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Nano b -Open Sets In Nano Tri Star Topological Spaces

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Abstract

In this paper a new kind of topology is called NanoTri star topology induced by two nano bitopology and is denoted by NT^*_{123}. NT^*_{123}b opensets, NT^*_{123} gb open sets are introduced and studied separation axioms of Nano Tri star topology

Keywords: *NT^*_{123} open sets, NT^*_{123} interior, NT^*_{123} closure , NT^*_{123} b opensets, NT^*_{123} gb open sets*

1. Introduction

The concept of a bitopological space was first introduced by Kelly [4] in 1963. A nonempty set X with two topologies T_1, T_2 is called a bitopological space, where the topology is defined as $T_1 \cup T_2$ and denoted by T_1T_2. As an extension of bitopological space, tri topological space was first initiated by Kovar[2] in 2000, where a nonempty set X with three topology is called a tri topological space. [5]In 2014 Palaniammal and Somasundaram introduced a topology $T_1 \cap T_2 \cap T_3$ in the tri topological space (X, T_1, T_2, T_3) and studied several properties of this topology .

I.N.F. Hameed and Moh. Yahya Abid gives the definition of 123 open set in tri topological spaces .U.D. Tapi , R. Sharma and B. Deole introduce semi open set and pre open set in tri topological space. Stella Irene Mary J introduce a new topology called Tri star topology induced by two bitopology and is denoted by T^*123. notion of Nano topology was introduced by Lellis Thivagar[**10**] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets ,Nano-interior and Nano-closure.He has also defined Nano continuous functions , Nano open mapping , Nano closed mapping and Nano Homeomorphism. K.Buvaneshwari[11] etal S.Chandrasekar[8] et al contributed in Nanobitopological spaces

In this paper, we introduce a new topology called Nano Tri star topology induced by two nano bitopology and is denoted by NT^*_{123}. The various concepts of Nano b open sets in NT^*_{123}- topological space are analyzed.

2. Preliminaries

Definition 2.1: A topology on a non empty set X is a collection T of subsets of X having the following the properties:

1) X and Φ are in T.
2) The union of the elements of any sub collection of T is in T.
3) The intersection of the elements of any finite sub collection of T is in T.

A set X for which a topology T has been specified is called a Topological space.

Definition 2.2.

A subset A of a topological space (X , τ) is called
(i)b-open set[1] if A □ cl (int (A)) ∪ int (cl (A)).
(ii)a generalized b- closed set (briefly gb- closed) [1] if bcl (A) □ U whenever A □ U and U is open in X.

DEFINITION 2.3,[10]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$ .That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$ (iii) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

DEFINITION 2.4 [10]

If (U, R) is an approximation space and X, Y ⊆ U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\phi) = U_R(\phi) = \phi$
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever X ⊆Y
(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(ix) $U_R\, U_R(X) = L_R\, U_R(X) = U_R(X)$
(x) $L_R\, L_R(X) = U_R\, L_R(X) = L_R(X)$

DEFINITION2.5[10] Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X ⊆Y. $\tau_R(X)$ satisfies the following axioms

(i) U, $\phi \in \tau_R(X)$
(ii)The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
(iii)The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call(U, $\tau_R(X)$)) as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets.

DEFINITION2.6

If $(U, \tau_{R_{1,2,3}}(X))$ is a Nano tri topological space with respect to U where and if then

(i) The Nano (1,2)* interior of A is defined as the union of all Nano (1,2)* open subsets of A contained in A and it is denoted by $N\tau_{1,2}int(A)$. $N\tau_{1,2}int(A)$ is the largest Nano (1,2)* open subset of A.

(ii) The Nano (1,2)* closure of A is defined as the intersection of all Nano (1,2)* closed Sets containing A and it is denoted by $N\tau_{1,2}cl(A)$.$N\tau_{1,2}cl(A)$ is the smallest Nano (1,2)*Closed set containing A.

3. NANO TRI STAR TOPOLOGICAL SPACE

In this section we introduce a new topology in (X, T_1,T_2, T_3)

3.1. NT123*-OPEN SETS

Throughout this article we consider nano bitopological spaces $(U, \tau_{R_{1,2}}(X))$ and $(U, \tau_{R_{2,3}}(X))$ for which the nano bitopology elements form a topology.

Definition 3.2:

Let $(U, \tau_{R_1}(X), \tau_{R_2}(X), \tau_{R_3}(X)) = (U, \tau^*_{R_{1,2,3}}(X))$ be a tri topological space. We define a new topology NT123*-called NanoTri star topology induced by two nano bitopology, as follows

$NT^*_{123}\ O\ (X) = [\tau_{R_1}(X) \cup \tau_{R_2}(X)] \cap [\tau_{R_2}(X) \cup \tau_{R_3}(X)]$ where $\tau_{R_1}(X) \cup \tau_{R_2}(X)$ and $\tau_{R_2}(X) \cup \tau_{R_3}(X)$ are nano bitopology defined on the nano bitopological spaces $(U, \tau_{R_{1,2}}(X))$ and $(U, \tau_{R_{2,3}}(X))$ respectively.

Example 3.3.

Let U = {p, q , r, s, t}, U/R_1= { {p} ,{q , r, s}, {t}}.

LetX_1={p,q}⊆U.Then $\tau_{R_1}(X)$={U, ϕ,{p},{p,q,r,s}, {q,r,s}

LetX_2={p,r}⊆U.Then $\tau_{R_2}(X)$={U, ϕ,{p},{p,q,r,s} ,{q,r,s }}.

Let X_3 = {q , r}⊆ U. Then $\tau_{R_3}(X)$={U, ϕ ,{q,r,s} }.

$NT^*_{123}\ O(X) = (\tau_{R_1}(X) \cup \tau_{R_2}(X)) \cap \tau_{R_2}(X) \cup \tau_{R_3}(X))$

Then $NT^*_{123}\ O(X)$= {U, ϕ ,{p},{p,q,r,s},{q,r,s}}

$NT^*_{123}\ C(X)$= {U, ϕ ,{t},{p,t},{q,r,s,t}}

Definition 3.4:

A□$(U, \tau^*_{R_{1,2,3}}(X))$ is called T^*_{123}-open in U, if $A \in [\tau_{R_1}(X) \cup \tau_{R_2}(X)] \cap [\tau_{R_2}(X) \cup \tau_{R_3}(X)]$. The union of all NT^*_{123}-open sets contained in A is called the NT^*_{123}-interior of A and denoted by NT^*_{123}-intA. We say A is NT^*_{123}-closed in U if A^C is NT^*_{123}-open, and the intersection of NT^*_{123}-closed sets containing A is called NT^*_{123}-closure of A and it is denoted by NT^*_{123}-cl(A) .

Definition 3.5
A subset A of a nano Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$ is called NT^*_{123} neighborhood of a point x∈U if and only if there exists an T^*_{123} open set U such that x∈U⊂A.

Remark 3.6
We will denoted to the NT^*_{123} interior (resp. NT^*_{123} closure) of any subset, say A of U by NT^*_{123}intA (resp.NT^*_{123}clA), where NT^*_{123}intA is the union of all NT^*_{123} open sets contained in A, and NT^*_{123} is the intersection of all NT^*_{123}closed sets containing A.

Definition 3.7
A subset A of a space U is said to be NT^*_{123} b open set
if A□ NT^*_{123}cl(NT^*_{123}int A) ∪ NT^*_{123} int(NT^*_{123}clA).

Remarks 3.8
(i)The complement of NT^*_{123}b open set is called NT^*_{123}b closed set. Thus A⊂U is NT^*_{123}b closed if and only if NT^*_{123}cl(NT^*_{123}intA) ∩ NT^*_{123}int(NT^*_{123}cl*A*) ⊂A.
(ii)The intersection of all NT^*_{123}b closed sets of U containing a subset A of U is called NT^*_{123}b closure of A and is denoted by NT^*_{123}clb(A). Analogously the NT^*_{123}b interior of A is the union of all NT^*_{123}b open sets contained in A denoted by NT^*_{123}intb(A) .

Definition 3.9
A subset *A* of a Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$ is called to be NT^*_{123}*gb*-closed if NT^*_{123}*b*cl(A) ⊂ U whenever A ⊂ U and U is NT^*_{123}open.

Remark 3.10
(i)The complement of NT^*_{123}gb -closed is NT^*_{123}gb-open.
(ii)The intersection of all NT^*_{123} -closed sets of U containing a subset A of U is called NT^*_{123} gb-closure of A and is denoted by NT^*_{123}clgb(A) Analogously the NT^*_{123}gb -interior of A is the union of all 123gb-open sets contained in A denoted by NT^*_{123} intgb(A)

Example 3.11.
The relationships between the concepts NT^*_{123} closed set, NT^*_{123}b closed set and NT^*_{123}gbclosed summarized in the following diagram:

NT^*_{123}closedset ⟶ NT^*_{123}bclosedset ⟶ NT^*_{123}gbclosed

Let U = {a, b , c, d, e}, U/R_1= { {a} ,{b , c, d}, {e}}.
Let X_1 ={a , b}⊆ U. Then
$\tau_{R_1}(X)$={U, ϕ ,{a},{a,b,c,d},{b,c,d}}
Let X_2 = {a , c}⊆ U. Then
$\tau_{R_2}(X)$={U, ϕ ,{a},{a,b,c,d} ,{ b,c,d }}.
Let X_3 = {b , c}⊆ U. Then $\tau_{R_3}(X)$={U, ϕ ,{b,c,d} }.
$NT^*_{123}(X)$= ($\tau_{R_1}(X)$∪ $\tau_{R_2}(X)$) ∩ ($\tau_{R_2}(X)$∪ $\tau_{R_3}(X)$)
Then NT^*_{123} O(X)= {U, ϕ ,{a},{a,b,c,d},{b,c,d}}
NT^*_{123} C(X)= {U, ϕ ,{e},{a,e},{b,c,d,e}}

NT^*_{123} bC(X)= {U, ϕ ,{a},{b},{c},{d},{e},{a,e},{b,c},{b,d},{b,e},{c,d},{d,e},{c,e},{a,b,e},{a,c,e},{a,d,e},{b,c,e},{b,c,d},{b,d,e},{c,d,e},{b,c,d,e}}

NT^*_{123} gbC(X)= {U, ϕ {a},{b},{c},{d},{e},{a,e},{a,c},{b,c},{b,d},{b,e},{c,d},{d,e},{c,e},{a,b,e},{a,c,e},{a,d,e},{b,c,e},{b,c,d},{b,d,e},{c,d,e},{a,b,d,e},{a,b,c,e},{a,c,d,e},{b,c,d,e}}
Now, we will prove every pointed in the above diagram in the following propositions:

Proposition 3.12.
Every NT^*_{123} closed subset of a Nano tri star topological space U is NT^*_{123}b closed.

Proof:
Let A⊂U be NT^*_{123} closed set,
since A°⊂ NT^*_{123}clA°,
hence T^*_{123}intA°⊂NT^*_{123}int(NT^*_{123}clA°),
but NT^*_{123}int A⊂A for any subset A,
hence A°⊂ NT^*_{123}int (NT^*_{123}clA°), and
A°⊂ NT^*_{123}int (NT^*_{123}clA°)∪ NT^*_{123}cl(NT^*_{123}int A°) hence A° is NT^*_{123}b open set, hence A is NT^*_{123}b open set .

Proposition 3.14
Every NT^*_{123}b closed subset of a Nano tri-topological space U is NT^*_{123} gb-closed.

Proof:
Let *A* be a NT^*_{123}b-closed subset of U , and let A⊂G , where G is NT^*_{123}b -open, since *A* is NT^*_{123}*b* -closed set, hence NT^*_{123}int(NT^*_{123}cl(*A*)) ∩ NT^*_{123}cl(NT^*_{123}int(*A*)) ⊂ *A* ,
NT^*_{123}int(NT^*_{123}cl(*A*)) ∩ NT^*_{123}cl(NT^*_{123}int(*A*))⊂G

since $NT^*_{123}clb(A)$ is the smallest $NT^*_{123}b$ -closed set containing A ,
i.e. A is $NT^*_{123}cl(A) = A \cup NT^*_{123}int(NT^*_{123}cl(A)) \cap NT^*_{123}cl(NT^*_{123}int(A)) \subset A$,
$\subset A \cup \Pi$
$\subset U$
i.e. A is $NT^*_{123}gb$-closed.
Now, we will give some examples to show that the inverse pointed in the diagram (2.1) is not True

Example 3.15

NT^*_{123}-bclosed set ⟶/ NT^*_{123} -closed set.

Let U ={1, 2 ,3, 4, 5}, U/R_1= { {1} , {2 , 3, 4}, {5}}. LetX_1={1,2}⊆U.Then
$\tau_{R_1}(X)$={U, ϕ ,{1},{1,2,3,4},{2,3,4}}
Let X_2 = {1 , 3}⊆ U. Then
$\tau_{R_2}(X)$={U, ϕ ,{1},{1,2,3,4} ,{ 2,3,4 }}.
Let X_3 = {2 , 3}⊆ U. Then $\tau_{R_3}(X)$={U, ϕ ,{2,3,4} }.
$NT^*_{123}(X) = (\tau_{R_1}(X) \cup \tau_{R_2}(X)) \cap (\tau_{R_2}(X) \cup \tau_{R_3}(X))$
Then $NT^*_{123}\,O(X)$= {U, ϕ ,{1},{1,2,3,4},{2,3,4}}
$NT^*_{123}\,C(X)$= {U, ϕ ,{5},{1,5},{2,3,4,5}}
$NT^*_{123}bC(X)$= {U, ϕ {1},{2},{3},{4},{5},{1,5},{2,3},{2,4},{2,5},{3,4},{4,5},{3,5},{1,2,5},{1,3,5},{1,4,5},{2,3,5},{2,3,4},{2,4,5},{3,4,5},{2,3,4,5}}
{{1},{2},{3},{4},{2,3},{2,4},{2,5},{3,4},{4,5},{3,5},{1,2,5},{1,3,5},{1,4,5},{2,3,5},{2,3,4},{2,4,5},{3,4,5}} are NT^*_{123} b -closed sets but not NT^*_{123} -closed sets. ⟶/

Example 3.16

$NT^*_{123}gb$ closed set ⟶/ $NT^*_{123}b$ -closed set.

In example (3.11), the sets {a,c},{a,b,c,e},{a,c,d,e},{b,c,d,e}are $NT^*_{123}gb$ -closed but it is not $NT^*_{123}b$

$NT^*_{123}gb$ T_kSpaces, k =0,1,2

In this section we will introduce new types of separation axioms which we called $NT^*_{123}gb$ T_k spaces for k =0,1,2 , for the sake of convenience, we begin with definition the concepts NT^*_{123} T_k spaces for k = 0,1,2

Definition 3.17

A Nano Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$ is called:

(i) NT^*_{123} –T_0 if and only if to each pair of distinct points x , y in U , there exists an NT^*_{123} – open set containing one of the points but not the other.
(ii)NT^*_{123}–T_0 if and only if to each pair of distinct points x , y of U , there exist a pair of 123 - open sets one containing x but not y and the other containing y but not x .
(iii)NT^*_{123} –T_1 if and only if to each pair of distinct points x , y of U , there exist a pair of disjoint NT^*_{123}-open sets one containing x and the other containing y .
(iv)NT^*_{123} -regular if and only if to each NT^*_{123} closed set F and each point x ∉ F , there exist disjoint NT^*_{123}-open sets G and H such that x∈G and F ⊂H .

Definition 3.17.

A Nano tri star topological spaces $(U, \tau^*_{R_{1,2,3}}(X))$ is said to be $NT^*_{123}gb$ –T_0 space if and only if to each pair of distinct points x , y in U , there exists a $NT^*_{123}gb$ -open set containing one of the points but not the other. Now we proceed to prove that every tri- topological space is $NT^*_{123}gb$ –T_0 space.

Proposition 3.18.

If $\{x\}$ is $NT^*_{123}gb$ -open for some $x \in U$, then x ∉cl({y})$NT^*_{123}clgb$, for all $y \neq x$.

Proof:

Let $\{x\}$ be $NT^*_{123}gb$ -open for some x∈U , then $U - \{x\}$ is $NT^*_{123}gb$ -closed, and x∈U $-\{x\}$. If $x \notin NT^*_{123}$ clgb({y}) for some $y \neq x$, then y, x both are in all the $NT^*_{123}gb$ - closed sets containing y , so $x \in$ U−{x} which is contraction, hence x ∉NT^*_{123} clgb({y})

Proposition 3.19.

In any Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$, any distinct points have distinct $NT^*_{123}gb$ - closures.

Proof:

Let x, $y \in$ U with $x \neq y$, and let $A = \{x\}^c$,hence $NT^*_{123}cl(A) = A$ or U . Now, if $NT^*_{123}cl(A) = A$, then A is NT^*_{123}-closed, hence it is $NT^*_{123}bt$ -closed, so U − A = {x} is $NT^*_{123}gb$ -open and not containing y . So by proposition (3.3), x∉NT^*_{123} clgb({y}) and y ∈ NT^*_{123} clgb({y}),which implies that NT^*_{123} clgb({y}) and NT^*_{123} clgb({x }) are distinct. If $NT^*_{123}cl(A)$ = U ,then A is $NT^*_{123}gb$ -open, hence {x} is $NT^*_{123}gb$ - closed, which mean that

NT^*_{123} clgb($\{x\}$) = $\{x\}$ which is not equal to NT^*_{123} clgb($\{y\}$)

Proposition 3.20

In any Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$, if distinct points have distinct NT^*_{123}gb -closures then U is NT^*_{123}gb – T_0 space

Proof:

Let x, y ∈ U with x ≠ y , with NT^*_{123} clgb($\{y\}$) is not equal to NT^*_{123}clgb($\{x\}$), hence there exists z ∈ U such that z ∈ NT^*_{123}clgb($\{x\}$), but z ∉ NT^*_{123} clgb($\{y\}$) or z ∈ NT^*_{123}clgb($\{y\}$),
z ∈ NT^*_{123}clgb($\{x\}$), but z ∉ NT^*_{123}clgb($\{x\}$) .Now without loss of generality,
let z ∈ NT^*_{123}clgb($\{x\}$) but z ∉ NT^*_{123}clgb($\{y\}$).If z ∈ NT^*_{123}clgb($\{x\}$) ,then NT^*_{123}clgb($\{x\}$)is contained in NT^*_{123}clgb($\{y\}$), hence z ∈ NT^*_{123}clgb($\{y\}$), which is a contradiction, this mean that x ∉ NT^*_{123}clgb($\{y\}$) .hence x ∈ NT^*_{123}clgb($\{y\}^c$), hence U is
NT^*_{123}gb – T_0 space.

Proposition 3.21

Every Nano tri star topological space is NT^*_{123}gb – T_0space.

Proof:

Follows from propositions (3.19) and (3.20).

Definition 3.22

A Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$ is said to be NT^*_{123}gb − T space if and only if to each pair of distinct points x , y in X with x ≠ y , there exist two NT^*_{123}gb -open sets G , H such that x ⊂ G, y ∉ G and y ∈ H, x ∉ H .

Proposition 3.23

Every NT^*_{123}gb -T_1space is NT^*_{123}gb - T_0 space.

Proof:

Follows from the definition of NT^*_{123}gb − T_1space.

Proposition 3.24

In a Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$, the following statements are equivalent:

(i)U is NT^*_{123}gb – T_1space.

(ii)For each x ∈ U ,{x} is NT^*_{123}gb -closed in U .

(iii)Each subset of U is the intersection of all NT^*_{123}gb -open sets containing it.

(iv) The intersection of all NT^*_{123}gb -open sets containing the point x ∈ U is {x}.

Proof:

(i) ⟹ (ii)U is NT^*_{123}gb-T_1 space and let x, y ∈ U and x ≠ y , then there exists an NT^*_{123}gb-open set, say $G_y$such that y ∈ G_y .
Hence y ∈ G_y ⊂ $\{x\}^c$ so y ∈ G_y ⊂ $\{x\}^c$ = ∪{ G_y :y ∈ $\{x\}^c$ }which is NT^*_{123}gb -open,
so {x} is NT^*_{123}gb -closed in U .

(ii) ⟹ (iii)Let A ⊂ U and y ∉ A. Hence A ⊂ $\{y\}^c$ and
$\{y\}^c$ is NT^*_{123}gb -open in U and
A = ∩ { $\{y\}^c$:y ∈ A^c } which is the intersection of all NT^*_{123}gb-open sets containing A .

(iii) ⟹ (iv)Obvious.

(iv) ⟹ (iii) Let x, y ∈ U and x ≠ y . By assumption, there exist at least an NT^*_{123}gb -open set containing x but not y also an NT^*_{123}gb -open set containing y but not x .
i.e. U is NT^*_{123}gb – T_1 space.

Definition 3.25

A Nano gb − tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$ is said to be NT^*_{123}gb-T_2 if and only if for x, y ∈ U , $x \neq y$,there exist two disjoint NT^*_{123}gb -open sets G , H in U such that x ∈ G and y ∈ H

Proposition 3.26

Every NT^*_{123}gb –T_2 space is NT^*_{123}gb – T_1space.

Proof:

Let U is a NT^*_{123} space and let x,y in G with x ≠ y , so by hypothesis there exist two disjoint NT^*_{123}gb – T_2 space, say G , H such that x ∈ G and y ∈ H , but G ∩ H = ϕ , hence x ∉ H and y ∉ G, i.e. U is NT^*_{123}gb – T_1 space.

Definition 3.27

A subset A of a Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$ called NT^*_{123}gb -neighborhood of a point $x \subset X$ if and only if there exists an NT^*_{123}gb -open set G such that x ∈ G ⊂ A.

Proposition 3.28

In a Nano tri star topological space $(U, \tau^*_{R_{1,2,3}}(X))$, the following statements are equivalent:

(i) X is NT^*_{123}gb – T_2 space.

(ii) If x ∈ X , then for each y ≠ x , there is an NT^*_{123}gb -neighborhood M(x) of x such that y ∉ NT^*_{123}gb (M(x))

(iii) For each x ⊂ {NT^*_{123}gb(M)}={x} , where M is an NT^*_{123}gb -neighborhood of x .

Proof:

(i) ⟹ (ii)Let x ∈ U , if y ∈ V with x ≠ y ,then there

exist disjoint $NT^*_{123}gb$ -open sets G ,H in U such that $x\in U$ and $y\in V$.
Then $x\in G\subset U-H$, hence $U-H$ is an $NT^*_{123}gb$ - neighborhood of x , but $U-H$ is an $NT^*_{123}gb$ -closed and $y\notin U-H$. Now let $M(x) = U-H$, i.e. $y\notin NT^*_{123}gb\,(M(x))$.

(ii) $\Rightarrow$ (iii) Obvious.

(iii) $\Rightarrow$ (i) Let x, y $\in$ U and $x\neq y$. By assumption, there exist at least an $NT^*_{123}gb$ – neighborhood M of x such that $y\notin NT^*_{123}gb(M)$, so $x\in U\text{-}NT^*_{123}gb\,(M)$ is $T^*_{123}gb$ -open, but M $NT^*_{123}gb$ -neighborhood of x , hence there exists an $NT^*_{123}gb$ -open set U such that $x\in G\subset M$ and $G\cap U$ $NT^*_{123}gb$ (M). i.e. U is $NT^*_{123}gb-T_2$ space.

Definition 3.29

A Nano tri-topological spaces $\left(U,\tau^*_{R_{1,2,3}}(X)\right)$ is said to be $NT^*_{123}gb$ -regular space if and only if for each $NT^*_{123}gb$ -closed set F and each point $x\notin F$, there exist disjoint $NT^*_{123}gb$-open sets G and H such that $x\in G$ and $F\in H$.

Proposition 3.30

A $NT^*_{123}gb-T_0$ space is $NT^*_{123}gb-T_1$ space if it is $NT^*_{123}gb$ -regular space.

Proof:

Let U be $NT^*_{123}gb-T_0$ space and $NT^*_{123}gb$ -regular space. And let x, $y\in U$ and $x\neq y$, hence there exists an $NT^*_{123}gb$ -open, say G such that G contains one of x and y , say x but not y , so $U-G$ is an $NT^*_{123}gb$ -closed and $x\in U-G$, but U is $NT^*_{123}gb$ -regular space, hence there exist disjoint $NT^*_{123}gb$ -open sets H_1 and H_2 such that $X\in H_1$ and $U-G\subset H_2$, hence $x\in H_1$ and $y\in H_2$, i.e.U is $NT^*_{123}gb-T_2$.

Definition3.31

A map $f:\left(U,\tau^*_{R_{1,2,3}}(X)\right)\rightarrow\left(V,\sigma^*_{R_{1,2,3}}(X)\right)$ is called $NT^*_{123}gb$ -irresolute if the inverse image of every $NT^*_{123}gb$ -open set in V is $NT^*_{123}gb$ -open in U .

Proposition 3.32

If (f : $\left(U,\tau^*_{R_{1,2,3}}(X)\right)\rightarrow\left(V,\sigma^*_{R_{1,2,3}}(X)\right)$ is an injective and $NT^*_{123}gb$ -irresolute map and V is $NT^*_{123}gb$ T_2 space then U is $NT^*_{123}gb-T_2$ space.

Proof:

Let x, $y\in U$ and $x\neq y$, since f is injective, then $f(x)\neq f(y)$, and since V is $NT^*_{123}gb-T_2$, then there exist disjoint $NT^*_{123}gb$-open sets U and V such that $f(x)\in U$ and $f(y)\in V$. Now let $P=f^{-1}(G)$ and $Q=f^{-1}(H)$ hence $x\in P$, $y\in Q$ and P , Q are $NT^*_{123}gb$ -open sets, with $P\cap Q = f^{1}(P)\cap f^{1}(Q) = f^{1}(P\cap Q)=\phi$ i.e. U is $NT^*_{123}gb-T_2$space.

5. Conclusions

In this paper we introduced new type topology is called Nano Tri star topology. and also we introduce the concepts of NT^*_{123} open, NT^*_{123}closed, NT^*_{123}bclosed set andNT^*_{123}gbclosed set and some of their properties are disscussed detaild.. Finally, we hope that this paper is just a beginning of new classes of functions, it will be necessary to carry out more theoretical research to investigate the relations between

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