International Journal of Scientific Engineering and Applied Science (IJSEAS) – Volume-3, Issue-3,March 2017
ISSN: 2395-3470
www.ijseas.com

Stochastic Model of a Mutualistic System: Non-Equilibrium Fluctuation and Stability

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Abstract

In the present paper i represent a critical analysis of non-equilibrium fluctuation and stability of a mutualistic system. It also include the comparative study of both deterministic and stochastic criteria of stability of the system on the basis of the statistical linearization of stochastic differential equation.

Keywords: Mutualistic System. Statistical Linearization. Non-Equilibrium Fluctuation. Stability. Decartes' rule.

1 Introduction:

Stability analysis of an ecosystem is one of the most important problems of population ecology. There are different aspects of stability in ecology and correspondingly different theory and criteria of stability [14]. Deterministic dynamical methods fail to provide a precise and unambiguous definition of stability of an ecosystem under the influences of a randomly fluctuating environment. Stochasticity plays vital role in the study of stability of the ecosystems. A biological or ecological community is considered to be stable when the number of components of the population does not undergo sharp population [14]. This is equivalent to the motion of stochastic stability in the sense of second order moments [1, 2].

The object of the present paper is to make a critical analysis of local stability of a mu-tualistic Lotka-Volterra ecosystem under random perturbation . Such a system can be modeled by a stochastic differential equation describing a system under the influence of a randomly fluctuating environmental . These stochastic equation are in general non-linear. There are different techniques of statistical linearization of stochastic differential

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equation [3,5,6,8,12,13]. Baishy and Chakrabarti [7] were the first to apply the technique of Valsakumar [6] to the problem of statistical linearization technique in the ecology. The above technique was used extensively later [4,7,9,10,15]. The present paper is a critical analysis of non-equilibrium fluctuation and stability of a Lotka-Volterra mutualistic system based on the above technique.

2 Mutualistic System: Deterministic Model and Analysis:

Let us consider a mutualistic model ecosystem [16] govern by the system of deterministic equations [16]

$$\frac{dN_1}{dt} = \frac{r_1}{k_1} N_1(k_1 - N_1 + N_2) \qquad (2.1a)$$

$$\frac{dN_2}{dt} = \frac{r_2}{k_2} N_2(k_2 - N_2 + bN_1) \qquad (2.1b)$$

where k_1 , k_2 are the carrying capacity and r_1 , r_2 are the growth rate of the species $N_1(t)$ and $N_2(t)$ respectively. b is the measure of mutualism effect of each and other. In this model each of them benefits from the presence of other species and grows logistically in absence of other species. Before we go to the stochastic extension and analysis of the system, let us first go to the deterministic behavior of the system (2:1). The stationary states of the system (2:1) are as follows:

$$E_0: (N_1^*, N_2^*) = (0,0) \qquad (2.2a)$$

$$E_1: (N_1^*, N_2^*) = (k_1, 0) \qquad (2.2b)$$

$$E_2: (N_1^*, N_2^*) = (0, k_2) \qquad (2.2c)$$

$$E^*: (N_1^*, N_2^*) = \left(\frac{k_1 + k_2}{(1-b)}, \frac{k_2 + bk_1}{(1-b)}\right) \qquad (2.2d)$$

The existence criterion of the steady-state E (N_1 ; N_2) population (i.e. non-negativity)demands the parameter b lies between 0 and 1. Let us study the effect of small perturbation of the steady-states. Let $x_1(t)$ and $x_2(t)$ be the perturbation such that

$$N_1(t) = N_1 + x_1(t) \qquad (2.3a)$$

$$N_2(t) = N_2 + x_2(t) \qquad (2.3b)$$

where x_1, x_2 are sufficiently small quantities. Linearising the system of equations (2.1) about the stationary states , we get,

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 \qquad (2.4a)$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 \qquad (2.4b)$$

Where $$a_{ij} = \left[\frac{\partial}{\partial N_i}\left(\frac{dN_i}{dt}\right)\right]_{(N_1^*,N_2^*)} \qquad (i,j = 1,2) \qquad (2.5)$$

The necessary and sufficient conditions of stability of the system about the steady-states (N_1 ; N_2) are [16]

$$a_{11} + a_{22} < 0 \qquad (2.6a)$$

and $$a_{11}a_{22} - a_{12}a_{21} > 0 \qquad (2.6b)$$

Let us first consider the trivial steady state $E_0(0,0)$.Then although $a_{11}a_{22} - a_{12}a_{21} > 0$ but $a_{11} + a_{22} = r_1 + r_2 > 0$ so that the steady state $E_0(0,0)$ is unstable. For the steady -state $E_1(k_1, 0)$, $a_{11}a_{22} - a_{12}a_{21} = -\frac{r_1 r_2}{k_2}(k_2 + bk_1) < 0$ so the steady state $E_1(k_1, 0)$ is also unstable.For the steady state $E_2(0, k_2)$, $a_{11}a_{22} - a_{12}a_{21} = -\frac{r_1 r_2}{k_1}(k_2 + k_1) < 0$ so the steady state $E_2(0, k_2)$ is also unstable. For the steady state $E^*\left(\frac{k_1+k_2}{(1-b)}, \frac{k_2+bk_1}{(1-b)}\right)$, $a_{11} + a_{22} = -[\frac{r_1}{k_1}N_1^* + \frac{r_2}{k_2}N_2^*] < 0$ and $a_{11}a_{22} - a_{12}a_{21} = \frac{r_1 r_2}{k_1 k_2}(1 - b)N_1^* N_2^* > 0 \; as \; 0 < b < 1$ so the steady state $E^*\left(\frac{k_1+k_2}{(1-b)}, \frac{k_2+bk_1}{(1-b)}\right)$ is stable.

3 Statistic Model: Statistical Linearization and Moment Equations:

Let us study the effect of stochastic perturbation on the mutualistic ecosystem (2.1). For that we extend the system of equation (2.1) to Ito type of stochastic differential equations:

$$\frac{dN_1}{dt} = \frac{r_1}{k_1}N_1(k_1 - N_1 + N_2) + \eta_1(t) \qquad (3.1a)$$

$$\frac{dN_2}{dt} = \frac{r_2}{k_2}N_2(k_2 - N_2 + bN_1) + \eta_2(t) \qquad (3.1b)$$

where the stochastic perturbation $\eta_1(t)$ and $\eta_1(t)$ are assumed to be Gaussian white noises satisfying the conditions:

$$<\eta_i(t)=0>, <\eta_i(t)\eta_j(t')>=2\epsilon_i\delta_{ij}(t-t') \ \ (i,j=1,2) \qquad (3.2)$$

where ϵ_i is the strength or intensity of the random perturbation and the bracket <> represents the ensemble average. The system of equation (3.1) are non-linear Langevin type and are in general very diffcult to solve. However, for the study of local stability about the stationary states, we can linearize them without loss of information.

Let us study about the endemic steady state $E^*:(N_1^*,N_2^*)=\left(\frac{k_1+k_2}{(1-b)},\frac{k_2+bk_1}{(1-b)}\right)$.

Let (x_1,x_2) be the deviation from the steady state $E^*:(N_1^*,N_2^*)=\left(\frac{k_1+k_2}{(1-b)},\frac{k_2+bk_1}{(1-b)}\right)$ *so that* $N_1=N_1^*+x_1$, $N_2=N_2^*+x_2$. In this case the system of equations (3.1) reduces to the form

$$\frac{dx_1}{dt}=-\frac{r_1N_1^*}{k_1}x_1+\frac{r_1N_1^*}{k_1}x_2-\frac{r_1}{k_1}x_1^2+\frac{r_1}{k_1}x_1x_2+\eta_1(t) \qquad (3.3a)$$

$$\frac{dx_2}{dt}=\frac{br_2N_2^*}{k_2}x_1-\frac{r_2N_2^*}{k_2}x_2-\frac{r_2}{k_2}x_2^2+\frac{br_2}{k_2}x_1x_2+\eta_2(t) \qquad (3.3b)$$

where η_1 and η_2 are described as in (3.2). Let us linearize the system of equation (3.3) statistically. The statistical linaerization consists of replacing the system (3.3) by the system of linear equations:

$$\frac{dx_1}{dt}=\alpha_1x_1+\beta_1x_2+c_1+\eta_1(t) \qquad (3.4a)$$

$$\frac{dx_2}{dt}=\alpha_2x_1+\beta_2x_2+c_2+\eta_2(t) \qquad (3.4b)$$

where the errors in the linearization are

$$e_1=-\frac{r_1N_1^*}{k_1}x_1+\frac{r_1N_1^*}{k_1}x_2-\frac{r_1}{k_1}x_1^2+\frac{r_1}{k_1}x_1x_2-\alpha_1x_1-\beta_1x_2-c_1 \qquad (3.5a)$$

$$e_2=\frac{br_2N_2^*}{k_2}x_1-\frac{r_2N_2^*}{k_2}x_2-\frac{r_2}{k_2}x_2^2+\frac{br_2}{k_2}x_1x_2-\alpha_2x_1-\beta_2x_2-c_2 \qquad (3.5b)$$

The unknown coefficients $\alpha_1,\alpha_2,\beta_1,\beta_2$ of the equation (3.4) are determined from the minimization of the averages of the squares of the errors (3.5). These coefficients are in general functions of the parameters k_1, k_2, r_1, r_2 and also the different moments involving x_1 and x_2 [6]. Simple calculations leads to the system of equations of the first moments:

$$\frac{d\langle x_1\rangle}{dt} = -\frac{r_1 N_1^*}{k_1}\langle x_1\rangle + \frac{r_1 N_1^*}{k_1}\langle x_2\rangle - \frac{r_1}{k_1}\langle x_1^2\rangle + \frac{r_1}{k_1}\langle x_1 x_2\rangle \quad (3.6a)$$

$$\frac{d\langle x_2\rangle}{dt} = \frac{br_2 N_2^*}{k_2}\langle x_1\rangle - \frac{r_2 N_2^*}{k_2}\langle x_2\rangle - \frac{r_2}{k_2}\langle x_2^2\rangle + \frac{br_2}{k_2}\langle x_1 x_2\rangle \quad (3.6b)$$

$$\frac{d\langle x_1^2\rangle}{dt} = 2[-\frac{r_1 N_1^*}{k_1}\langle x_1^2\rangle + \frac{r_1 N_1^*}{k_1}\langle x_1 x_2\rangle - \frac{r_1}{k_1}\langle x_1^3\rangle + \frac{r_1}{k_1}\langle x_1^2 x_2\rangle + \varepsilon_1] \quad (3.6c)$$

$$\frac{d\langle x_2^2\rangle}{dt} = 2[\frac{br_2 N_2^*}{k_2}\langle x_1 x_2\rangle - \frac{r_2 N_2^*}{k_2}\langle x_2^2\rangle - \frac{r_2}{k_2}\langle x_2^3\rangle + \frac{br_2}{k_2}\langle x_1 x_2^2\rangle + \varepsilon_2] \quad (3.6d)$$

$$\frac{d\langle x_1 x_2\rangle}{dt} = -(\frac{r_1 N_1^*}{k_1} + \frac{r_2 N_2^*}{k_2})\langle x_1 x_2\rangle + \frac{br_2 N_2^*}{k_2}\langle x_1^2\rangle + \frac{r_1 N_1^*}{k_1}\langle x_2^2\rangle + (\frac{r_1}{k_1} - \frac{r_2}{k_2})\langle x_1 x_2^2\rangle + (\frac{br_2}{k_2} - \frac{r_1}{k_1})\langle x_1^2 x_2\rangle \quad (3.6e)$$

where we have used the relations[6]:

$$\langle x_1\eta_1\rangle = \varepsilon_1, \quad \langle x_1\eta_2\rangle = 0, \quad \langle x_2\eta_1\rangle = 0, \quad \langle x_2\eta_2\rangle = \varepsilon_2 \quad (3.7)$$

Let us assume that the system size expansion is valid such that all correlations and variances are of the order of $(\frac{1}{N})$ compared to the averages, so that [6,11]

$$\langle x_1 x_2\rangle \propto \circ[\frac{\langle x_1\rangle}{N}] \quad or \quad \circ[\frac{\langle x_1\rangle}{N}] \quad (3.8a)$$

$$\langle x_1^2\rangle \propto \circ[\frac{\langle x_1\rangle}{N}] \quad (3.8b)$$

$$\langle x_2^2\rangle \propto \circ[\frac{\langle x_2\rangle}{N}] \quad (3.8c)$$

where N is the population size of the system. We also assume that the correlation ϵ_1 and ϵ_2 given by (3.7) decreases with the increase of the population size and they are assumed to be of the order of the inverse of the population size N.

$$\varepsilon_i \propto \circ[\frac{1}{N}]_{i=1,2} \quad (3.9)$$

Let us the relation (Valsakumar et al,1983)

$$\langle x_1^2 x_2\rangle = 2\langle x_1\rangle[\langle x_1x_2\rangle - \langle x_1\rangle\langle x_2\rangle] + \langle x_1^2\rangle\langle x_2\rangle \quad (3.10a)$$
$$\langle x_1 x_2^2\rangle = 2\langle x_2\rangle[\langle x_1x_2\rangle - \langle x_1\rangle\langle x_2\rangle] + \langle x_1\rangle\langle x_2^2\rangle \quad (3.10b)$$
$$\langle x_1^3\rangle = 2\langle x_1\rangle\langle x_1^2\rangle - 2\langle x_1\rangle^3 \quad (3.10c)$$

and keeping the lowest order term, we get the following reduced equations:

$$\frac{d\langle x_1^2\rangle}{dt} = -\frac{2r_1N_1^*}{k_1}\langle x_1^2\rangle + \frac{2r_1N_1^*}{k_1}\langle x_1x_2\rangle - \frac{6r_1}{k_1}\langle x_1\rangle\langle x_1^2\rangle + \frac{4r_1}{k_1}\langle x_1\rangle^3 + \frac{4r_1}{k_1}\langle x_1\rangle\langle x_1x_2\rangle - 4\langle x_1^2\rangle\langle x_2\rangle + 2\frac{r_1}{k_1}\langle x_1^2\rangle\langle x_2\rangle + \varepsilon_1 \quad (3.11a)$$

$$\frac{d\langle x_2^2\rangle}{dt} = \frac{2br_2N_2^*}{k_2}\langle x_1x_2\rangle - \frac{2r_2N_2^*}{k_2}\langle x_2^2\rangle - \frac{6r_2}{k_2}\langle x_2\rangle\langle x_2^2\rangle + \frac{4r_2}{k_2}\langle x_2\rangle^3 + \frac{4br_2}{k_2}\langle x_2\rangle\langle x_1x_2\rangle - \frac{4br_2}{k_2}\langle x_1\rangle\langle x_2\rangle^2 + \frac{2br_2}{k_2}\langle x_1\rangle\langle x_2\rangle^2 + \varepsilon_2 \quad (3.11b)$$

$$\frac{d\langle x_1x_2\rangle}{dt} = -(\frac{r_1N_1^*}{k_1} + \frac{r_2N_2^*}{k_2})\langle x_1x_2\rangle + \frac{br_2N_2^*}{k_2}\langle x_1^2\rangle + \frac{r_1N_1^*}{k_1}\langle x_2^2\rangle + 2(\frac{br_2}{k_2} - \frac{r_1}{k_1})\{\langle x_1\rangle\langle x_1x_2\rangle - \langle x_1\rangle^2\langle x_2\rangle\} + (\frac{br_2}{k_2} - \frac{r_1}{k_1})\langle x_1^2\rangle\langle x_2\rangle + 2(\frac{r_1}{k_1} - \frac{r_2}{k_2})\langle x_2\rangle\langle x_1x_2\rangle - \langle x_1\rangle\langle x_2\rangle^2\} + (\frac{r_1}{k_1} - \frac{r_2}{k_2})\langle x_1\rangle\langle x_2^2\rangle \quad (3.11c)$$

We shall now replace the average $< x_1 >$ and $< x_2 >$ in equations (3.11) by their steady equilibrium vales given by

$$\langle x_1\rangle = 0, \quad \langle x_2\rangle = 0 \quad (3.12)$$

Then the equations (3.11) become

$$\frac{d\langle x_1^2\rangle}{dt} = -\frac{2r_1N_1^*}{k_1}\langle x_1^2\rangle + \frac{2r_1N_1^*}{k_1}\langle x_1x_2\rangle \quad (3.13a)$$
$$\frac{d\langle x_2^2\rangle}{dt} = \frac{2br_2N_2^*}{k_2}\langle x_1x_2\rangle - \frac{2r_2N_2^*}{k_2}\langle x_2^2\rangle \quad (3.13b)$$
$$\frac{d\langle x_1x_2\rangle}{dt} = -(\frac{r_1N_1^*}{k_1} + \frac{r_2N_2^*}{k_2})\langle x_1x_2\rangle + \frac{br_2N_2^*}{k_2}\langle x_1^2\rangle + \frac{r_1N_1^*}{k_1}\langle x_2^2\rangle \quad (3.13c)$$

Equations (3.13)are the required moment equations which are ordinary coupled-linear equations.

4 Non-equilibrium Fluctuation and Stability Analysis:

To solve the system of equation (3.13) for the steady state $E^*: (N_1^*, N_2^*) = \left(\frac{k_1+k_2}{(1-b)}, \frac{k_2+bk_1}{(1-b)}\right)$ we rewrite them as

$$[D + \frac{2r_1N_1^*}{k_1}]\langle x_1^2\rangle = \frac{2r_1N_1^*}{k_1}\langle x_1x_2\rangle \quad (4.1a)$$

$$[D + \frac{2r_2N_2^*}{k_2}]\langle x_2^2\rangle = \frac{2br_2N_2^*}{k_2}\langle x_1x_2\rangle \quad (4.1b)$$

$$[D + \frac{r_1N_1^*}{k_1} + \frac{r_2N_2^*}{k_2}]\langle x_1x_2\rangle = \frac{br_2N_2^*}{k_2}\langle x_1^2\rangle + \frac{r_1N_1^*}{k_1}\langle x_2^2\rangle \quad (4.1c)$$

Multiplying (4.1c) by $[D+\frac{2r_1N_1^*}{k_1}]$ and $[D+\frac{2r_2N_2^*}{k_2}]$, using (4.1b) and (4.1a) we get ,

$$[(D+\frac{r_1N_1^*}{k_1}+\frac{r_2N_2^*}{k_2})(D+\frac{2r_1N_1^*}{k_1})(D+\frac{2r_2N_2^*}{k_2})]\langle x_1x_2\rangle = \frac{4br_1r_2N_1^*N_2^*}{k_1k_2}[D+\frac{r_1N_1^*}{k_1}+\frac{r_2N_2^*}{k_2}]\langle x_1x_2\rangle \quad (4.2)$$

$$[D^3+3(\frac{r_1N_1^*}{k_1}+\frac{r_2N_2^*}{k_2})D_2+2\{(\frac{r_1N_1^*}{k_1}+\frac{r_2N_2^*}{k_2})^2+\frac{2(1-b)r_1r_2N_1^*N_2^*}{k_1k_2}\}D + \frac{4(1-b)r_1r_2N_1^*N_2^*}{k_1k_2}(\frac{r_1N_1^*}{k_1}+\frac{r_2N_2^*}{k_2})]\langle x_1x_2\rangle = 0 \quad (4.3)$$

Now for simplicity of the calculation we set , $u=\frac{r_1N_1^*}{k_1}>0$ and $v=\frac{r_2N_2^*}{2}>0$, then the equation (4.3) reduces to

$$[D^3+3(u+v)D^2+2\{(u+v)^2+2(1-b)uv\}D+4(1-b)uv(u+v)]\langle x_1x_2\rangle = 0 \quad (4.4)$$

Taking $<x_1x_2>=Ae^{mt}$ as a trial solution, we get auxiliary equation as

$$[m^3+3(u+v)m^2+2\{(u+v)^2+2(1-b)uv\}m+4(1-b)uv(u+v)]\langle x_1x_2\rangle = 0 \quad (4.5)$$

To find the nature of the roots we can rewrite it as

$$a_1m^3+3a_2m^2+3a_3m+a_4=0 \quad (4.6)$$

Where $a_1=1, a_2=u+v, a_3=\frac{2}{3}[(u+v)^2+2(1-b)uv]$ and $a_4=4(1-b)(u+v)uv$. Now eliminate the second degree term from the equation (4.6) we get

$$z^3+3Hz+G=0 \quad (4.7)$$

where $z=a_1m+a_2$, $H=a_1a_3-a_2^2$ and $G=a_1^2a_4-3a_1a_2a_3+2a_2^3$ i.e.,

$$z=m+(u+v) \quad (4.8)$$

$$H=-\frac{1}{3}[(u-v)^2+4buv] \quad (4.9)$$

$$G=2(1-b)(u+v)uv \quad (4.10)$$

Then we calculate the expression $G^2+4H^3=-\frac{4}{27}[\{(u-v)^2+4buv\}^3-27(1-b)^2(u+v)^2u^2v^2]$, which determine the roots of the equation (4.7).

Case-I: If $\{(u-v)^2+4buv\}^3>27(1-b)^2(u+v)^2u^2v^2$ then all the roots are real. Then from the equation (4.6) we set

$$P(m)=a_1m^3+3a_2m^2+3a_3m+a_4 \quad (4.11)$$

where $a_i > 0$,$i = 1, 2, 3, 4$.

$$P(-m) = -a_1m^3 + 3a_2m^2 - 3a_3m + a_4 \tag{4.12}$$

Then by the Descartes' rule of sign the number of change of sign in $P(m)$ is zero. So there is no positive real solution. But the number of change of sign in $P(-m)$ is three. So the equation $P(m) = 0$ has exactly three negative real solutions. In this case the required solution is of the form

$$\langle x_1x_2\rangle = L_1e^{\alpha_1 t} + L_2e^{\alpha_2 t} + L_3e^{\alpha_3 t} \tag{4.13}$$

$$\langle x_2^2\rangle = M_1e^{\alpha_1 t} + M_2e^{\alpha_2 t} + M_3e^{\alpha_3 t} + c_1 \tag{4.14}$$

$$\langle x_1^2\rangle = N_1e^{\alpha_1 t} + N_2e^{\alpha_2 t} + N_3e^{\alpha_3 t} + c_2 \tag{4.15}$$

where L_i's ,M_i's and N_i's are constants (i=1,2,3). c_1 and c_2 are integrating constants. When $t \longrightarrow \infty$ then $\langle x_1x_2\rangle$, $\langle x_2^2\rangle$, $\langle x_1^2\rangle$ tend to finite numbers i.e., the moment remains finite after a large time. So the system is stable.

Case-II: If $\{(u-v)^2 + 4buv\}^3 > 27(1-b)^2(u+v)^2u^2v^2$ then one root is real and other two roots are imaginary, both of them are conjugate as the coefficients are real. From the equation (4.7) we get

$$z^3 + 3Hz + G = 0 \tag{4.16}$$

To find the solution of (4.16) we get

$$\begin{aligned} z &= p + q \qquad (4.17)\\ or,\ z^3 &= p^3 + q^3 + 3pqz \\ or,\ z^3 - 3pqz - (p^3+q^3) &= 0 \end{aligned} \tag{4.18}$$

Comparing (4.16) with (4.18) we get $pq = -H$ and $p^3 + q^3 = -G$. After simple calculation we get

$$p^3 = \frac{1}{2}\{-G + \sqrt{G^2 + 4H^3}\} \tag{4.19}$$

and

$$q^3 = \frac{1}{2}\{-G - \sqrt{G^2 + 4H^3}\} \tag{4.20}$$

Let β be any value of $[\frac{1}{2}\{-G + \sqrt{G^2 + 4H^3}\}]^{\frac{1}{3}}$, which is real and negative. Then other two values of p are $\omega\beta$,$\omega^2\beta$.

Now it is clear that $Re(\omega\beta) > 0$, $Re(\omega^2\beta) > 0$
where ω is a complex cube root of unity.

Similarly, Let γ be any value of $[\frac{1}{2}\{-G - \sqrt{G^2 + 4H^3}\}]^{\frac{1}{3}}$, which is real and negative. Then other two values of q are $\omega\gamma$,$\omega^2\gamma$.

Now it is clear that
$Re(\omega\gamma) > 0$, $Re(\omega^2\gamma) > 0$
where ω is a complex cube root of unity.

As $pq > 0$ and $p^3+q^3 < 0$ then we said that $\{(\beta,\gamma),(\omega\beta,\omega\gamma),(\omega^2\beta,\omega^2\gamma)\}$ or $\{(\beta,\gamma),(\omega\beta,\omega^2\gamma),(\omega^2\beta,\omega\gamma)\}$ are two set of solution of z of the form (p,q).

So one real root is negative and the real part of two complex roots are positive. In this case the required solution is of the form

$$\langle x_1x_2\rangle = A_1e^{(\beta+\gamma)t} + e^{\lambda_1 t}[A_2\cos(w_1t) + A_3\sin(w_1t)] \tag{4.21}$$

$$\langle x_2^2\rangle = B_1e^{(\beta+\gamma)t} + e^{\lambda_1 t}[B_2\cos(w_1t) + B_3\sin(w_1t)] + C \tag{4.22}$$

$$\langle x_1^2\rangle = C_1e^{(\beta+\gamma)t} + e^{\lambda_1 t}[C_2\cos(w_1t) + C_3\sin(w_1t)] + D \tag{4.23}$$

or of the form

$$\langle x_1x_2\rangle = A_1e^{(\beta+\gamma)t} + e^{\lambda_2 t}[A_2\cos(w_2t) + A_3\sin(w_2t)] \tag{4.24}$$

$$\langle x_2^2\rangle = B_1e^{(\beta+\gamma)t} + e^{\lambda_2 t}[B_2\cos(w_2t) + B_3\sin(w_2t)] + C \tag{4.25}$$

$$\langle x_1^2\rangle = C_1e^{(\beta+\gamma)t} + e^{\lambda_2 t}[C_2\cos(w_2t) + C_3\sin(w_2t)] + D \tag{4.26}$$

where $A_i's$, $B_i's$ and $C_i's$ are constants (i=1,2,3). C and D are integrating constants. $\lambda_i \pm w_i$ are the complex roots where $.\lambda_i > 0$. In this case $< x_1x_2 >. < x_1^2 > and < x_2^2 >$ diverges with increasing time. Consequently the system is stochastically unstable in the sense of second order moments.

5 Conclusion

In the present paper i have made comparative study of stability of ecological system under both deterministic and stochastic perturbation. The system under consideration is a mutualistic Lotka-Volterra system. The interior equilibrium point $E^*:(N_1^*, N_2^*) = \left(\frac{k_1+k_2}{(1-b)}, \frac{k_2+bk_1}{(1-b)}\right)$ of the system is stable deterministically when$0 < b < 1$ which result from the existence condition of the equilibrium. But when the system is perturbed ,then under certain condition described in case-I the system is stochastically stable and in the reverse condition described in case-II the system is unstable. In a particular case if $r_1 = k_1 = r_2 = k_2 = 1$ then for 0 < b < 0:58 the system is unstable and for 0:58 < b < 1 the system is stochastically stable. The parameter b is thus a bifurcation parameter. Thus the system behavior changes when b crossing the bifurcation point b = 0:58.

Acknowledgement
The author wishes to thank Prof. C.G. Chakrabarti, Ex S.N.Bose Professor of Applied Mathematics, Calcutta University for giving me valuable suggestion to prepare the paper.

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