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A NEW DEFINITION OF CURVATURE FOR POLYGONS, CONVEX POLYHEDRA AND HYPERSOLIDS

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ABSTRACT

Considering a balance between pressure drop and wall shear stress in a duct of arbitrary cross-section shape for the laminar flow of a Newtonian liquid, we proposed a definition of curvature for polygons based on the hydraulic radius. We extended this definition in 3D for the case of regular convex polyhedra i.e. the five Platonic Solids. As found for polygons and according to isoperimetric theorem, curvature radius corresponds to the radius of inscribed sphere. We tried then to find a general form available for dimensions n>3 based on the cases of hypersphere and hypercube. Finally, results are discussed considering the influence of ducts cross-section curvature on the laminar flow stability in ducts of complex cross-section shape. Results obtained appeared in good agreement with experimental results obtained for turbulence birth in such channels.

1. INTRODUCTION.

In an important paper on partial differential equations in Physics [1], H. Poincaré insisted on the importance of solids shape geometrical description when using diffusion equations for heat (Fourier), momentum (Newton)and solute (Fick) transfers. Considering heat diffusion Fourier problem, he clearly showed how the ratio of surface to volume: S/V (m^{-1}) is important.

For momentum transfer (Newton law) in the laminar flow regime, Navier-Stokes equation reduces to a diffusion equation where temperature (scalar field) is replaced by velocity (vector field). Diffusion equations are of major interest in Physics and H. Poincaré's approach remains today of very important to understand mathematical aspects of these second order partial differential equations.

Considering the Laplacian operator involved in all diffusion equation, it can be interpreted as a local value of the difference between local value and mean value of the field and as the mean value of local field curvature. Mathematical interpretation of Δf also gives the ratio S/V introduced by H. Poincaré.

In Fluid Mechanics, depending on ducts cross-section geometry or shape, the fully established velocity field can be very simple (laminar flow in a tube gives a parabolic velocity profile: Poiseuille flow) or much more complex (even for the laminar flow in a duct of rectangular cross section). In recent papers, Delplace [2] and Delplace & Srivastava [3] showed that ducts cross section curvature is a major parameter for the viscous liquid flow characterisation using the well-known Reynolds dimensionless number. The consequence of this approach is the great importance of shapes curvature definition for 1D (duct of circular cross-section), 2D (ducts of any cross-section shape but of cylindrical geometry) and 3D (non-cylindrical ducts) flows. Curvature definition of polygons and polyhedra is then of major interest even if this problem remains unsolved [4,5,6].

Moreover, a rigorous definition of polygons and polyhedra curvature must agree with well-known isoperimetric inequalities. In 2D, the famous isoperimetric quotient is: $Q_{2D} = 4\pi S/P^2$ and in 3D, we have: $Q_{3D} = 36\pi V^2/S^3$. In these relationships, P is the perimeter of the compact geometry, S its surface and V its volume. These ratios are always < 1 for n-gons, equality is only obtained for discus (2D) and sphere (3D) giving the largest surface and volume for respectively the smallest perimeter and surface.

In the first chapter of the present paper, we will deduce, from a balance between pressure and viscous friction stress in a pipe of arbitrary cross-section shape a new definition of curvature whatever is the planar 2D geometry. This definition will be analysed and discussed through examples and isoperimetric quotient Q_{2D} given above.

In the second chapter, we will try to extend the previous approach to 3D compact convex geometries made of the five Platonic solids: tetrahedron, cube or regular hexahedron, octahedron, dodecahedron and icosahedron. Using isoperimetric quotient Q_{3D} given above, a definition of Platonic solids curvature will be proposed and discussed.

Finally, in the last chapter, we will propose an essay for the definition of curvature in dimension $n > 3$.

2. A CURVATURE DEFINITION FOR POLYGONS.

In Fluid Mechanics, the laminar flow of a viscous Newtonian or non-Newtonian liquid in a regular or often called cylindrical duct of arbitrary cross-section shape is a major problem widely discussed in literature [6,7,8,9]. Under steady state conditions, meaning that $\partial\vec{u}/\partial t = 0$, where $\vec{u}$ is the liquid local velocity, the scalar velocity field $u(x, y)$ in Cartesian coordinates (z coordinate being taken in the fluid flow direction) can be obtained by solving the following Poisson partial differential equation:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{\Delta P}{\eta L} \qquad (1)$$

In this equation, ΔP (Pa) is the pressure drop in the pipe of length L (m) and η (Pa.s) is the Newtonian liquid dynamic viscosity or shear viscosity. Equation (1) can be solved analytically for some simple polygonal geometries like rectangular or triangular ducts but solutions are always of complex mathematical form [7]. For example, in a duct of rectangular cross-section with cross-section length $2b$ and cross-section width $2a$, we obtain by use of Saint-Venant [8] resolution method:

$$u(x, y) = \frac{16\Delta P a^2}{\eta L \pi^3} \sum_{n=1,3,5,\dots}^{+\infty} \frac{(-1)^{(n-1)/2}}{n^3} \left\{1 - \frac{ch\left(\frac{n\pi y}{2a}\right)}{ch\left(\frac{n\pi b}{2a}\right)} \cos\left(\frac{n\pi x}{2a}\right)\right\} \qquad (2)$$

This equation allows all Mechanical characteristics of the flow to be calculated and of course the calculation of pressure drop along the pipe as reported in [2] and [6]. But it has the inconvenient to be only available for a restricted number of geometries where an analytical solution of (1) exists.

Another way (more rough) to consider this problem is to make a balance between normal pressure loses in the pipe and tangential viscous stress as described in the following figure:

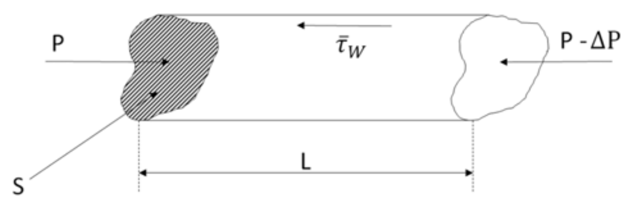
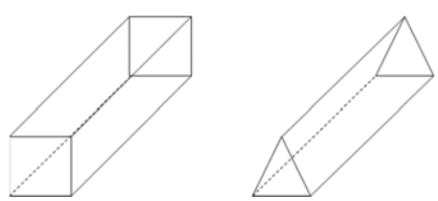


Figure 1: Pressure drop and viscous friction stress for a pipe of arbitrary cross section.

In this figure, S (m²) is the arbitrary cross section area, P (Pa) is the liquid pressure inside the pipe, ΔP (Pa) is the pressure drop after flow length L (m) and $\bar{\tau}_W$ (Pa) is the average wall shear stress caused by viscosity and the existence of a velocity gradient at the wall.



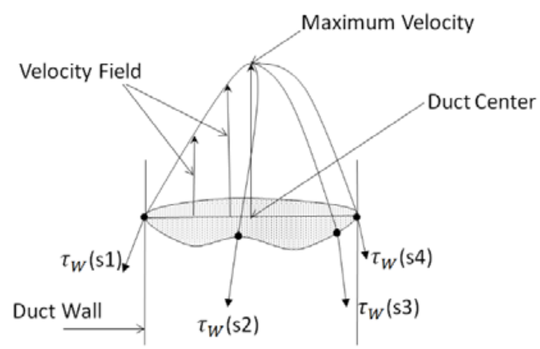
In this schematic representation, the duct is considered cylindrical meaning that all its axis along length L are parallel to each other and its cross-section geometry can be of any shape. We can of course consider regular cross sections like a square duct or an equilateral triangular duct as illustrated in the following figure.

Figure 2: Square and equilateral triangular ducts as examples of regular 2D geometries.

These regular geometries are important because it exists analytical solutions of equation (1) as showed above, moreover, experimental results were obtained through pressure drop and flow-rate measurements with Newtonian and non-Newtonian liquids [9]. These results are of major importance because they validate theoretical and numerical calculations and they give a strong basis for all the approach of the laminar flow of viscous liquids in ducts of complex cross section shape reported for example in [6,10].

The balance of pressure and viscous friction stress at the wall can easily be written as follows:

$$S\,.\Delta P = \oint \tau_W(s)\, L\, ds \qquad (3)$$



In this equation, s is the curve coordinate along the surface S perimeter of length C and $\tau_W(s)$ is the wall shear stress at each point of the cross-section perimeter. The following figure illustrates $\tau_W(s)$ along an arbitrary cross-section shape.

Figure 3: Schematic representation of $\tau_W(s)$ along an arbitrary cross section.

The mean value of function $\tau_W(s)$ along perimeter of length C is defined as:

$$\bar{\tau}_W = \frac{1}{C}\oint \tau_W(s)\; ds \quad \Rightarrow \quad \oint \tau_W\,(s)\,ds = C\,\bar{\tau}_W$$

Reporting in equation (3) gives:

$\Delta P\ S = L\ C\ \bar{\tau}_W$

And then:

$$\bar{\tau}_W = \frac{\Delta P\ S}{L\ C} \qquad (4)$$

When applied in the case of a circular cross-section, this equation corresponds to the case of Poiseuille flow and because of the symmetry of parabolic velocity field, the mean wall shear stress is equal to the local value and we have the following relationship:

$$\forall\ s\ \in [0, \pi D] \quad \bar{\tau}_W = \tau_W(s) = \frac{\Delta P\ D}{4\ L} \qquad (5)$$

Comparison of equations (4) and (5) gives in Fluid Mechanics the well-known definition of hydraulic diameter called D_H:

$$D_H = \frac{4\ S}{C} \qquad (6)$$

Using this quantity in equation (4) gives the general form of average wall shear stress in a cylindrical duct of arbitrary cross-section shape whatever the shape:

$$\bar{\tau}_W = \frac{\Delta P\ D_H}{4\ L} \qquad (7)$$

From this equation, we propose to define $R_H = D_H/2$ as the radius of curvature of any cross-section and then of any polygon. Curvature in 2D of any shape can then be defined as:

$$C_H = \frac{1}{R_H} = \frac{C}{2\ S} \qquad (8)$$

Let us use equation (8) in the case of regular convex polygons with n sides $n \geq 3$. If we call a the side length and h the radius of inscribed circle, we have the following classical relationships $\forall\ n \geq 3$:

$C = n\ a$

$$S = n\, a\, \frac{h}{2}$$

With $h = \frac{a}{2\tan(\pi/n)}$

Then, from equation (8), curvature of any regular convex polygon can be defined as:

$$C_H = \frac{C}{2\,S} = \frac{n\,a}{n\,a\,h} = \frac{1}{h} = \frac{2\tan(\pi/n)}{a}$$

With $\lim_{n\to+\infty} C_H = 0$

This last result is of major interest, it means that when you increase the number of sides having the same length a , curvature reaches zero corresponding to a flat geometry. This result could be interpreted as a scaling effect well known in fractal geometry. Roughly speaking, curvature tends to vanish for regular convex polygons having a very large number of sides.

But this definition of curvature has also to agree with isoperimetric theorem and particularly with the isoperimetric ratio Q_{2D}:

$$Q_{2D} = \frac{4\,\pi\,S}{C^2} \qquad (9)$$

As recalled in the introduction of this paper, $Q_{2D} < 1$ for any polygon and $Q_{2D} = 1$ for the circle. Using our definition of hydraulic radius (equation 8) in equation (9) we have:

$$Q_{2D} = \frac{2\,\pi\,R_H}{C} = \frac{C_i}{C} \qquad (10)$$

In the case of a regular convex polygon, Q_{2D} is then the ratio of inscribed circle perimeter C_i to polygon perimeter C which can be rewritten as followed:

$$Q_{2D} = \frac{2\,\pi\,a}{2\tan(\pi/n)\,n\,a} = \frac{\pi}{n\tan(\pi/n)}$$

Considering that: $\tan x \sim x + \frac{x^3}{3} + o(x^5)$ gives

$$\lim_{n\to+\infty} Q_{2D} = \frac{\pi}{n\,\frac{\pi}{n}} = 1$$

This surprising result shows that when $n \to +\infty$ the polygon perimeter and the inscribed circle perimeter are identical. Associated with the previous result of a null curvature of a regular convex polygon also when $n \to +\infty$ it gives important topological information about the behaviour of compact convex surfaces.

Finally, considering analytical solutions of equation (1) obtained for simple regular convex polygons and Fluid Mechanics experimental results obtained by Delplace et al. [9] for also regular geometries, definition of hydraulic radius $R_H = 2\,S/C$ as the possible radius of curvature of any compact curve of surface S and perimeter C appears possible. Moreover, this definition is well in agreement with both Delplace [2] analysis of Reynolds number in terms of curvature and isoperimetric theorem.

2. A CURVATURE DEFINITION FOR REGULAR CONVEX POLYHEDRA.

The case of 3D solids discussed in the introduction of this publication when considering H. Poincaré's analysis of diffusion problems is clearly much more complex than the previous one. But in Fluid Mechanics and more specifically in Chemical Engineering, a notion of hydraulic diameter was required for specific problems encountered in complex transport phenomena like those in porous media or packed beds. Definition given in equation (6) was then extended for practical use in the case of non-cylindrical ducts:

$$D_H = 4\,\frac{V}{S} \qquad (11)$$

This definition involves the ratio V/S as expected from H. Poincaré's calculations and Laplacian operator definition as a mean value and a numerical factor equal to 4 is used by analogy with the definition of hydraulic diameter in 2D given by equation (6). But the difference between equations (6) and (11) is major, equation (6) comes from a rigorous balance between normal pressures and tangential friction stress as reported in chapter 1 but equation (11) comes from an analogy or extension of equation (6).

It is then interesting and important to test for the case of well-known solids having perfectly defined geometrical characteristics i.e. the five Platonic Solids. These five 3D geometries are the unique regular convex polyhedra. Considering a Platonic solid with n faces, A being the area of each face and R being the in-radius of the solid i.e. the radius of the inscribed sphere, we have:

The volume V and total surface S defined by,

$V = \frac{n\,A\,R}{3}$ and $S = n\,A$

Then, considering equation (11),

$D_H = \frac{4}{3}\,R$ and $R_H = \frac{D_H}{2} = \frac{2}{3}\,R$

Per our definition of curvature in 2D as the inverse of inscribed circle radius, we can calculate the curvature of the five Platonic Solids as the inverse of the square of inscribed sphere radius:

$$C_H = \frac{1}{R_H^2} = \frac{9/4}{R^2} \qquad (12)$$

This curvature definition must agree with isoperimetric ratio in 3D given in the introduction of this paper i.e.:

$$Q_{3D} = \frac{36\,\pi\,V^2}{S^3} \qquad (13)$$

From equation (11), equation (13) can be rewritten as followed:

$$Q_{3D} = \frac{9\,\pi\,R_H^2}{S} = \frac{S_H}{S} \qquad (14)$$

This result gives surface S_H of the equivalent sphere equal to $9\,\pi\,R_H^2$ which is of course impossible because sphere surface is equal to $4\,\pi\,R_H^2$. The consequence is that the definition of hydraulic diameter given by equation (11) is not right. This result shows how powerful and important is the use of isoperimetric theorem and isoperimetric ratio for establishment of a curvature definition as expected in the introduction of this paper. Let us then define the hydraulic diameter in 3D by the general following form:

$$D_H = k \, \frac{V}{S}$$

Then, we have: $\frac{V^2}{S^2} = \frac{D_H^2}{k^2}$ we can replace in (13) giving:

$$Q_{3D} = \frac{36\,\pi}{k^2} \cdot \frac{4\,R_H^2}{S}$$

And because $S_H = 4\,\pi\,R_H^2$ we have:

$$k = 6$$

This important result shows that hydraulic diameter in 3D must be defined as:

$$D_H = 6\,\frac{V}{S} \qquad (15)$$

To agree with isoperimetric theorem in 3D.

From this result giving hydraulic diameter definition different from the usual value given by equation (11), we can easily determine the equivalent radius of a Platonic Solid:

$$R_H = 3\,\frac{V}{S} = 3\,\frac{n\,A\,R}{3}\,\frac{1}{n\,A} = R \qquad (16)$$

As obtained for regular convex polygons, hydraulic radius of Platonic Solids corresponds to the radius of inscribed sphere and therefore, we can define curvature as the inverse of square in-radius:

$$C_{3D} = \frac{1}{R^2} = \frac{S^2}{9\,V^2} \qquad (17)$$

Using equation (17) and well-known geometrical characteristics of Platonic Solids (numbered from 1: Tetrahedron to 5: Icosahedron), it is then of interest to compare them and to analyse the results. The following table contains formula for R (in-radius or radius of inscribed sphere), R' (out-radius or radius of circumsphere), S the surface, V the volume, Q_{3D} the isoperimetric ratio being also the ratio of circumsphere volume and Solid volume and C_{3D}. The quantity called a is the side length of each face of the solid and φ is the gold number equal to $(1+\sqrt{5})/2$.

	R	R'	S	V	Q_{3D}	C_{3D}
Tetrahedron 1	$\frac{a}{2\sqrt{6}}$	$\frac{\sqrt{3}}{2\sqrt{2}}a$	$\sqrt{3}a^2$	$\frac{\sqrt{2}}{12}a^3$	$\frac{36\pi}{216\sqrt{3}}$	$\frac{24}{a^2}$
Hexahedron 2	$\frac{a}{2}$	$\frac{\sqrt{3}}{2}a$	$6a^2$	a^3	$\frac{36\pi}{216}$	$\frac{4}{a^2}$
Octahedron 3	$\frac{a}{\sqrt{6}}$	$\frac{a}{\sqrt{2}}$	$2\sqrt{3}a^2$	$\frac{\sqrt{2}}{3}a^3$	$\frac{72\pi}{216\sqrt{3}}$	$\frac{6}{a^2}$
Dodecahedron 4	$\frac{\sqrt{25+11\sqrt{5}}}{2\sqrt{10}}a$	$\frac{\varphi^2}{2}a$	$3\sqrt{25+10\sqrt{5}a^2}$	$\frac{15+7\sqrt{5}}{4}a^3$	$\pi\frac{5^{0.75}}{75}\varphi^{3.5}$	$\frac{40/(25+11\sqrt{5})}{a^2}$
Icosahedron 5	$\frac{3+\sqrt{5}}{4\sqrt{3}}a$	$\sqrt{\frac{5+\sqrt{5}}{2}\frac{a}{2}}$	$5\sqrt{3}a^2$	$5\,\frac{3+\sqrt{5}}{12}\,a^3$	$\frac{\pi}{15\sqrt{3}}\varphi^4$	$\frac{48/(3+\sqrt{5})^2}{a^2}$

The Q_{3D} column shows that isoperimetric ratio increases from a minimum value $\cong 0.302$ for Tetrahedron to a maximum value $\cong 0.829$ for Icosahedron. This well-known result representing the filling capacity of Platonic Solids, means that for a given surface S, they always have a volume lower than the sphere volume.

Considering C_{3D} formula, highest curvature is found for Tetrahedron and lowest value for Dodecahedron. This result agrees with well-known property of Dodecahedron having a lower in-sphere radius than Icosahedron. The filling ration being $\frac{\varphi}{\pi}\sqrt{\frac{5}{3}} \cong 0.665$ for Dodecahedron and $\frac{\sqrt{\varphi}\,\sqrt[4]{5}}{\pi} \cong 0.605$ for Icosahedron. Curvature values follow filling ratio tendency for all Platonic Solids. The lowest filling giving the highest curvature.

Considering now the three Platonic Solids having the same face geometry (equilateral triangle), curvature increases when the number of faces decreases from 20 for Icosahedron to 8 for Octahedron and 4 for Tetrahedron. These three polyhedra are often characterized by their type of volume which is identical and called P3.

Finally, definition of polyhedra curvature given in equation (15) allows to generalise Appolonium rule established for Icosahedron (number 5 in the table) and Dodecahedron (number 4 in the table). If they have the same in-radius, then:

$$\frac{V_5}{V_4} = \frac{S_5}{S_4} = \sqrt{\frac{3}{10}\left(5 - \sqrt{5}\right)} \qquad (18)$$

From our definition of curvature, this result is straightforward and available for all Platonic Solids. Two regular convex polyhedra having the same in-radius will have the same curvature and the same ratio S/V.

Another case of interest to test how powerful is equation (15) is the cylinder geometry. Let us consider a compact cylinder of radius R and length L: we have

$V = \pi\, R^2\, L$ and $S = 2\,\pi\, R\, L + 2\,\pi\, R^2$

Then: $D_H = 6\,\frac{V}{S} = \frac{6\,\pi\, R^2\, L}{2\,\pi\, R\, L + 2\,\pi\, R^2}$ and $R_H = \frac{3\, R\, L}{2\, L + 2\, R}$ => $C_{3D} = \frac{1}{R_H^2} = \frac{4}{9\, R^2} + \frac{8}{9\, R\, L} + \frac{4}{9\, L^2}$ (20)

If we consider now the cylinder of length $L = 2\, R$ able to contain an inscribed sphere of radius R as represented in the following figure:

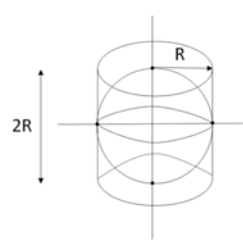


Figure 4: sphere inscribed in a cylinder.

Then using $L = 2\, R$ in equation (20) gives:

$$C_{3D} = \frac{4}{9\, R^2} + \frac{4}{9\, R^2} + \frac{1}{9\, R^2} = \frac{1}{R^2}$$

Which is the curvature of inscribed sphere as expected.

To conclude this chapter on curvature of regular convex polyhedra, we would like to come back to H. Poincaré's original work [1] and more accurately on page 77 where he defined an important quantity for solving the Fourier problem for heat diffusion in solids. The following ratio we called K is of major interest whatever is the convex solid:

$$K = \frac{6\, K_0\, V}{\pi\, \lambda^5} \qquad (19)$$

With K_0 being a numerical constant and λ the largest distance between two points of the solid surface. Dimension of K is m^{-2} i.e. the dimension of a 3D curvature and, for a sphere, using $K_0 = 4$, we obtain $K = 1/R^2$. This relationship is well in agreement with equation (17) which could be then available for any convex solid.

3. AN ESSAY ON HYPERSOLIDS CURVATURE DEFINITION.

The objective is an attempt to generalize in $\mathbb{R}^n$ curvature definitions obtained for 2D and 3D and given by equations (8) and (15).

As demonstrated above, curvature definition always involves the inverse of inscribed circle or sphere radius. Two geometries appeared particularly well adapted to find a generalized form of equations (8) and (17): the hypersphere and the hypercube. These figures have well established geometrical characteristics i.e. n-volume and n-surface; moreover, it is well-known that inscribed n-sphere in a n-cube will take less volume fraction as n increases. For example, normal sphere ($n = 3$) will take 52.3% of cube volume but only 0.25% for $n = 10$.

Let us then consider the 4-cube with inscribed 4-sphere (volume ratio is 30.8% in that case), a being the 4-cube side length, V being its external surface and W its volume. Considering previous results, we can suppose:

$$C^4 = k\ \frac{V^3}{W^3} = \frac{1}{(a/2)^3} = \frac{8}{a^3} \qquad (21)$$

With C^4 the 4-cube curvature.

From 4-cube geometry, we have: $V = 8\,a^3$ and $W = a^4$. Then, equation (21) can be rewritten as followed:

$$C^4 = k\ \frac{\left(8a^3\right)^3}{(a^4)^3} = \frac{8}{a^3}$$

This relationship allows numerical constant k to be determined and we find: $k = \frac{1}{64}$; giving the following definition of 4-cube curvature:

$$C^4 = \frac{V^3}{64\,W^3} = \frac{1}{R^3} \qquad (22)$$

R being the radius of the inscribed 4-sphere.

From this result, it is then easy to build the general form of a n-curvature C^n equation:

$$C^n = \frac{L_{n-1}^{n-1}}{n^{n-1}\,L_n^{n-1}} = \frac{1}{R^{n-1}} \qquad (23)$$

For example, for $n = 3$, we have:

$$C^3 = \frac{L_2^2}{3^2\,L_3^2}$$

Taking: $L_2 = S$ and $L_3 = V$, it gives:

$$C^3 = \frac{S^2}{9\,V^2}$$

Which corresponds to curvature definition in 3D given by equation (17).

Using equation (23), it is possible to obtain a generalized form of hydraulic diameter in dimension n:

$$D_H^n = 2\,n\ \frac{L_n}{L_{n-1}} \qquad (24)$$

For $n = 2$ we have:

$D_H^2 = 4\ \frac{S}{C}$

Corresponding to equation (6).

These results show that equation (23) could be a way to calculate n-curvature of compact convex regular n-gons, moreover, considering that equation (6) is available whatever is the shape of the compact surface perimeter, it could be a simple and practical way to calculate a curvature for all n-shapes.

4. CONCLUSION

In this paper, we tried to use hydraulic diameter used in Fluid Mechanics as a mean to define a curvature for polygons, polyhedra and n-gons. From a balance between pressure drop and viscous friction in a cylindrical pipe of arbitrary cross-section shape, we established the 2D definition of hydraulic diameter. We extended this definition in 3D for Platonic Solids per the isoperimetric ratio and we obtained $D_H = 6\,V/S$ which is a result quite different from the usual definition used in Fluid Mechanics.

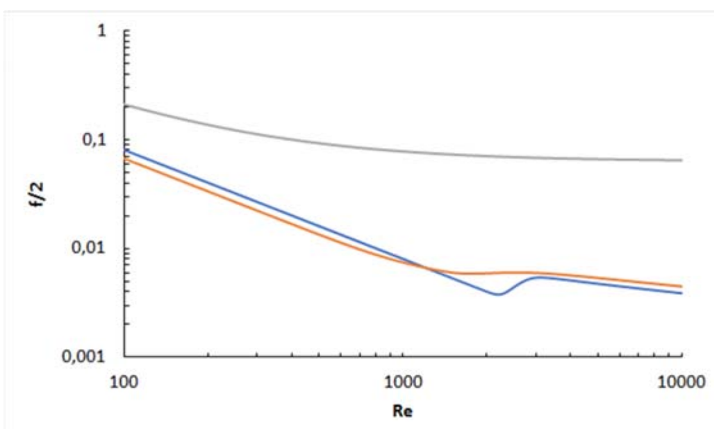
As for regular convex polygons, curvature of the five Platonic Solids involved the inverse of inscribed sphere radius. This important result is also available for a sphere inscribed in a cylinder and it agrees with the formula introduced by H. Poincaré for heat diffusion in solids. It could signify that this definition could be available for any convex solid.

In an essay, we tried to extend the approach to higher dimensions by considering the cases of the hypersphere and hypercube. A general definition of hydraulic diameter and n-curvature was then proposed.

From a practical point of view, these results indicate that for a given n-shape and a given n-volume, n-curvature will always increase as n-surface increase i.e. the perimeter in 2D and the surface in 3D.

Associated to fluid flow stability knowledge showing that turbulence appears more quickly when pipe walls rugosity increase (according to the famous Moody diagram), it could explain and signify a deep link between pipe walls curvature and turbulence birth. This mathematical result is in perfect agreement with Delplace et al. [9] experimental results showing for example, a lower critical Reynolds number in equilateral cross-section duct.

Finally, these results seem to indicate that "fractalization" of geometry is strongly linked to curvature according to Winter [11] approach. From this consideration, pipes wall roughness could be a "fractalization" giving rise to less stable flow and then to turbulence promoting. Moreover, the well-known static turbulence promoters could be considered as devices able to increase curvature and then to fractalize pipe overall geometry. Modified pipes walls being the large-scale way to promote turbulence in agreement with Delplace [9,12] experimental results. The following friction curves experimentally obtained by this author for a tube (blue line), a duct of equilateral triangular cross section (brown line) and a corrugated channel (grey line) are a perfect illustration of this strong link between curvature and fluid flow stability.



Critical Reynolds number value is 2100 for the tube, 800 for the equilateral triangular duct and 30 for the corrugated channel.

Further works are needed to establish the link between ducts curvature and fractal geometry in Fluid Mechanics.

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