \lambda_1\frac{\partial S_{\theta\theta}}{\partial t} = 0, \tag{28}$$

$$S_{\theta z} + \lambda_1\frac{\partial S_{\theta z}}{\partial t} = 0, \tag{29}$$

$$S_{zz} + \lambda_1\frac{\partial S_{zz}}{\partial t} = 0. \tag{30}$$

Note that, the treating of Eq. (25) gives $S_{rr} = f(t)/r$ while Eqs. (26), (28), (29) and (30) yields $S_{r\theta} = S_{\theta\theta} = S_{\theta z} = S_{zz} = g(r)\,\mathrm{Exp}(-t/\lambda_1)$ where $f(t)$ is an arbitrary function of time and $g(r)$ is an arbitrary function of r. The last components of the stress tensor decays exponentially with time. Therefore, the only remaining stress tensor component is:

$$S_{rz} + \lambda_1\frac{\partial S_{rz}}{\partial t} = \mu\left(\frac{\partial W}{\partial r} + \lambda_2\frac{\partial^2 W}{\partial r\partial t}\right). \tag{31}$$

2.4. The Blood Viscosity

The viscosity of blood has a basic influence on flow in the larger arteries, while the elasticity has basic influence in the arterioles and capillaries. The rheological behaviour of blood is governed by the concentration and the properties of RBCs, deformability, orientation and aggregation.

Giesekus model is characterized by four parameters μ, λ_1, λ_2 and "a". The inclusion of ($\underline{\underline{S}}_p \cdot \underline{\underline{S}}_p$) term in Eq. (16) gives a viscosity function that is much more realistic than any other model [17]. The analytical function for apparent viscosity due to this model which offers the best fit of experimental data for a quite large range of flow has been found to be:

$$\tilde{\mu} = \mu\left\{\frac{\lambda_2}{\lambda_1} + \left(1 - \frac{\lambda_2}{\lambda_1}\right)\frac{(1-f)^2}{1+(1-2a)f}\right\}, \tag{32}$$

where

$$f = \frac{1-\chi}{1+(1-2a)\chi}, \tag{33}$$

$$\chi^2 = \frac{\sqrt{1+16a(1-a)(\lambda_1\dot{\gamma})^2} - 1}{8a(1-a)(\lambda_1\dot{\gamma})^2}. \tag{34}$$

Equation (32) is used for evaluation of the variable viscosity by using Giesekus model, where the viscosity decrement depending on shear-rate. Giesekus parameters are obtained by fitting an experimental data [55], see Table 1. It is clear from Fig. (2) that, the viscosity decreases dramatically as the rate of shear increases.

When the RBCs are at very small shear rates, they tend to aggregate. With very low shear rates, the viscoelastic property of blood is dominated by the aggregation and cell deformability is relatively insignificant. As the shear rate increases the size of aggregates begins to decrease. With a further increase in shear rate, the cells will rearrange to provide channels for the plasma to pass through and for the cells to slide. Influence of aggregation on the viscoelasticity diminishes and the influence of red cell deformation begins to increase. As shear rates become large, RBCs will stretch or deform and align with the flow. Cell layers are formed, separated by plasma, and flow is now attributed to layers of cells sliding on layers of plasma. The cell layer allows for easier flow of blood and as such there is a reduced viscosity and reduced elasticity.

Table 1: Giesekus model parameters for human blood.

Hematocrit and Temperature	Giesekus model parameters			
	μ (mPa.s)	λ_1 (s)	λ_2/λ_1	a
$h = 40.5\%$, $T = 37\ ^{o}C$	0.0064	0.061	0.48	0.18
$h = 40\%$, $T = 37\ ^{o}C$	0.0071	0.050	0.45	0.20
$h = 45\%$, $T = 37\ ^{o}C$	0.0087	0.038	0.40	0.35
$h = 45\%$, $T = 23\ ^{o}C$	0.0170	0.035	0.20	0.60

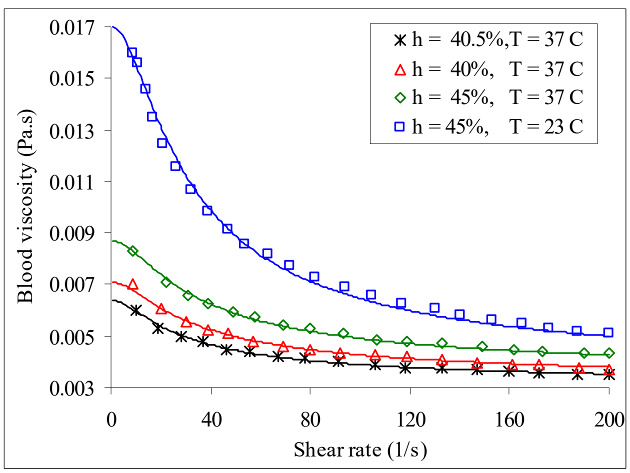


Fig. 2. Comparison of viscosity function $\tilde{\mu}(\dot{\gamma})$ for Giesekus model where dots represent the blood experimental data [55] and solid lines represent Giesekus model fit from Eq. (32).

3. Solution of the Problem

In view of all assumptions given in the last subsections, the unsteady flow of blood is governed by z-component of the momentum equation:

$$\frac{\partial S_{rz}}{\partial r}+\frac{1}{r}S_{rz}-\frac{\partial p}{\partial z}-\sigma B_o^2\, W+\Re_z=\rho\frac{\partial W}{\partial t}\ , \quad (35)$$

and the stress tensor component S_{rz} given in Eq. (31). The elimination of S_{rz} from these two equations shows that, the velocity field $W(r, t)$ is governed by:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial W}{\partial t}-\frac{\mu}{\rho}\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}\right)+\frac{\sigma B_o^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)W-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\Re_z=-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z}, \quad (36)$$

using Eq. (13) for Darcy's resistance expression, we can replace the last two terms in the LHS of Eq. (36). Rearranging the obtained equation we get:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial W}{\partial t}-\frac{\mu}{\rho}\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}\right)+\frac{\sigma B_o^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)W+\frac{\mu\phi}{\rho k}\left(1+\lambda_2\frac{\partial}{\partial t}\right)W=-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z}. \quad (37)$$

Equation (37) represents the governing flow equation with its boundary conditions:

$$W(R_1,t)=W(R_2,t)=0\,. \quad (38)$$

We nondimensionalize the problem by defining the following dimensionless variables:

$$\overset{*}{r}=\frac{r}{R_1},\quad \overset{*}{t}=\frac{\mu}{\rho R_1^2}t,\quad \overset{*}{W}=\frac{\mu L}{\Delta p R_1^2}W,\quad \overset{*}{\lambda}_1=\frac{\mu}{\rho R_1^2}\lambda_1,\quad \overset{*}{\lambda}_2=\frac{\mu}{\rho R_1^2}\lambda_2. \quad (39)$$

Using Eq. (39), the dimensionless governing equation and boundary conditions are obtained as follows (after dropping the dimensionless mark "*" for simplicity):

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial W}{\partial t}+MW\right)-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}\right)+\frac{1}{K}\left(1+\lambda_2\frac{\partial}{\partial t}\right)W=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\Psi(t), \quad (40)$$

$$W(1, t) = W(\gamma,\ t) = 0, \quad (41)$$

where $\gamma = R_2 / R_1$ is the defined as the gap parameter and

$$M = \frac{\sigma B_o^2 R_1^2}{\mu},\ \frac{1}{K} = \frac{\phi R_1^2}{k},\ \Psi(t) = \frac{-L}{\Delta p}\frac{\partial p}{\partial z}. \quad (42)$$

The parameter M is known as the magnetic parameter whereas K is the permeability parameter.

Pulsating flow is composed of a steady component and a superimposed periodical time varying component called oscillation. Oscillating flow itself is a special pulsating flow, which is governed by an oscillation only with a zero steady flow component. In this case the flow is supposed to generate impulsively due to cosine pulses applied periodically on the positive z-direction. Equation (40) must be solved subject to boundary conditions, Eq. (41), for pulsating pressure gradient. So, let the pressure gradient or the function $\Psi(t)$ oscillate with frequency ω and amplitude P_o, i.e.

$$\Psi(t) = P_o \cos \omega t = \mathrm{Re}\left(P_o e^{i\omega t}\right), \quad (43)$$

and assuming the velocity function has the form:

$$W(r, t) = \mathrm{Re}\left(H(r)\, e^{i\omega t}\right), \quad (44)$$

where Re( ) represents the real part of the complex function which follows. Substituting about $\Psi(t)$ and $W(r,\ t)$ into Eq. (40) and equating the harmonic terms, retaining coefficients of $e^{i\omega t}$, the corresponding equation becomes:

$$H'' + \frac{1}{r}H' - \left[\frac{(M+i\omega)(1+i\omega\lambda_1)}{(1+i\omega\lambda_2)} + \frac{1}{K}\right]H = -\frac{(1+i\omega\lambda_1)}{(1+i\omega\lambda_2)}P_o, \quad (45)$$

$$H'' + \frac{1}{r}H' - \beta^2 H = -\frac{\alpha_1}{\alpha_2}P_o, \quad (46)$$

where

$$\beta^2 = \frac{\alpha_1}{\alpha_2}(M+i\omega),\ \alpha_1 = (1+i\omega\lambda_1),\ \alpha_2 = (1+i\omega\lambda_2), \quad (47)$$

the boundary conditions, Eq. (41), may be rephrased in terms of the new functions, with the result

$$H(1) = H(\gamma) = 0. \quad (48)$$

Equation (46) can be easily recognized as Bessel function. The solution of this equation is:

$$H(r) = \frac{\alpha_1 P_o}{\alpha_2 \beta^2} + C_1 J_0(i\beta r) + C_2 Y_0(-i\beta r), \quad (49)$$

where $i = \sqrt{-1}$, J_0 and Y_0 are the Bessel functions of zero orders of first and second kind. Using the boundary conditions, Eq (48), C_1 and C_2 are obtained as follows:

$$C_1 = \frac{\alpha_1 P_o}{\alpha_2 \beta^2}\left[\frac{-Y_0(-i\beta) + Y_0(-i\beta\gamma)}{J_0(i\beta\gamma)\,Y_0(-i\beta) - J_0(i\beta)\,Y_0(-i\beta\gamma)}\right], \quad (50)$$

$$C_2 = \frac{\alpha_1 P_o}{\alpha_2 \beta^2}\left[\frac{-J_0(i\beta\gamma) + J_0(i\beta)}{J_0(i\beta\gamma)\,Y_0(-i\beta) - J_0(i\beta)\,Y_0(-i\beta\gamma)}\right]. \quad (51)$$

Hence, the velocity field is given by:

$$W(r, t) = \mathrm{Re}\left\{\left[\frac{\alpha_1 P_o}{\alpha_2 \beta^2} + C_1 J_0(i\beta r) + C_2 Y_0(-i\beta r)\right]e^{i\omega t}\right\}. \quad (52)$$

4. Blood Flow Rate

The dimensionless volumetric flow rate of blood through the gap between the two concentric cylinders is equal to:

$$Q = 2\pi \int_1^{\gamma} W(r)\, r\, dr, \quad (53)$$

by using the velocity functions from Eq. (52), the integral in Eq. (53) can be calculated. Therefore, the rate of blood flow is now calculated as:

$$Q = \mathrm{Re}\left\{\frac{2\pi}{\beta}\left[\frac{\alpha_1 P_o(\gamma^2 - 1)}{2\alpha_2 \beta} + \left(\gamma I_1(\beta\gamma) - I_1(\beta)\right)C_1 - i\left(Y_1(-i\beta) - \gamma Y_1(-i\beta\gamma)\right)C_2\right]\right\}, \quad (54)$$

where I_1 and Y_1 are the modified Bessel functions of first order and C_1, C_2 are given by Eqs. (50) and (51).

5. Results and Discussion

The results of this paper may help for the studies in the field of medical research. The obtained results could be useful in the treatment of human blood flow problems in wide and narrow arteries. The effects of the various parameters on the flow characteristics such as geometry, viscoelasticcity, magnetic and permeability parameters have been studied. The variation of velocity field with r is calculated from Eq. (52) for different values of the included physical parameters at $P_o = 1$ and for fixed λ_1, λ_2, M, K, ωt. The following figures are classified into tow categories "a" and "b". Category "a" is drawn for gap parameter $\gamma = 2$ and figures in category "b" is drawn for $\gamma = 3$. The effect of geometry is clear from the comparison between two categories "a" and "b".

5.1. Viscoelasticity Effects

Initially, the effects of viscoelastic parameter λ_1 for fixed λ_2, M, K and ωt on the velocity profile are examined graphically in Figs. (3a) and (3b). Then the effects of λ_2 for fixed λ_1, M, K and ωt are examined in Figs. (4a) and (4b). We observe that the velocity increases with increasing λ_1 and decreasing with increasing λ_2.

5.2. Magnetic field Effects

The application of magnetic field could be useful in the treatment of some health problems such as cardiovascular diseases and blood flow in narrow arteries especially when blood clots are formed in the lumen of the coronary artery. The value of M is an index to the relative importance of magnetic forces. When $M = 0$, magnetic forces are absent; when M increases, the magnetic force becomes increasingly important. The influence of magnetic parameter M on the blood velocity profile is given in Figs. (5a) and (5b) for fixed $\lambda_1 = 3.3$, $\lambda_2 = 0.15$, $K = 0.2$ and $\omega t = \pi / 4$. Figures (5c) and (5d) show the same behaviour in tree dimensions. In those figures we observe that the axial blood velocity decreases with the increase in the value of M.

5.3. Porous Medium Effects

This subsection displays the effect of porous medium on the axial velocity field. The values of K are an index to the relative importance of permeability of the porous medium. The value $K = 0$ is for solid gap; and when $K \to \infty$ (or $1/K \to 0$), the resistance offered by porous medium is absent, which means that the gap between the two cylinders becomes a hollow gap. The influence of K on the blood velocity field is given in Figs. (6a) and (6b) for fixed values of λ_1, λ_2, M and ωt. These figures show that, the increase in K yields an effect opposite to that of the magnetic field. We observe that, the velocity is showing increasing behaviour with the decrement in values of K. This is because β decreases as K increases. Figures (6c) and (6d) show the same behaviour in tree dimensions.

5.4. Pulsating Pressure Effects

Pulsatile flow of blood through cardiovascular especially on artery has drawn the attention to the researchers for a long time due to its great importance in medical sciences. Under normal conditions, blood flow in human circulatory system depends upon the pumping action of the heart and this produces a pressure gradient through the arterial network. The variation of blood velocity field with r is given in Figs. (7a) and (7b) for different values of ωt. Figures show that, the blood velocity decreases with increasing ωt. The oscillatory response of the velocity to a pulsating pressure gradient can be seen in Figs. (7c) and (7d) (which shows, in three dimensions, the effect of ωt on the velocity distribution) for $M = 0.5$.

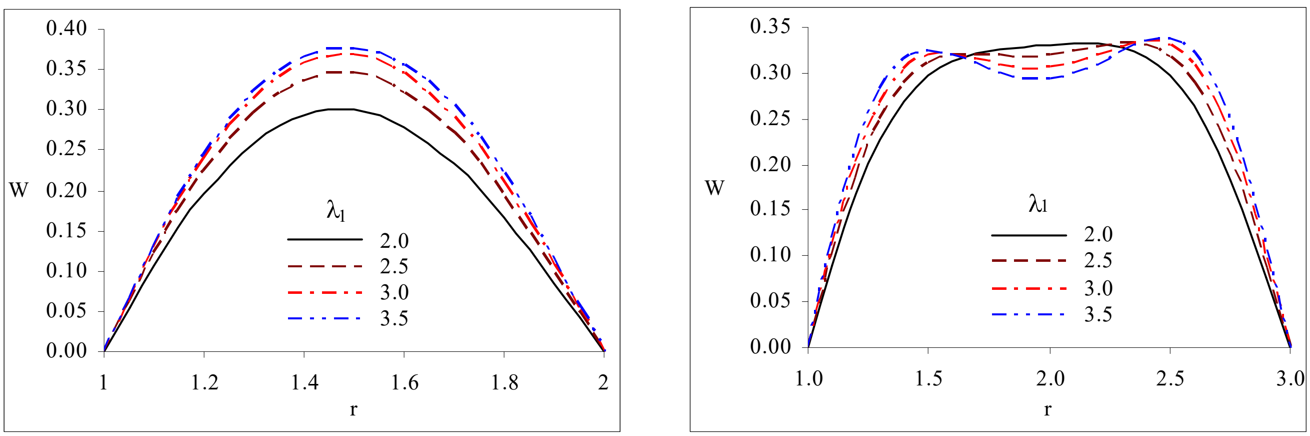
5.5. Volume Flow Rate

Figure (8a) shows the variation of volume flow rate Q with magnetic parameter M for $K = 0.5, 1, 10$, $\lambda_1 = 0.3$, $\lambda_2 = 0.15$ and Fig. (8b) represents the effects of permeability parameter K on the flow rate for $M = 0, 0.5, 1.5$, $\lambda_1 = 0.3$, $\lambda_2 = 0.15$. The three dimensions variation of volume flow rate Q with magnetic parameter M and permeability parameter K is given in Fig. (8c) for fixed values of P_o, λ_1, λ_2. We observe that Q decreases with increment in M and increases with increment in K. in the annular region of concentric cylinders.

6. Conclusions

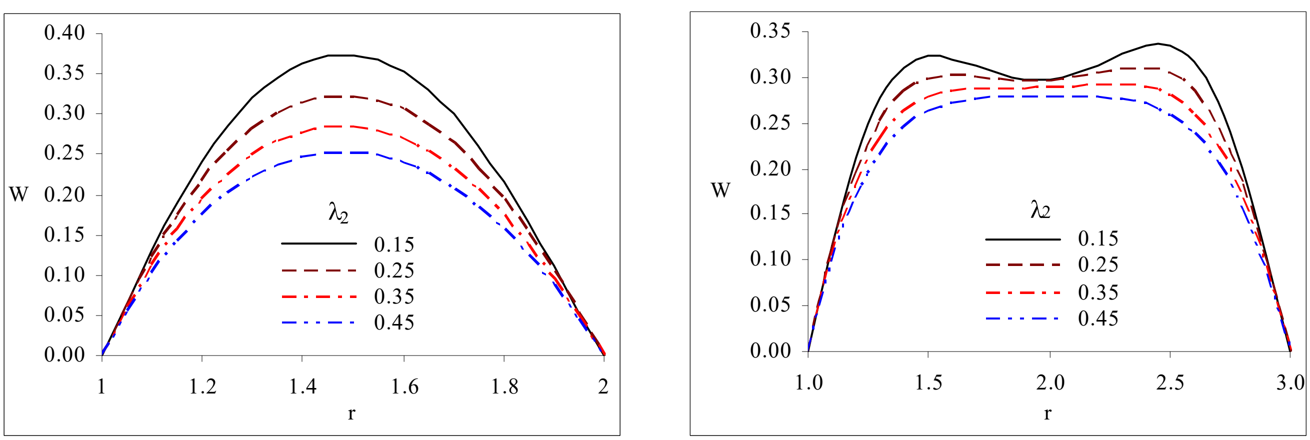
In this work the mathematical analysis for the effects of circular magnetic field on the blood velocity profiles in the gap between concentric cylinders filled with porous medium have been studied. The pulsatile flow of blood is characterized in this model by several parameters: the gape, parameter, the magnetic parameter, the permeability parameter and the rheological parameters of blood, i.e., the viscosity, relaxation time and retardation time. The effects of these parameters on the velocity field are studied. Therefore, we conclude the following remarks:

- We can use magnetic field to control blood velocity.
- By using an appropriate magnetic field it is possible to control blood pressure and also it is effective for conditions such as poor circulation.
- An increasing in magnetic parameter reduces both the velocity profile and the flow rate due to the effect of magnetic force against the flow direction.
- An increase in the permeability yields an effect opposite to that of the magnetic parameter.
- The present model gives a most general form of velocity expression from which the other models can be obtained



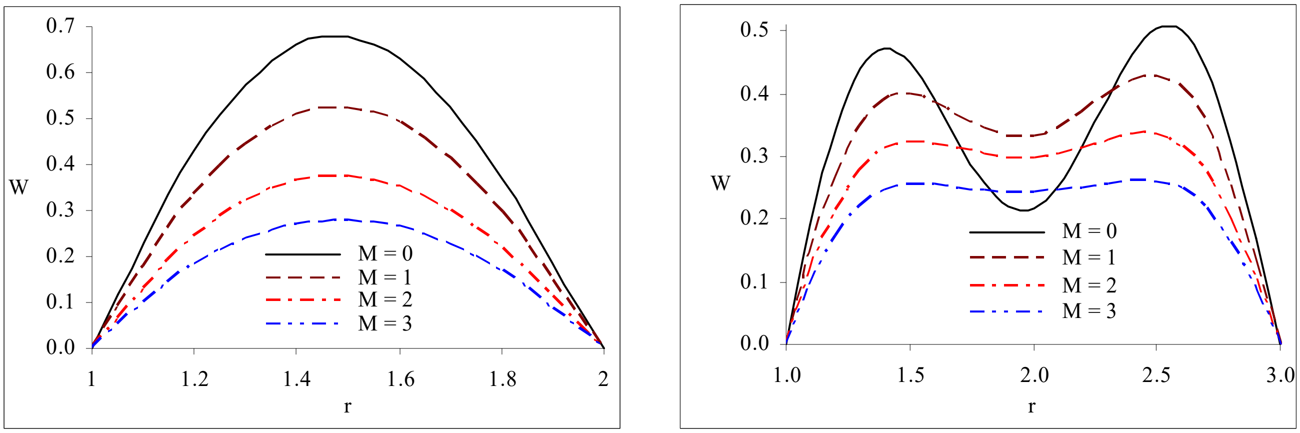
(a) $\gamma = 2$ (b) $\gamma = 3$

Fig. 3. Velocity field for various values of λ_1 and for fixed $\lambda_2 = 0.15$, $M = 2$, $K = 0.2$ and $\omega t = 3\pi/4$.



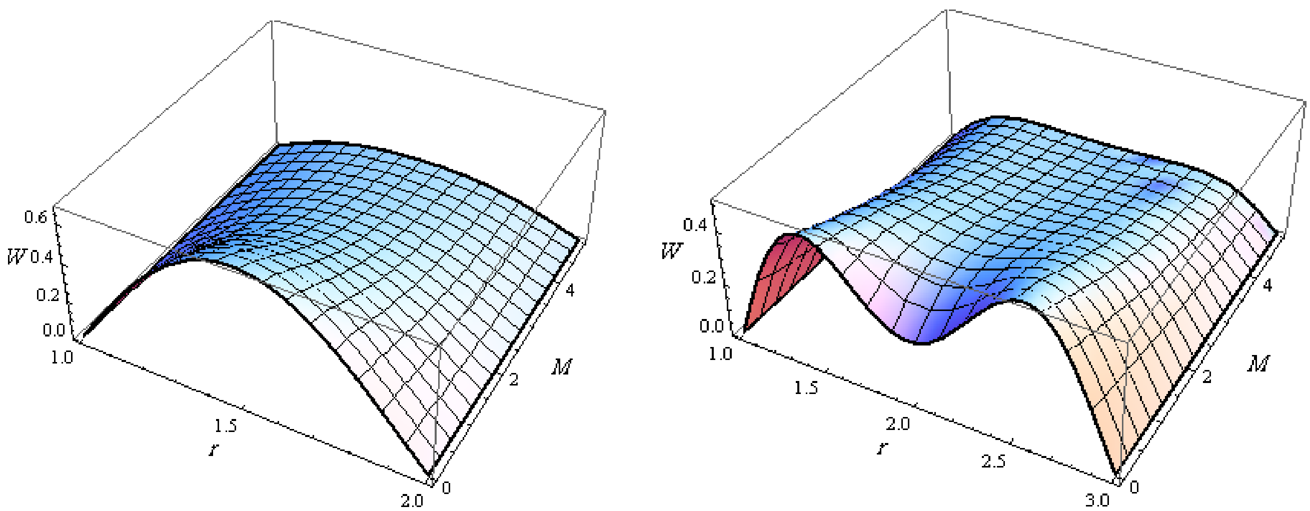
(a) $\gamma = 2$ (b) $\gamma = 3$

Fig. 4. Velocity field for various values of λ_2 and for fixed $\lambda_1 = 3.3$, $M = 2$, $K = 0.2$ and $\omega t = 3\pi/4$.



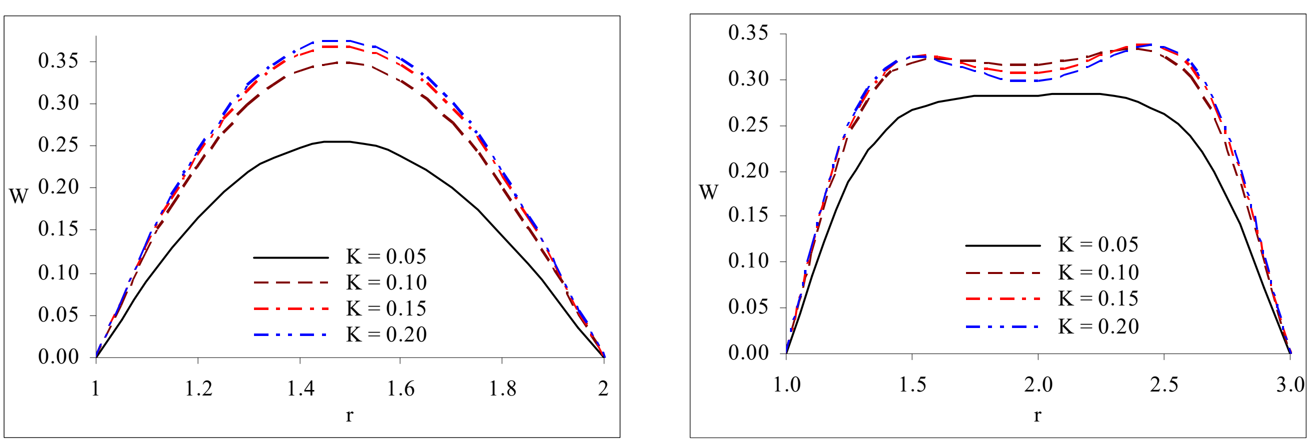
(a) $\gamma = 2$ (b) $\gamma = 3$

Fig. 5. Velocity field for various values of M and for fixed $\lambda_1 = 3.3$, $\lambda_2 = 0.15$, $K = 0.2$ and $\omega t = 3\pi/4$.



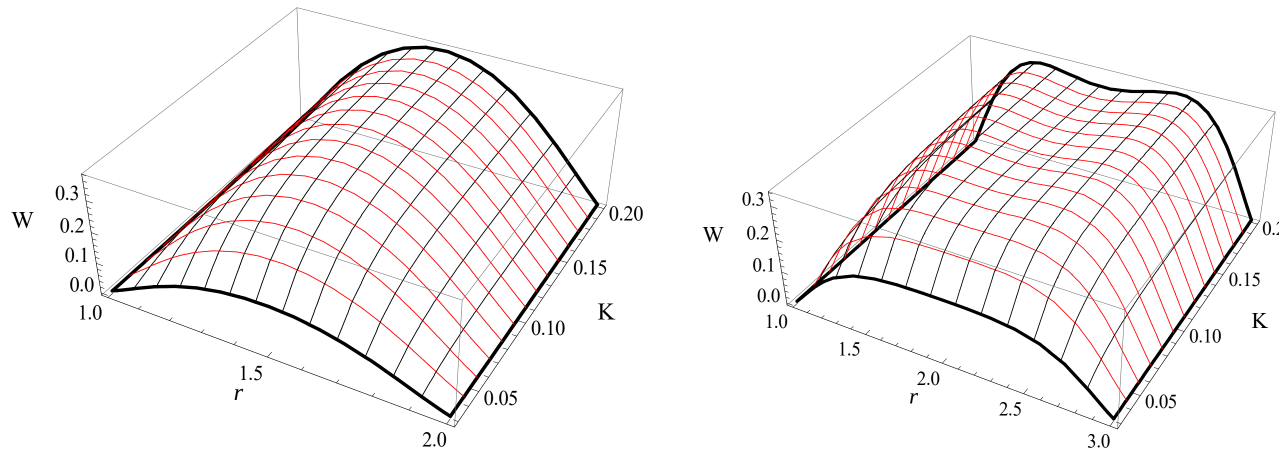
(c) $\gamma = 2$ (d) $\gamma = 3$

Fig. 5. Three dimension velocity field for fixed $\lambda_1 = 3.3$, $\lambda_2 = 0.15$, $K = 0.2$ and $\omega t = 3\pi/4$.



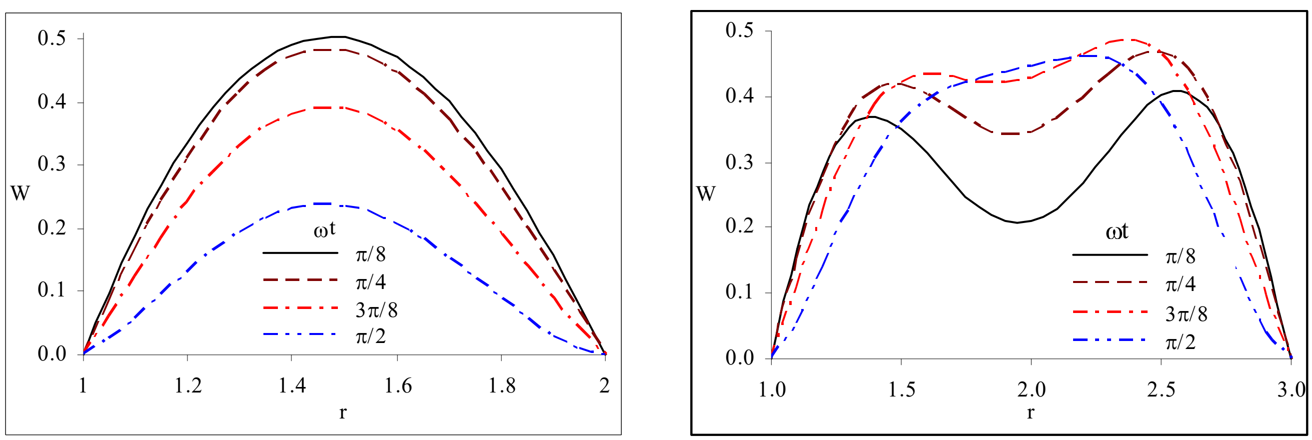
(a) $\gamma = 2$ (b) $\gamma = 3$

Fig. 6. Velocity field for various values of K and for fixed $\lambda_1 = 3.3$, $\lambda_2 = 0.15$, $M = 2$ and $\omega t = 3\pi/4$.



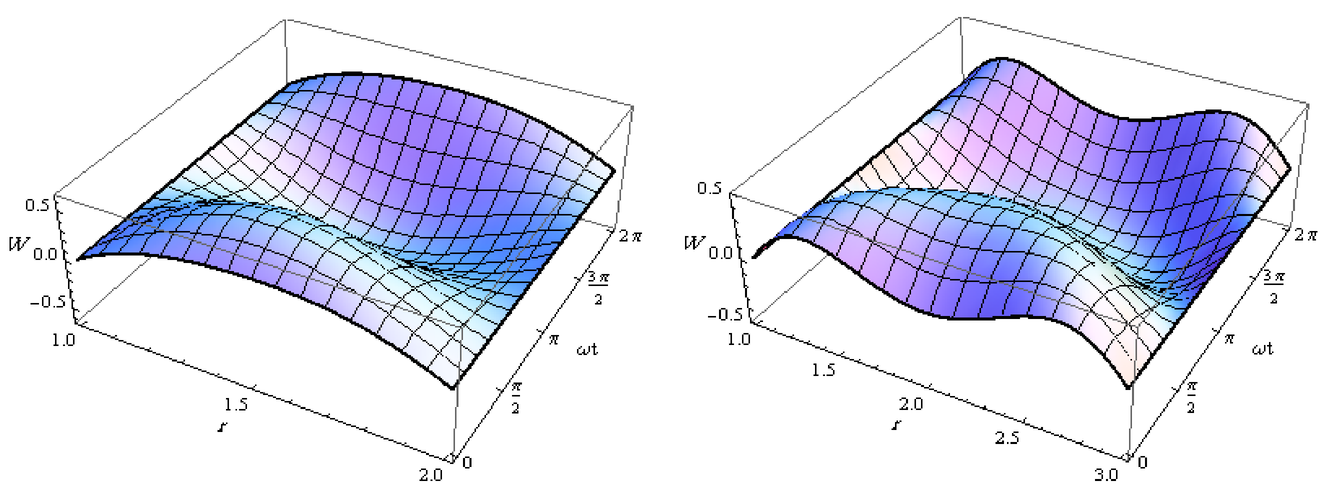
(c) $\gamma = 2$ (d) $\gamma = 3$

Fig. 6. Three dimension velocity field for fixed $\lambda_1 = 3.3$, $\lambda_2 = 0.15$, $M = 2$ and $\omega t = 3\pi/4$.



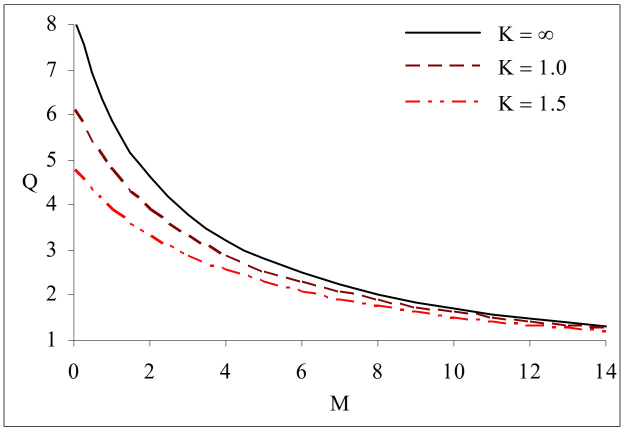
(a) $\gamma = 2$ (b) $\gamma = 3$

Fig. 7. Velocity field for various values of ωt and for fixed $\lambda_1 = 3.3$, $\lambda_2 = 0.15$ and $M = 2$.

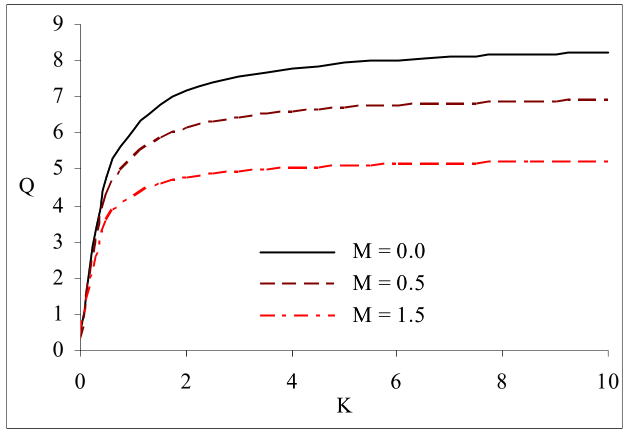


(c) $\gamma = 2$ (d) $\gamma = 3$

Fig. 7. Three dimension velocity field for fixed $\lambda_1 = 3.3$, $\lambda_2 = 0.15$, $M = 0.5$ and $K = 0.2$.



(a)



(b)

Fig. 8. Variation of volumetric flow rate with M for different K (in a) and with K for different M (in b) when $\lambda_1 = 0.3$, $\lambda_2 = 0.15$, $\gamma = 3$

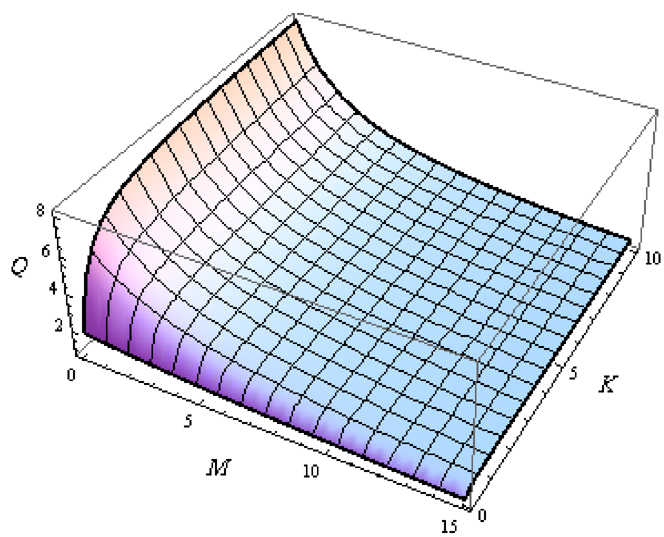


Fig. 8c. Three dimensions variation of volume flow rate Q with magnetic parameter M and permeability parameter K for $P_o = 1$, $\lambda_1 = 0.3$, $\lambda_2 = 0.15$, $\gamma = 3$.

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