

# TRANSIENT HEAT AND MASS TRANSFER OF MICROPOLAR FLUID BETWEEN POROUS VERTICAL CHANNEL WITH BOUNDARY CONDITIONS OF THIRD KIND

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## Abstract

An investigation of unsteady free convective micro polar fluid flow of heat and mass transfer during the motion of incompressible fluid through a porous medium and the presence of thermal radiation bounded by an infinite vertical porous plate. The fluid is considered to be a gray, absorbing-emitting but non scattering medium, and the Cogley-Vincent-Gilles formulation is adopted to simulate the radiation component of heat transfer. The governing partial differential equations are reduced into coupled non-linear ordinary differential equation using suitable similarity transformation. The resulting systems of equations are solved numerically with Crank-Nicolson implicit finite difference method. The effect of various physical parameter such as transient, micro polar parameter, radiation parameter, Prandtl number, Biot number, Reynolds number, Schmidt number on the velocity, temperature and concentration field are discussed graphically.

**Keywords:** Thermal Radiation, Micro polar Fluid, Heat and Mass transfer.

Internal heat generation plays a vital role in many engineering applications geophysics and energy related problems. Eringen [1] developed the theory of micro polar fluids for the case where only micro rotational effects and micro rotational inertia exist. Since, Navier-Stokes theory does not describe precisely the physical properties of polymer fluids, colloidal solutions, suspension solutions, liquid crystals and fluids containing small additives. Eringen [2] extended the theory of thermo micro polar fluids and derived the constitutive laws for fluids with microstructure. His theory of micro fluids has opened up new areas in research in the physics of fluid flow. By Eringen's definition, a simple micro fluid is a fluent medium whose properties and behavior are affected by the local motions of the material particles contained in each of its volume elements such a fluid possesses local inertia. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium. Simple problems on the flow of such fluids were studied by a number of researchers and a review of this work was given by Ariman et al. [3].

The study of flow and heat transfer for an electrically conducting micro polar fluid past a porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as magneto hydrodynamic (MHD) generators, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and the boundary

## 1. Introduction

The micro polar fluids have great interest because the Navier-Stokes equation for Newtonian fluids cannot describe the characteristics of fluid with suspended particles. The fully developed heat and mass transfer by mixed convection of a micro polar fluid in a vertical channel for equal and unequal wall temperatures, concentrations and Biot numbers.

layer control in the field of aerodynamics. It obtained approximate solutions for the two dimensional flow of an incompressible, viscous fluid past an infinite porous vertical plate with constant suction velocity normal to the plate, the difference between the temperature of the plate and the free stream is moderately large causing the free convection currents. The problem of the flow past an impulsively started isothermal infinite vertical plate with mass transfer effects investigated by Soundalgekar and Takhar [4]. Agarwal and Dhanapal [5] have analyzed the effect of temperature dependent heat sources on the fully developed free convection micro-polar fluid flow when a constant suction (or injection) is applied on the plates and the fluid. The extension of above type of flows to include magneto hydrodynamic effects has become important due to several engineering applications such as in MHD generators, designing cooling system for nuclear reactors, flow meters, etc. where the micro concentration provides an important parameter for deciding the rate of heat flow. By simulating it, one can obtain the desired temperature in such equipments. Several investigators have made theoretical and experimental studies of micro-polar flow in the presence of a transverse magnetic field during the last three decades. Umavathi and Malashetty [6] have studied the problem of combined free and forced mixed convection flow in a vertical channel with symmetric and asymmetric boundary heating in the presence of viscous and Joulean dissipations.

Hassanien and Hamad [7] introduced new similarity solutions of flow of and heat transfer in a micro polar fluid along a vertical plate in a thermally stratified medium. The general analysis is developed in their study for the case of ambient temperature that varies exponentially with time as well as being uniform and varying with the position. Aung [8], Miyatake and Fujii [9] studied the natural convection heat transfer for fully

developed flow between vertical parallel plates with asymmetric boundary conditions. Nelson and Wood [10] examined the fully developed heat and mass transfer by natural convection between vertical parallel plates with asymmetric boundary conditions. Recently, Chamkha et al. [11] studied the fully developed free convection of a micro polar fluid in a vertical channel. Ibrahim et al. [12] studied the effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. This study indicates that, the volumetric heat generation term that arises due to heat source exert a strong influence on the heat transfer and as a consequence, also on the fluid flow. The study of radiation effects on the various types of flows is quite complicated. In the recent years, many authors have studied radiation effects on the boundary layer of radiating fluids past a

plate. Radiation effect on heat transfer in an electrically conducting fluid at a stretching surface with a uniform free stream has been analyzed by Abo-Eldahab and Ghonaim [13]. The thermal radiation interaction of the boundary layer flow of micro polar fluid past a heated vertical porous plate embedded in a porous medium with variable suction as well as heat flux at the plate by Rahman and Sultan [14]. The unsteady free convection flow of an incompressible electrically conducting micro polar fluid, bounded by two parallel infinite porous vertical plates submitted to an external magnetic field and the thermal boundary condition of forced convection by Zueco et al. [15]. The effects of Hall current and radiation absorption on MHD free convection mass transfer flow of a micro polar fluid in a rotating frame of reference. A uniform magnetic field acts perpendicular to the porous surface in which absorbs micro polar fluid with a constant suction velocity. The entire system rotates about the axes

normal to the plate with uniform angular velocity has been analyzed by Satya Narayanan et al.[16]. D.Prakash and M.Muthamilselvan [17] investigated the effect of radiation on transient MHD flow of micro polar fluids between porous vertical channel with boundary conditions of third kind. The governing equations are solved by Crank-Nicolson implicit finite difference method which reduces temperature of the fluid by applying thermal radiation. Bakr [18] presented an analysis of MHD free convection and mass transfer adjacent to moving vertical plate for micro polar fluid in a rotating frame of reference in the presence of heat generation absorption and a chemical reaction. An analysis of MHD free convection and mass transfer adjacent to moving vertical plate for micro polar fluid in a rotating frame of reference in the presence of heat generation absorption and a chemical reaction and also with thermal radiation effects studied Das [19].

In the present work, we have analyzed the effect of temperature depends on radiation parameter, Prandtl number and concentration depends on Schmidt number on the fully developed free convection electrically conducting micro polar fluid between two parallel porous vertical plates. The governing equations are solved numerically by Crank-Nicolson Method.

## 2. Mathematical Formulation

Consider an unsteady fully developed laminar flow of a micro polar fluid between two vertical plates. The vertical plates are separated by a distance  $L$ . The inner surface of the left plate at  $y=0$  and the inner surface of the right plate at  $y=L$ . The flow is fully developed, the transverse velocity is zero and the flow depends only on the transverse co-ordinate  $y$ . The fluid properties are assumed to be constant except for density variation in the buoyancy force term. The micro

polar fluid is assumed to be a gray, absorbing-emitting, but non-scattering medium. There is a component of micro rotation in the direction normal to  $x$  and  $y$ ,  $(0, 0, n)$  and neglecting the effect of viscous dissipation. The governing equations can be written as,

Continuity Equation

$$\frac{\partial v}{\partial y} = 0$$

(1)

Momentum Equation

$$\frac{\partial u}{\partial t} + \nu_0 \left( \frac{\partial u}{\partial y} \right) = \left( \nu + \frac{K}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \beta g(T - T_0) + \beta g(C - C_0) + \left( \frac{K}{\rho} \right) \frac{\partial n}{\partial y}$$

(2)

Angular Momentum Equation

$$\rho j \left( \frac{\partial n}{\partial t} \right) + \nu_0 \rho j \left( \frac{\partial n}{\partial y} \right) = \gamma \left( \frac{\partial^2 n}{\partial y^2} \right) - K \left( 2n + \frac{\partial u}{\partial y} \right) \quad (3)$$

Energy Equation

$$\rho c_p \left( \frac{\partial T}{\partial t} \right) + \nu_0 \rho c_p \left( \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial y^2} \right) - \frac{\partial q_r}{\partial y}$$

(4)

Species Concentration Equation

$$\left( \frac{\partial C}{\partial t} \right) + \nu_0 \left( \frac{\partial C}{\partial y} \right) = D \left( \frac{\partial^2 C}{\partial y^2} \right)$$

(5)

where  $\beta$  is the thermal expansion coefficient,  $\rho$  is the density,  $c_p$  is the specific heat at constant pressure,  $k$  is the thermal conductivity of the fluid,  $\nu$  is the kinematic viscosity,  $K$  is the gyroviscosity,  $\gamma$  is the material constant,  $j$  is the microinertia,  $T_0$  is the temperature in hydrostatic state,  $C$  and  $C_0$  are the concentration of species, and  $D$  is the molecular diffusivity.

The advantage and limitations of the Cogley et al. [20],

- (i) It does not require an extra transport equation for the incident radiation.
- (ii) It can only be used for an optically thin, near equilibrium and non-gray gas.

Cogley model is well suited for

- (i) Surface-to-surface radiation heating or cooling,
- (ii) Coupled radiation, convection or conducting heat transfer and
- (iii) Radiation in glass processing, glass fibre drawing and central processing.

Cogley et al. [20] have shown that the radiative heat flux is represented by the following form,

$$\frac{\partial q_r}{\partial y} = 4(T - T_0) \int_0^\infty K_{\lambda h} \left( \frac{\partial e_{\lambda p}}{\partial T} \right) d\lambda \quad (6)$$

Where  $K_{\lambda h}$  is the absorption coefficient,  $\lambda$  is the wave length,  $e_{\lambda p}$  is the Planck's function,  $T$  is the temperature of the walls at time  $t \leq 0$ .

On the use of (6) in (4) becomes,

$$\rho c_p \left( \frac{\partial T}{\partial t} \right) + v_0 \rho c_p \left( \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial y^2} \right) - 4(T - T_0)I \quad (7)$$

Where

$$I = \int_0^\infty K_{\lambda h} \left( \frac{\partial e_{\lambda p}}{\partial T} \right) d\lambda \quad (8)$$

The walls of the channel are assumed to have a negligible thickness and exchange the heat with an external fluid by convection. At  $y = 0$ , the external convection coefficient  $h_1$ ,  $h_3$  is considered uniform and the fluid in the region  $y < 0$  is assumed to have a uniform reference temperature  $T_1$  and concentration  $C_1$ . At  $y = L$ , the

corresponding constant values are  $h_2, h_4, T_0$  and  $C_0$  ( $T_0 \geq T_1$ ,  $C_0 \geq C_1$ )

The appropriate initial and boundary conditions are written as

$$\text{For } t \leq 0; \quad u = 0, \quad n = 0, \quad T = T_0, \quad C = C_0 \quad (9)$$

$$\begin{aligned} \text{For } t \geq 0; \quad u = 0, \quad v = v_0, \quad n = 0, \\ -k_w \frac{\partial T}{\partial y} = h_1 [T_1 - T], \quad -D \frac{\partial C}{\partial y} = h_3 [C_1 - C] \quad \text{at } y = 0 \quad (10) \\ u = 0, \quad v = v_0, \quad n = 0, \\ -k_w \frac{\partial T}{\partial y} = h_2 [T - T_0], \quad -D \frac{\partial C}{\partial y} = h_4 [C - C_0] \quad \text{at } y = L \quad (11) \end{aligned}$$

Where  $v_0$  is the constant suction or injection of the fluid through the porous limiting surface, with  $v_0 \leq 0$  implies injection at  $y = L$  and suction at  $y = 0$ , while the opposites occurs for  $v_0 > 0$

We introduce the dimensionless variable to dimensionalize the governing equations,

$$\begin{aligned} Y = \frac{y}{L}, \quad U = \frac{u \rho g \beta L^2}{k}, \quad \tau = \frac{t v}{L^2}, \quad \theta = \frac{(T - T_0) \rho^2 g^2 \beta^2 L^4}{k \mu}, \\ B = \frac{j}{L^2}, \quad \phi = \frac{(C - C_0) \rho^2 g^2 \beta^2 L^4}{k \mu}, \quad N = \frac{n \rho g \beta L^3 v m}{k}, \quad Pr = \frac{v}{\alpha}, \\ Re = \frac{v_0 \rho L}{\mu}, \quad \lambda_1 = \frac{K}{\mu}, \quad \lambda_2 = \frac{\gamma}{\mu L^2}, \quad Sc = \frac{v}{D} \quad (12) \end{aligned}$$

Where  $\lambda_1$  is the micro polar parameter,  $\lambda_2$  and  $B$  are the micro polar material constants and  $Re$  is the cross flow Reynolds number.

With the help of (12), the governing equations can be rewritten as

$$\left( \frac{\partial U}{\partial \tau} \right) + Re \left( \frac{\partial U}{\partial Y} \right) = (1 + \lambda_1) \left( \frac{\partial^2 U}{\partial Y^2} \right) + \theta + \phi + \lambda_1 \left( \frac{\partial N}{\partial Y} \right) \quad (13)$$

$$B \left( \frac{\partial N}{\partial \tau} \right) + Re B \left( \frac{\partial N}{\partial Y} \right) = \lambda_2 \left( \frac{\partial^2 N}{\partial Y^2} \right) - \lambda_2 \left( \frac{\partial U}{\partial Y} + 2N \right) \quad (14)$$

$$Pr \left( \frac{\partial \theta}{\partial \tau} \right) + RePr \left( \frac{\partial \theta}{\partial Y} \right) = \left( \frac{\partial^2 \theta}{\partial Y^2} \right) - N_R \theta \quad (15)$$

$$Sc \left( \frac{\partial \phi}{\partial \tau} \right) + ReSc \left( \frac{\partial \phi}{\partial Y} \right) = \left( \frac{\partial^2 \phi}{\partial Y^2} \right) \quad (16)$$

and the dimensionless initial and boundary conditions become,

$$\tau \leq 0 ; U = 0, N = 0, \theta = 0, \phi = 0$$

$$(17) \tau > 0 ; U = 0, N = 0$$

$$- \frac{\partial \theta}{\partial Y} = Bi_1 [\xi - \theta], - \frac{\partial \phi}{\partial Y} = Bi_3 [\eta - \phi] \text{ at } Y = 0$$

$$(18) \tau > 0 ; U = 0, N = 0$$

$$- \frac{\partial \theta}{\partial Y} = Bi_2 [\theta - \epsilon \xi], - \frac{\partial \phi}{\partial Y} = Bi_4 [\phi - \eta \zeta] \text{ at } Y = 1$$

$$(19)$$

Where  $\xi = \frac{Pr Gr \beta g L}{c_p}$  is the dimensionless group,

$\eta = \frac{Pr G m \beta g L}{c_p}$  is the dimensionless group,

$Gr = \frac{g \beta L^3 (T - T_0)}{\nu}$  is a Grashof number,

$Gm = \frac{g \beta L^3 (C - C_0)}{\nu}$  is a modified Grashof number,

$\epsilon = \frac{T_2 - T_0}{T_1 - T_0}$  is a non dimensional parameter,

$\zeta = \frac{C_2 - C_0}{C_1 - C_0}$  is a non dimensional parameter,

$Bi_1 = \frac{h_1 L}{k_w}, Bi_2 = \frac{h_2 L}{k_w}, Bi_3 = \frac{h_3 L}{D_w}, Bi_4 = \frac{h_4 L}{D_w}$

are the Biot numbers,  $k_w$ : Thermal conductivity of the walls,  $D_w$ : Molecular diffusivity,  $N_R = \frac{4L^2 I}{k}$  is the thermal radiation parameter.

The shear couple stresses, heat flux on the walls and the local mass flux of the plate are defined,

$$\tau_w = (\mu + K) \left. \frac{\partial u}{\partial y} \right|_{y=0,1}, \tau_m = \gamma \left. \frac{\partial n}{\partial y} \right|_{y=0,1},$$

$$q_w = -k \left. \frac{\partial T}{\partial y} \right|_{y=0,1}, q_m = -D \left. \frac{\partial C}{\partial y} \right|_{y=0,1}$$

### 3. Numerical procedure

The coupled system of (13) - (16) is highly non-linear. Problems involving non-linearity are difficult to solve exactly. So, the coupled system

of (13) - (16) and the initial and boundary conditions (17) - (19) are solved by Crank-Nicolson implicit finite difference technique. The computational domain  $(0 < \tau < \infty) - (0 < Y < 1)$  is divided into a mesh of lines parallel to  $\tau$  and  $Y$  axes. The finite difference approximations of (13) - (16) are substituting for the approximations by derivatives. The system of equations at the  $i, j$ th level, by using forward difference approximations (20)-(21) and by using the central differencing to evaluate the second order derivatives (22).

$$\left. \frac{\partial u}{\partial t} \right|_{i,j} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \quad (20)$$

$$\left. \frac{\partial u}{\partial Y} \right|_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta y} \quad (21)$$

$$\left. \frac{\partial^2 u}{\partial Y^2} \right|_{i,j} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} \quad (22)$$

By using (20)-(22) into the governing equations, the following appropriate sets of finite difference equations are obtained.

$$\begin{aligned} & \left( \frac{U_{i,j+1} - U_{i,j}}{\Delta \tau} \right) + Re \left( \frac{U_{i+1,j} - U_{i,j}}{\Delta Y} \right) \\ & = (1 \\ & + \lambda_1) \left( \frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{(\Delta Y)^2} \right) \\ & + \theta_{i,j} + \phi_{i,j} + \lambda_1 \left( \frac{N_{i+1,j} - N_{i,j}}{\Delta Y} \right) \end{aligned}$$

(23)

$$\begin{aligned} & B \left( \frac{N_{i,j+1} - N_{i,j}}{\Delta \tau} \right) + ReB \left( \frac{N_{i+1,j} - N_{i,j}}{\Delta Y} \right) = \\ & (\lambda_2) \left( \frac{N_{i-1,j} - 2N_{i,j} + N_{i+1,j}}{(\Delta Y)^2} \right) - \lambda_1 \left( \frac{U_{i+1,j} - U_{i,j}}{\Delta Y} + \right. \\ & \left. 2N_{i,j} \right) \quad (24) \end{aligned}$$

$$Pr \left( \frac{\theta_{i,j-1} - \theta_{i,j}}{\Delta \tau} \right) + RePr \left( \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta Y} \right) = \left( \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta Y)^2} \right) - N_R \theta_{i,j} \quad (25)$$

$$Sc \left( \frac{\phi_{i,j-1} - \phi_{i,j}}{\Delta \tau} \right) + ReSc \left( \frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta Y} \right) = \left( \frac{\phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j}}{(\Delta Y)^2} \right) \quad (26)$$

they associated initial and boundary conditions may be expressed as,

$$U_{i,1} = 0, N_{i,1} = 0, \theta_{i,1} = 0, \phi_{i,1} = 0 \quad (27)$$

$$U_{1,j} = 0, N_{1,j} = 0, -\frac{\theta_{1,j-1} - \theta_{1,j}}{\Delta Y} = Bi_1 [\xi - \theta_{1,j}],$$

$$-\frac{\phi_{1,j-1} - \phi_{1,j}}{\Delta Y} = Bi_3 [\psi - \phi_{1,j}] \quad (28)$$

$$U_{N,j} = 0, N_{N,j} = 0, -\frac{\theta_{N,j-1} - \theta_{N,j}}{\Delta Y} = Bi_2 [\theta_{N,j} - \epsilon \xi],$$

$$-\frac{\phi_{N,j-1} - \phi_{N,j}}{\Delta Y} = Bi_4 [\phi_{N,j} - \varsigma \eta] \quad (29)$$

Where  $\Delta Y$  and  $\Delta \tau$  are the mesh sizes along  $Y$  and time directions, respectively. The computational domain ( $0 < \tau < \infty$ ) is divided into intervals and the step size  $\Delta \tau = 0.002$  for time ( $\tau$ ). In finite difference scheme, the computation carried out for different grids ( $0 < Y < 1$ ). The numerical solutions are obtained with the grid size 201.

#### 4. Results and discussion

An numerical solution for the problem of mixed convective flow of heat and mass transfer of micro polar fluid in a vertical channel is analyzed numerically. The results are presented for a range values of the non-dimensional parameter such as cross flow Reynolds number  $Re$ , micro polar parameter  $\lambda_1$ , Prandtl number  $Pr$ , Schmidt number  $Sc$ , radiation parameter  $N_R$ , and Biot

numbers  $Bi_1, Bi_2, Bi_3$  and  $Bi_4$ . It considered in all cases studied that  $Bi_1 = 10, Bi_2 = 1, Bi_3 = 5, Bi_4 = 1, \xi = 1.0, \epsilon = 1.2, \eta = 1.0, \varsigma = 1.2, B = 0.001, Sc = 0.96, Re = 2.0, N_R = 1.0, \lambda_1 = 3.0, \lambda_2 = 1.0$ . The numerical solutions are evaluated for different values of governing parameters and the results are presented graphically in Figs:1-12.

##### 4.1 Hydrodynamic Aspect:

Fig. 3 illustrates that the transient ( $\tau = 0.1, 0.2, 0.3$ ) and the variation of Reynolds number for  $Re = 1$  and  $Re = 10$ , it will increases the velocity profile ( $U$ ) to reach the steady state for fixed time parameters. The influence of cross flow of Reynolds number and the micro polar parameter on the velocity profile is analyzed in Fig. 4. As the velocity profile ( $U$ ) is increases with an increase in Reynolds number. It is noted that the velocity profile decreases when the micro polar parameter increases.

Fig. 5 depicts the velocity distribution along the spatial coordinate  $Y$  for different values of radiation parameter. The velocity distribution decreases with an increase in the radiation parameter. Also the maximum velocity is displaced toward  $Y=1$  for higher Reynolds number  $Re=10$ . To analyze the transient response in the angular velocity distribution. In Fig. 6, the angular velocity increases with time, the values of micro rotation are negative between  $Y=0$  and  $Y=0.5$  approximately, whereas they are positive between  $Y=0.5$  and  $Y=1$  approximately besides, as Reynolds number increases the amplitude of the angular velocity.



Fig. 7 portrays the angular velocity distribution for different values of micro polar parameter. It is seen that the micro polar parameter strongly influences the angular velocity. In the case of higher Reynolds number, the micro polar parameter enforces to increase the angular velocity almost full length of the channel. The influence of the radiation parameter on the angular velocity profile is depicted in Fig. 8 It can be seen that the radiation effect lead to increase the angular velocity for the first half of the channel and decrease it for the remaining. Further, an increase in the Reynolds number is to decrease the angular velocity throughout the channel.

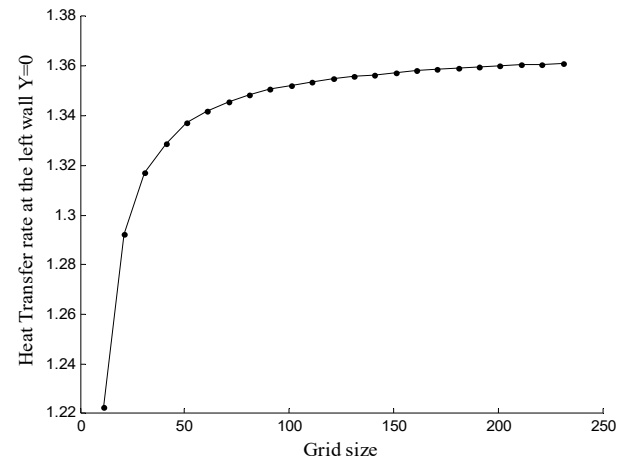
#### 4.2 Heat and mass transfer Aspects:

Dimensionless temperature profiles are presented in Fig.9 for different values of transient ( $\tau = 0.1, 0.2, 0.3$ ) and the fixed values of Reynolds number ( $Re$ ). Fig. 10 represents the temperature distribution for different values of radiation parameter  $N_R$ . It is observed that an increase in the radiation parameter decreases the temperature distribution in the thermal boundary layer. As Reynolds number increases, temperature profile decreases between two plates while opposite trend is observed for radiation parameter.

Fig. 11 temperature is maximum at the plate than it falls exponentially and finally tends to zero for both air ( $Pr = 0.72$ ) and water ( $Pr = 1$ ). Temperature is greater for air than water. It is because that of thermal conductivity of fluid decreases with increasing  $Pr$ , therefore thermal boundary layer thickness decreases with increasing  $Pr$ .

It exhibits the effects of Reynolds number ( $Re$ ) and transient ( $\tau$ ) on concentration. It is observed that concentration increases with increasing time shown in Fig. 12. The concentration profiles for different Schmidt numbers are given in Fig. 13 which shows that,

increasing values of Schmidt number ( $Sc = 0.96$ ) and ( $Sc = 1$ ) implies the decrease in



concentration profile.

Figure 1: Heat transfer rate at the left wall  $Y = 0$  for different grid systems with  $Re = -2$ ,  $\tau = 0.2$ ,  $\lambda_1 = 3$ ,  $N_R = 1$ ,  $Pr = 0.7$ ,  $Bi_1 10$ ,  $Bi_2 = 1$ ,  $Bi_3 = 5$ ,  $Bi_4 = 1$

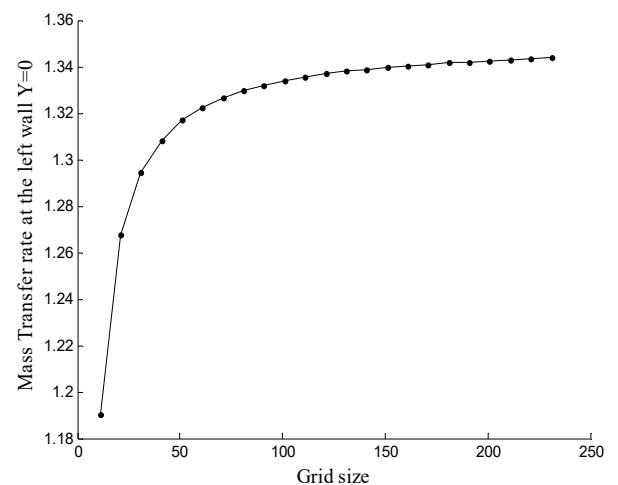


Figure 2: Mass transfer rate at the left wall  $Y = 0$  for different grid systems with  $Re = -2$ ,  $\tau = 0.2$ ,  $\lambda_1 = 3$ ,  $N_R = 1$ ,  $Pr = 0.72$ ,  $Bi_1 = 10$ ,  $Bi_2 = 1$ ,  $Bi_3 = 5$ ,  $Bi_4 = 1$

values of micro polar parameter  $\lambda_1$  and Reynolds number.

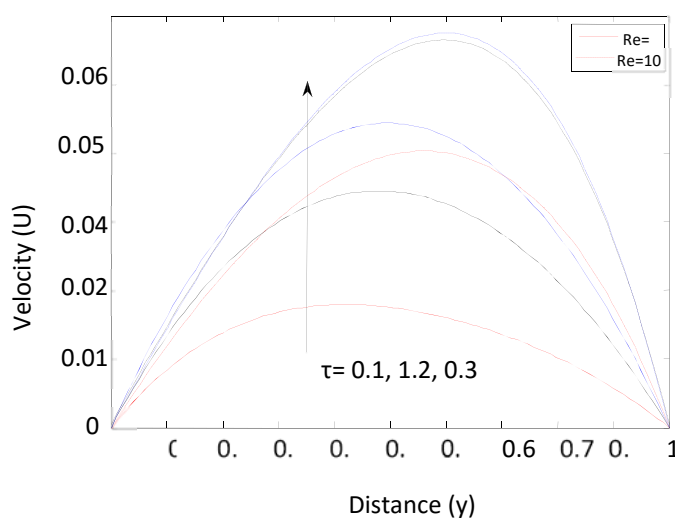


Figure 3: Velocity profile for different values of transient and Reynolds number (Re).

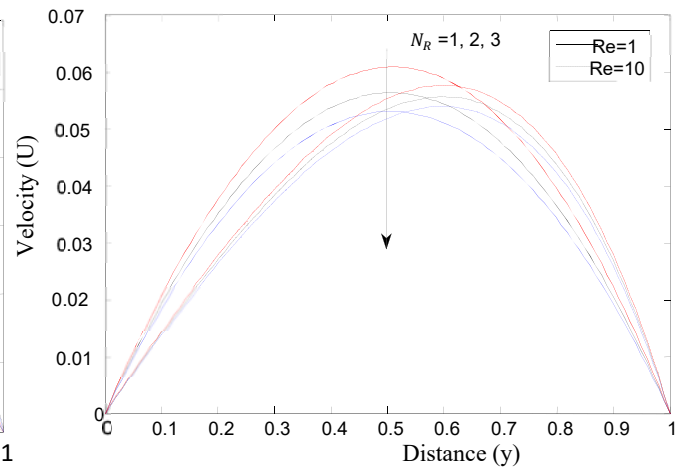


Figure 5: Velocity profile for different values of radiation parameter ( $N_R$ ) and Reynolds number (Re).

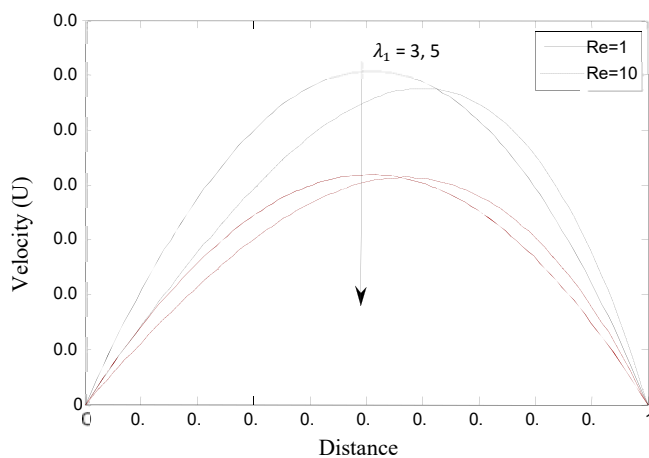


Figure 4: Velocity profile for different

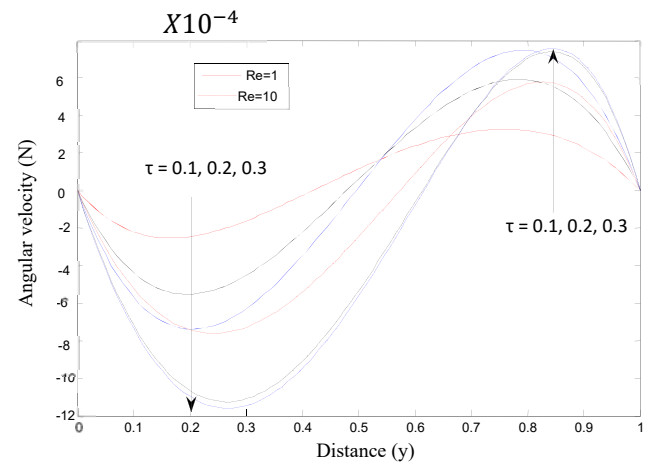




Figure 6: Angular Velocity profile for different values of transient ( $\tau$ ) and Reynolds number ( $Re$ ).

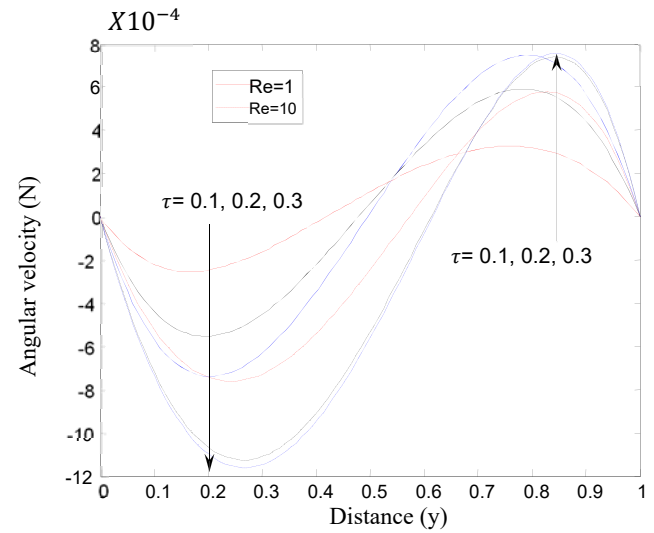
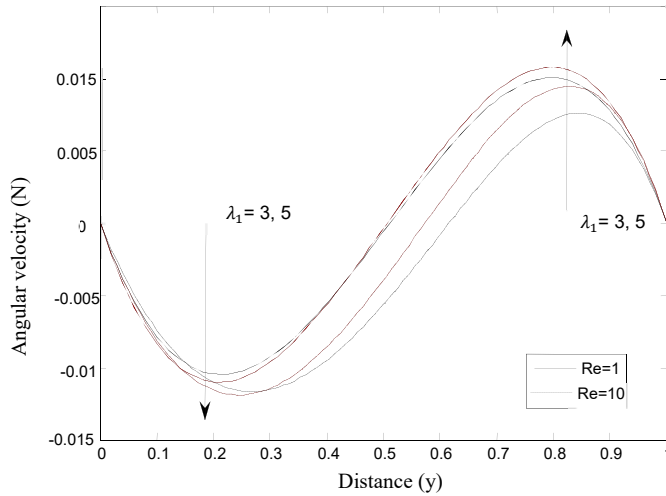


Figure 9: Angular Velocity profile for different values of transient ( $\tau$ ) and Reynolds number ( $Re$ ).

Figure 7: Angular Velocity profile for different values of micro polar parameter ( $\lambda_1$ ) and Reynolds number ( $Re$ ).

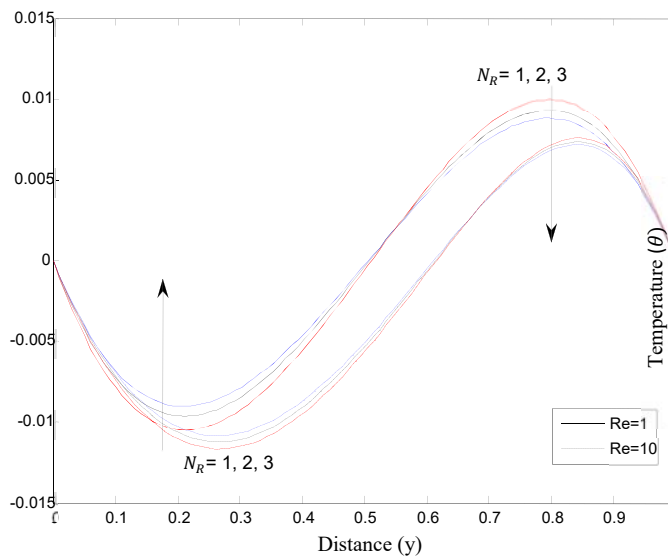
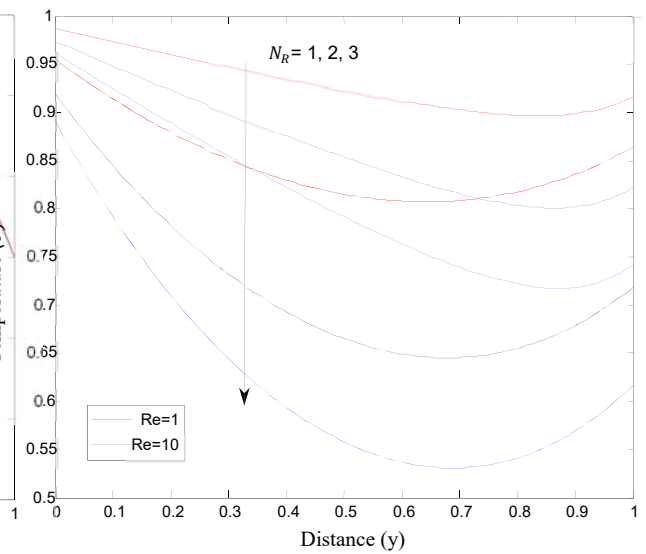


Figure 8: Angular Velocity profile for different values of radiation parameter  $N_R$  and Reynolds number ( $Re$ ).

Figure 10: Temperature profile for different values of radiation parameter  $N_R$  and Reynolds number ( $Re$ ).



Reynolds number ( $Re$ ).

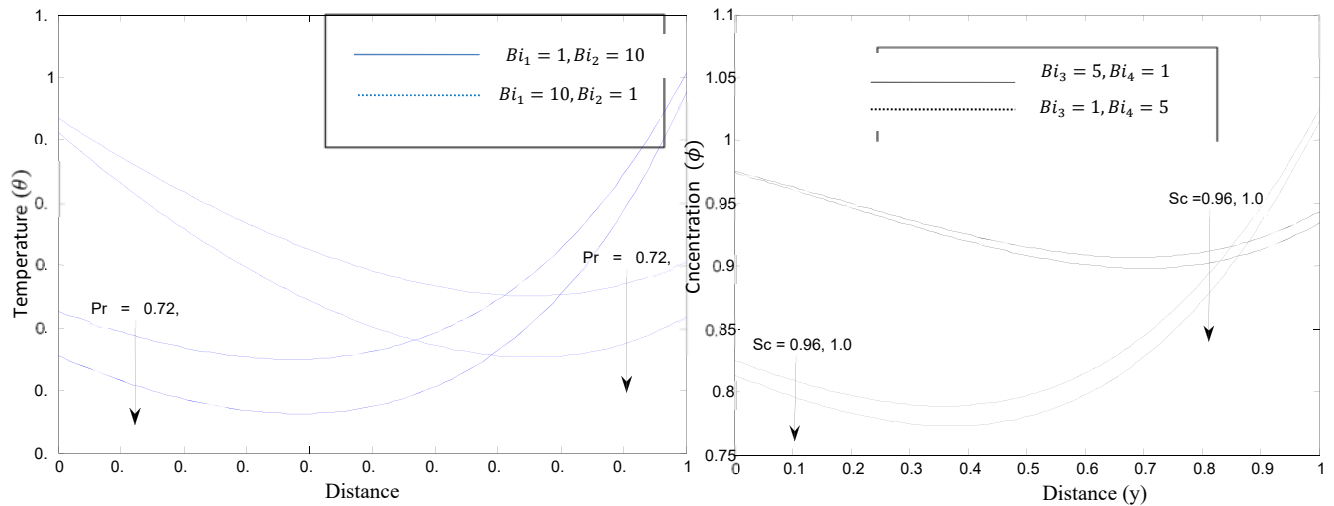
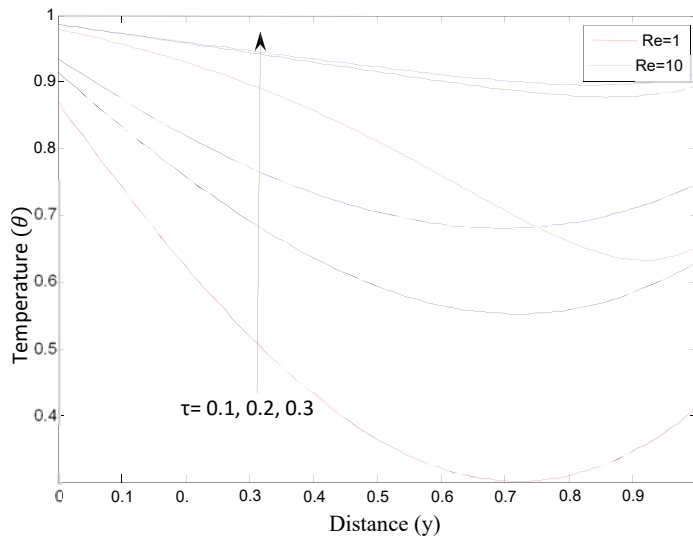


Figure 11: Temperature profile for different values of Prandtl number ( $Pr$ ) and Reynolds number ( $Re$ ) with different type of heating.

Figure 12: Concentration profile for



different values of transient ( $\tau$ ) and Reynolds number ( $Re$ ).

Figure 13: Concentration profile for different values of Schmidt number ( $Sc$ ) and Reynolds number ( $Re$ ) with different type of heating.

## 5. Conclusion

To obtained numerical solution of a unsteady boundary layer flow and heat transfer and mass transfer of a micro polar fluid with porous medium and in the presence of radiation has been the subject of this work. The non-linear governing equations are solved numerically by developing a suitable numerical technique with the help of Crank-Nicolson finite difference method.

The following conclusions can be

drawn on the basis of the numerical results.

1. An increase in the Reynolds number is to increase the velocity, temperature and the concentration, but decrease the angular velocity of the fixed values of time parameter.
2. An increase in the micro polar is to decrease the velocity and angular velocity. An increase of chemical

reaction parameters is to increase the velocity and concentration, but the decrease of angular velocity.

3. An increase in the radiation is to decrease the velocity and temperature, but increases the angular velocity.
4. An increase in the radiation parameter or Prandtl number is to decrease the temperature. An increase in the Schmidt number is to decrease the concentration.

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