ANALYSIS OF A QUEUEING MODELS IN THE FOREST

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ABSTRACT

Modeling in queing systems with nature of the forest system. Key areas include their limiting distributions, asymptotic behaviors, modeling issues and applications. The onset of asymptotic modeling for the nature of the forest single server queuing models and then processed to multi server models supporting diffusion approximations developed recently. Our model shows that queues with nature flows have limiting distribution and extreme value maximum. In addition, the diffusion approximation can conveniently model permanent characters such as the queue length or the time distributions in these systems.

Keywords : Key areas, multi server models, approximations.

INTRODUCTION

Queuing theory is the mathematical study of waiting lines, or queues. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted. Queuing theory is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide service. Queuing theory started with research by Agner Krarup Erlang when he created models to describe the Copenhagen telephone exchange. The ideas have since seen applications including telecommunications, traffic engineering, computing and the design of factories, shops, offices and hospitals. Etymology of Queuing System: The word queue comes, via French, from the Latin cauda, meaning tail. The spelling "queuing" over "queuing" is typically encountered in the academic research field. In fact, one of the flagship journals of the profession is named Queuing Systems. Application of Queuing Theory: The public switched telephone network (PSTN) is designed to accommodate the offered traffic intensity with only a small loss. The performance of loss systems is quantified by their grade of service, driven by the assumption that if sufficient capacity is not available, the call is refused and lost. Alternatively, overflow systems make use of alternative routes to divert calls via different paths — even these systems have a finite traffic carrying capacity. However, the use of queuing in PSTNs allows the systems to queue their customers' requests until free resources become available. This means that if traffic intensity levels exceed available capacity, customer's calls are not lost; customers instead wait until they can be served. This method is used in queuing customers for the next available operator. A queuing discipline determines the manner in which the exchange handles calls from customers. It defines the way they will be served, the order in which they are served, and the way in which resources are divided among the customers. Here are details of four queuing disciplines: First in first out: This principle states that customers are served one at a time and that the customer that has been waiting the longest is served first.

Last in first out: This principle also serves customers one at a time; however the customer with the shortest waiting time will be served first. Also known as a stack. Processor sharing: Service capacity is shared equally between customers. Priority: Customers with high priority are served first. Queuing is handled by control processes within exchanges, which can be modeled using state equations. Queuing systems use a particular form of state equations known as a Markov chain that models the system in each state. Incoming traffic to these systems is modeled via a Poisson distribution and is subject to Erlang's queuing theory assumptions viz. • Pure-chance traffic – Call arrivals and departures are random and independent events. • Statistical equilibrium – Probabilities within the system do not change. • Full availability – All incoming traffic can be routed to any other customer within the network. • Congestion is cleared as soon as servers are free. Classic queuing theory involves complex calculations to determine waiting time, service time, server utilization and other metrics that are used to measure queuing performance. Queuing networks: Networks of queues are systems a number of queues are connected by customer routing. When a customer is serviced at one node it can join another node and queue for service, or leave the network. For a network of m the state of the system can be described by an m–dimensional vector $(x_1,x_2,...,x_m)$ where xi represents the number of customers at each node. The first significant results in this area were Jackson networks, for which an efficient product-form stationary distribution exists and the mean value analysis which allows average metrics such as throughput and sojourn times to be computed. If the total number of customers in the network remains constant the network is called a closed network and has also been shown to have a product–form stationary distribution in the Gordon–Newell theorem. This result was extended to the BCMP network where a network with very general service time, regimes and customer routing is shown to also exhibit a product-form stationary distribution. Networks of customers have also been investigated; Kelly networks where customers of different classes experience different priority levels at different service nodes.

TREE PLANTING AND TREE CUTTING PROCESS

Let x(t) be the number of the trees at time, trees at time t] in which three types of events occur namely, tree planting, tree cutting and sapling fail to grow.

Then the continuous time discrete random process {X(t)}. With state space {0,1,2,….} is called a Tree planting, tree cutting and sapling fail to grow if the following postulates are satisfied.

If $X(t) = n$, that is the size of the forest trees at time t is n or the system is in state n. Let λ_n, n = 0,1,2,…. be the rate at which tree planting occurs in the state n and μ_n, n = 1,2,… be the rate at which tree cutting occur in state n, and ψ_n,n = 1,2, … be the rate of which sapling fail to grow .

i. P[1 tree planting in (t, t+h)] = $\lambda_n h + O(h)$
ii. P [0 tree planting in (t, t +h)] = $1- \lambda_n h + O (h)$
iii. P [2 or more tree planting in (t,t+h)] = O (h)
iv. Tree planting in (t,t+h) are independent of time since the last tree planting
v. P[1 tree cutting in (t,t+h)] = $\mu_n h+ O (h)$
vi. P [0 tree cutting in (t, t+h)] = $1- \mu_n h+ O(h)$
vii. P [2 or more tree cutting in (t, t+h)] = O (h)
viii. Tree cutting occurring in (t, t+h) are independent of time since the last tree cutting
ix. p [1 sapling fails to grow in (t, t+h) = $\Psi_n h + 0(h)$
x. p [0 sapling fails to grow in (t, t+h) = $1-\Psi_n + 0(h)$
xi. p [2 or more sapling in (t, t+h) = 0(h)
xii. Sapling fails to grow occuring in (t, t+h) are independent of time since the last sapling fails to grow.
xiii. Tree planting, tree cutting and sampling fails to grow occur independently of each other at any time.

Probability Distribution of X(t)

Let X(t) be a tree planting, tree cutting and sapling fails to grow.

Let $X(t) = n$ be the event with correcponding probability $P_n(t)=P[X(t)=n]$

Then $P_n(t+h) = P[X(t+h) = n]$ be the probability that the size of the forest is n at time t+h

The event $X(t+h) = n$ means that the process will be in state n at time t+h if one of the following mutually exclusive and exhaustive events occurs.

i. $X(t) = n$ and no tree planting or tree cutting and sapling fails to grow in (t,t+h)
ii. $X(t) = n+1$; no tree planting, no tree cutting and no sapling in fails grow in (t,t+h)
iii. $X(t) = n-1$; no tree planting, one tree cutting and no sapling in fails grow in (t, t+h)
iv. $X(t) = n$; 1 tree planting and 1 tree cutting and no sapling fails to grow in (t, t+h)
v. $x(t) = n-1$; 1 tree planting, 1 tree cutting and 1 saplings fails to grow.
vi. $x(t) = n-2$; no tree planting, 1 tree cutting and 1 saplings fails to grow
vii. $x(t) = n$; 1 tree planting, no tree cutting and 1 sapling fails to grow.
viii. $x(t) = n-1$; no tree planting, no tree cutting and 1 sapling fails to grow.

Then , $P[X(t)=n] = P_n(t+h)$

$$\begin{aligned}
= \; & P_n(t)\,[1-\lambda_n h + O(h)]\,[1-\mu_n h+ O(h)]\,[1-\Psi_n h + 0(h)] + \\
& P_{n-1}(t)\,[\lambda_{n-1} h + O(h)]\,[1-\mu_{n-1} h + O(h)]\,[1-\Psi_{n-1}h + 0(h)]+ \\
& P_{n+1}(t)\,[1-\lambda_{n-1} h + O(h)]\,[\mu_{n+1} h + O(h)] + [1-\Psi_{n-1}h + 0(h)]+ \\
& P_n(t)\,[\lambda_n h + O(h)]\,[\mu_n h+ O(h)]\,[1-\Psi_n h + 0(h)] + \\
& P_{n-1}(t)\,[\lambda_{n-1} h + O(h)]\,[\mu_{n-1} h + O(h)]\,[\Psi_{n-1}h + 0(h)] + \\
& P_{n-2}(t)\,[\lambda_{n-2} h + O(h)]\,[\mu_{n-2} h + O(h)]\,[\Psi_{n-2}h + 0(h)] + \\
& P_n(t)\,[\lambda_n h + O(h)]\,[1-\mu_n h + O(h)]\,[\Psi_n h + 0(h)] + \\
& P_{n-1}(t)\,[1-\lambda_{n-1} h + O(h)]\,[1-\mu_{n-1} h + O(h)]\,[\Psi_{n-1}h + 0(h)]
\end{aligned}$$

[$\therefore P_n$ (t+h) = P[(i)+(ii)+(iii)+(iv)+(v) + (vi) + (vii) + (viii)]

$= \quad P_n(t)\,[1-\lambda_n h - \mu_n h - \Psi_n h] + O(h) + P_{n-1}(t)\,[\lambda_{n-1} h] + O(h) + P_{n+1}(t)\,[\mu_{n+1}(h)] + O(h) + P_n(t) + \ldots\ldots..$

Since $O(h)\,[1-\lambda_n h + O(h)] = O(h)$ & $\lambda_n\mu_n h^2 - \lambda_n O(h) + O(h) = O(h)$

$\Rightarrow P_n(t)\,[1-\lambda_n h - \mu_n h] + O(h) + P_{n-1}(t)\,[\lambda_{n-1} h + P_{n+1}(t)\,\mu_{n+1} h + O(h) + \ldots\ldots..$

$\therefore\ P_n(t+h) - P_n(t) = -P_n(t)\,(\lambda_n + \mu_n)h + P_{n-1}(t)\,\lambda_{n-1}(t) + \mu_{n+1}\,h + O(h) + \ldots\ldots..$

$$\Rightarrow \frac{P_n(t+h)-P_n(t)}{h} = -P_n(t)\,(\lambda_n+\mu_n) + P_{n-1}(t)\,\lambda_{n-1} + P_{n+1}(t)\,P_{n-1}(t)\,\mu_{n+1} + O(h)/h$$

$$\lim_{n\to\infty} \frac{P_n(t+h)-P_n(t)}{h} = -(\lambda_n+\mu_n+\Psi_n) + P_n(t) + \lambda_{n-1}P_{n-1}(t) + P_{n-1}(t)\,\mu_{n+1} + \ldots\ldots$$

i.e., $P_n^1(t) = -(\lambda_n+\mu_n+\Psi_n)\,P_n(t) + \lambda_{n-1}P_{n-1}(t) + P_{n+1}(t)\,\mu_{n+1} + P_{n-2}(t)\,\Psi_{n-2}$ …(1)

This is true of all $n \geq 1$, since the state space has 0, we find $P_o(t+h)$ by a similar reasoning.

$$P_0(t+h) = P_0(t)\,[\,1-\lambda_0 h + O(h)] + P_1(t)\,[\,1-\lambda_0 h + O(h)]\,[\mu_1 h + O(h)]$$

$$= P_0(t) - \lambda_0 hP_0(t) + P_1(t)\,\mu_1 h + O(h)$$

$$\Rightarrow P_0(t+h) - P_0(t) = -\lambda_0 h\,P_0(t) + \mu_1 hP_1(t) + O(h)$$

$$\frac{P_0(t+h) - P_0(t)}{t} = \lambda\,P_0(t) + \mu_1 P_1(t) + \frac{O(h)}{h}$$

$$\lim_{n\to\infty} \frac{P_0(t+h) - P_0(t)}{t} = \lambda_0\,P_0(t) + \mu_1 P_1(t)$$

i.e. $P_0^1(t) = -\lambda_0\,P_0(t) + \mu_1\,P_1(t)$ ………(2)

Solving (1) and (2) we get $P_n(t)$ for $n \geq 0$, which gives the probability distribution of X(t) .

Pure – tree planting and Pure – tree cutting Process

In a tree planting – tree cutting process if the tree cutting rates $\mu_n = 0$ for all n = 1,2,3…. then the process is called a pure – tree planting process and if the tree planting rates $\lambda_n = 0$ for all $n = 0,1,2,\ldots$ then the process sis called a pure – tree cutting process. That is if arrival alone is considered, the process is called pure tree planting process and if departure tree cutting alone is considered, the process is called pure – tree cutting process.

Probability $P_n(t)$ of Pure – Tree planting Process

Let X(t) is a pure tree planting process

$\therefore \mu_n = 0$ for all $n = 1,2,3\ldots$

Let the tree planting rate $\lambda_n = \lambda$ for all $n = 0,1,2,3\ldots$

The initial conditions are $P_0(0)= 1$, $P_j(0) = 0$ for $j \neq 0$

Then $P_n^1(t) = -\lambda P_n(t) + \lambda P_{n-1}(t)$; $n \geq 1$ ....(3)

[$\therefore$ by (1) and (2)]

and $P_0^1(t) = -\lambda P_0(t)$; $n = 0$ ......(4)

Now

Let $[P_n(t)] = \int_0^\infty e^{-st} P_n(t)\, dt = P_n(s)$

and $L[P_n^1(t)] = SP_n(s) - P_n(0)$

From (3),

$L[P_n^1(t)] = -\lambda L[P_n(t)] + \lambda L[P_{n-1}(t)]$

$\Rightarrow S P_{n(s)} - p n(0) = -\lambda P_n(s) + \lambda P_{n-1}(s)$

$\Rightarrow S P_n(s) = -\lambda P_n(s) + \lambda p_{n-1}(s)$ ($\therefore P_n(0) = 0$)

$\Rightarrow (\lambda+s) P_n(s) = \lambda P_{n-1}(s)$

$P_n(S) = \frac{\lambda}{\lambda+s} P_{n-1}(s)$ ......(5)

(4) $\Rightarrow$

$L[P_0^1(t)] = -\lambda L[P_0(t)]$

$\Rightarrow S P_0(s) = P_0(o) = -\lambda P_o(s)$

$\Rightarrow P_0(s)\ [\,s+\lambda] = 1$

$\Rightarrow P_o(s)\ = \frac{1}{s+\lambda}$

Now,

$$p_n(s) = \frac{\lambda}{\lambda+s} \cdot \frac{\lambda}{\lambda+s}\ P_{n-2}\ (s) \qquad (3) \qquad \text{(by (5))}$$

$$= \frac{\lambda^2}{(\lambda+s)^2} \cdot \frac{\lambda}{\lambda+s}\ P_{n-3}\ (s) \qquad (3)....$$

$$= \frac{\lambda^n}{(\lambda+s)^n}\ P_0\ (s)$$

$$= \frac{\lambda^n}{(\lambda+s)^n}\ \frac{1}{\lambda+s}$$

$$P_n(s) = \frac{\lambda^n}{(\lambda+s)^{n+1}} \quad ; n \geq 0$$

If y is the (n+1) stage Erlang random Variable then

$$f_y(t) = \frac{\lambda e^{-\lambda t}\ (\lambda t)^{a-1}}{\Gamma\, a} \quad ; \ \ y \geq 0\ \lambda > 0$$

$$\Rightarrow \quad f_y(t) = \frac{\lambda^{n+1} e^{-\lambda t}\ t^n}{\Gamma\ (n+1)} \qquad (\therefore\ a = n+1)$$

$$= \frac{\lambda^{n+1} e^{-\lambda t}\ t^n}{n!}$$

then $L\ [f_y(t)]$

$$= L\left[\frac{\lambda^{n+1}}{n!}\quad e^{-\lambda t}\ t^n\right] = \frac{\lambda^{n+1}}{n!}\quad L\ [e^{-\lambda t}\ t^n]$$

$$= \frac{\lambda^{n+1}}{n!}\ \ \frac{n!}{s+\lambda^{n+1}}$$

$$\therefore \left[L\ [e^{-at} t^n]\ \ \frac{n!}{(s+a)^{n+1}}\right]$$

$$= \frac{\lambda^{n+1}}{(s+\lambda)^{n+1}}$$

$$=> \left[\frac{L\ [f y(t)]}{\lambda}\right] = \frac{\lambda^n}{(s+\lambda)^{n+1}}$$

=> $L\left[\frac{1}{\lambda}\, fy(t)\right]$ = $P_n(s) = L\,[P_n(t)]$

=> $\frac{1}{\lambda}\, fy(t)$ = $P_n(t)$

$P_n(t)$ = $\frac{1}{\lambda}\ \frac{\lambda^{n+1}}{n!}\ \ e^{-\lambda t}\, t^n = \frac{\lambda^n}{n!}\ e^{-\lambda t}\, t^n$

⇨ $P\,[x(t) = n]$ = $\frac{e^{-\lambda t}\,(\lambda t)^n}{n!}$

which is a poisson process.

Probability Function of Pure – tree cutting Process

Let X(t) be a pure tree cutting Process

$\therefore\ \lambda_n = 0,\ n = 0, 1, 2 \ldots$..

Let the tree cutting rate $\mu_n = \mu$, for all $n = 1, 2 \ldots$..

$P_n^1\,(t) = -\,\mu_n P_n\,(t)$

$P_r^1\,(t) = -\,\mu P_r(t) + \mu\, P_{r+1}\,(t)\ ;\ r = 1, 2, \ldots\, n\text{-}1$

$P_0^1\,(t) = \mu\, P_1(t)$

Taking Laplace transform,

$P_r(s) = \frac{1}{\mu}\ \left[\frac{\mu}{s+\mu}\right]^{n-r+1}$ $\qquad r = 1, 2, \ldots n$

If y is an (n-r+1) state Erlang process with parameter μ, then

$f_y(t) = \mu e^{-\mu t}\ \frac{(\mu t)^{n-r}}{\Gamma_{(n-r+1)}} = e^{-\mu t}\ \frac{(\mu t)^{n-r+1}}{(n-r)!} = \frac{\mu^{n-r+1}}{(n-r)!}\ e^{-\mu t}\ t^{n-r}$

$L[f_y(t)] = L\left[\frac{\mu^{n-r+1}}{(n-r)!}\ e^{-\mu t}\ t^{n-r}\right] = \frac{\mu^{n-r+1}}{(n-r)!}\ L\ [e^{-\mu t}\ t^{n-r}]$

$$= \frac{\mu^{n-r+1}}{(n-r)!} \frac{(n-r)!}{(s+\mu)^{n-r+1}} = \frac{\mu^{n-r+1}}{(s+\mu)^{n-r+1}}$$

$$= \frac{1}{\mu} L[f_y(t)] = \frac{\mu^{n-r+1}}{(s+\mu)^{n-r+1}}$$

$$= L\left[\frac{1}{\mu} f_y(t)\right] = P_r(s)$$

$$= L\left[\frac{1}{\mu} f_y(t)\right] = L[P_r(t)] \frac{1}{\mu} f_y(t) = P_r(t)$$

$$P_r(t) = \frac{1}{\mu} f_y(t) = \frac{1}{\mu} \frac{\mu^{n-r+1}}{(n-r)!} e^{-\mu t} t^{n-r}$$

$$= \frac{\mu^{n-r+1}}{(n-r)!} e^{-\mu t} t^{n-r} = \frac{e^{-\mu t} (\mu t)^{n-r}}{(n-r)!}; r = 1, 2, 3, \ldots$$

CONCLUSION

The evaluation of queuing system in an establishment is necessary for the betterment of the establishment. As it concerns the case study company, the evaluation or analysis of their queuing system shows that the case study. This will also increase the efficiency of the establishment due to the appreciation in their serve to the customers as and at when due.

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