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A Note on Knuth's Algorithm for Computing Extended Greatest Common Divisor using SGN Function

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Abstract
In this note we gave new implementation of Knuth's Algorithm for Computing Extended Greatest Common Divisor using SGN Function (KACEGCDSF). Very intriguing and practical oriented tasks lead to problem of searching greatest common divisor [55, 37]. In our implementation we reduce the number of iterations and now they are 50% of wide spread implementation of KACEGCDSF. Fundamental books of classical algebra problems and basic algorithms deal with this algorithm [1-32], [34-55]. Visual C# 2017 programming environment is used.
Keywords: *extended greatest common divisor, SGN function, Euclidean Algorithm, Knuth's algorithm, reduced number of iterations.*

1. Introduction

Our work is continuation of research in [28-33].

All algorithms given here work when both *a* and *b* are positive or both *a* and *b* are negative. KACEGCDSF is known (see [37]):

Algorithm 1.

```
t = 0; u = 0;
if (a > 0) s = 1; else if (a < 0) s = -1;
if (b > 0) v = 1; else if (b < 0) v = -1;
do { if (a < 1) { x = u; y = v; break; }
     if (b < 1) { x = s; y = t; break; }
     if (a > b) { q = a / b; a -= q * b;
                  s -= q * u; t -= q * v; }
          else { q = b / a; b -= q * a;
                 u -= q * s; v -= q * t; } }
while (true);
gcd = ao * x + bo * y;
```

2. Main Results

Now we set the task to optimize KACEGCDSF. For testing we will use the following computer: processor - Intel(R) Core(TM) i7-6700HQ CPU 2.60GHz, 2592 Mhz, 4 Core(s), 8 Logical Processor(s), RAM 16 GB, Microsoft Windows 10 Enterprise x64.

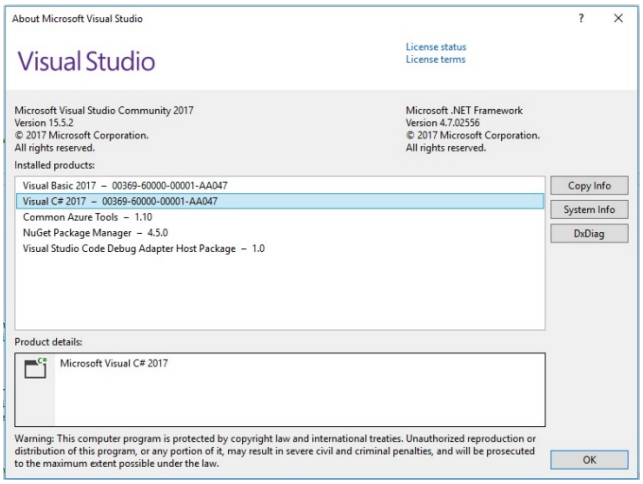

Fig. 1. Visual C# 2017

with the following programing environment (see Fig. 1.).

We introduce the following algorithms. Algorithm 2 is recursive implementation of Algorithm 1:

Algorithm 2.

```
static long Euclid(long a, long b, ref long x, ref long
y, long s, long t, long u, long v)
     { if (a < 1) { x = u; y = v; return b; }
       if (b < 1) { x = s; y = t; return a; }
      if (a > b) { long q = a / b; a -= q * b;
                   s -= q * u; t -= q * v; }
       else { long q = b / a; b -= q * a;
```

```
            u -= q * s; v -= q * t; }
        long d = Euclid(a, b, ref x, ref y, s, t, u, v);
        return d; }
```

And its calling:

```
t = 0; u = 0;
if (a > 0) s = 1; else if (a < 0) s = -1; if (b > 0) v = 1;
else if (b < 0) v = -1;
if (a > b) Euclid(a, b, ref x, ref y, s, t, u, v); else
Euclid(b, a, ref y, ref x, s, t, u, v);
gcd = x * ao + y * bo;
```

New realization of Algorithm 1:

Algorithm 3.

```
        t = 0; u = 0;
        if (a > 0) s = 1; else if (a < 0) s = -1;
        if (b > 0) v = 1; else if (b < 0) v = -1;
        if (a > b) do { q = a / b; a -= q * b;
                        s -= q * u; t -= q * v;
            if (a < 1) { x = u; y = v; break; }
                        q = b / a; b -= q * a;
                        u -= q * s; v -= q * t;
            if (b < 1) { x = s; y = t; break; } }
                while (true);
        else do { q = b / a; b -= q * a;
                        u -= q * s; v -= q * t;
            if (b < 1) { x = s; y = t; break; }
                        q = a / b; a -= q * b;
                        s -= q * u; t -= q * v;
            if (a < 1) { x = u; y = v; break; } }
                while (true);
gcd = ao * x + bo * y;
```

and its recursive presentation:

Algorithm 4.

```
static long Euclid(long a, long b, ref long x, ref long
y, long s, long t, long u, long v)
 { if (b < 1) { x = s; y = t; return a; }
   long q = a / b; a -= q * b; s -= q * u; t -= q * v;
   if (a < 1) { x = u; y = v; return b; }
   q = b / a; b -= q * a; u -= q * s; v -= q * t;
   long d = Euclid(a, b, ref x, ref y, s, t, u, v);
   return d; }
```

The calling of Algorithm 4 is:

```
t = 0; u = 0;
if (a > 0) s = 1; else if (a < 0) s = -1; if (b > 0) v = 1;
else if (b < 0) v = -1;
if (a > b) Euclid(a, b, ref x, ref y, s, t, u, v); else
Euclid(b, a, ref y, ref x, s, t, u, v);
gcd = x * ao + y * bo;
```

3. Numerical experiments

Part 1. We will use the following task:

```
    long a, b, ao, bo, x, y, u, s, t, v, gcd, d;
    long d;
    d = 0;
    for (int i = 1; i < 100000001; i++)
        { bo = i; ao = 200000002 - i;
          b = Math.Abs(bo); a = Math.Abs(ao);
    //here is the source code of
    //every one of Algorithms 1-4
    d += gcd; }
    Console.WriteLine(d);
```

Results of Part 1.:

Time of Algorithm 1:	37.434 sec.;
Time of Algorithm 2:	73.441 sec.;
Time of Algorithm 3:	33.912 sec.;
Time of Algorithm 4:	50.401 sec.

Part 2. We will use the following task where we swapped the values of ‘a’ and ‘b’:

```
    long a, b, ao, bo, x, y, u, s, t, v, gcd, d;
    long d;
    d = 0;
    for (int i = 1; i < 100000001; i++)
        { ao = i; bo = 200000002 - i;
          b = Math.Abs(bo); a = Math.Abs(ao);
    //here is the source code of
    //every one of Algorithms 1-4
    d += gcd; }
    Console.WriteLine(d);
```

Results of Part 2.:

Time of Algorithm 1:	36.878 sec.;
Time of Algorithm 2:	72.370 sec.;
Time of Algorithm 3:	33.852 sec.;
Time of Algorithm 4:	50.801 sec.

Part 3. Average time of performance

EN = (Part 1.Algorithm N + Part 2.Algorithm N) / 2, where N = 1 to 4 denotes using of Algorithms 1 to 4.

E1 = 37.156 sec.
E2 = 72.9055 sec.
E3 = 33.882 sec.
E4 = 50.601 sec.

Numerical experiments give view that new Algorithms 3 and 4 are much faster than Algorithm 1 and its recursive implementation Algorithm 2 respectively.

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