Stars Evolution, String Self Energy and Nonsingular Black Hole on the Basis of Generalized Special Relativity

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Abstract:
Generalized Special Relativity energy-momentum relation beside the positivity or negativity of energy was used to construct star evolution model. In the first approach short range repulsive beside long range attractive gravity force are assumed to contribute to the total energy. This shows the existence of finite self energy of matter in the form of string. It shows that the star radius is that of General Relativity black hole radius. The minimization of energy with respect to potential, radius and mass shows in all cases the string nature of matter building blocks. The star evolution to become supernova or black hole is shown to be related to the relation of thermal to attractive gravity force in the same sense shown by General Relativity.

Keywords: star evolution, black hole, string, self-energy, generalized special relativity.

1. Introduction:
General Relativity Theory(GR) is one of the most successful theory that describes the universe[1].The socalled big bang model describes the evolution of the universe[2].It states that the universe starts with singularity in space-time. It then expands, where matter, i.e. elementary particles is formed at early universe. These particles join together to form light atoms.Later on these particles are assembled in a cloud forming galaxies, stars, planets and all other astronomical objects[3].The formation of stars is one of most striking features of General Relativity(GR). These stars can be come white dwarfs or red giant stars, supernova or black holes [4]. However the evolution of stars suffers from noticeable setbacks, for instance the so called black holes results from space-time singularity which means break down of the laws of physics [5]. This draw back can be cured in this work by using Generalized Special Relativity (GSR).

This is done in Section two. Sections three and four are devoted for discussion and conclusion respectively.

2. Conditions of StarsEvolution:

Consider a short range repulsive gravity field derived by some others of the form [6]:

$$\varphi_s = \frac{c_1}{r} e^{-c_0 r} \quad (1)$$

$$\varphi_L = -\frac{GM}{r} \quad (2)$$

$$\varphi = \varphi_s + \varphi_L = \frac{c_1}{r} e^{-c_0 r} - \frac{GM}{r}$$

$$= \frac{1}{r}[c_1 e^{-c_0 r} - GM] (3)$$

$$E = mc^2 = m_0 c^2 \left(1 + \frac{2\varphi}{c^2}\right)\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{-1/2} \quad (4)$$

For small r or strictly speaking small$c_0 r$:

$$e^{-c_0 r} = 1 - c_0 r \quad (5)$$

$$\varphi = \frac{1}{r}[c_1 - c_1 c_0 r - GM] (6)$$

To makeφ finite, one needs

$$c_1 = GM \quad (7)$$

Thus

$$\varphi = -\frac{c_1 c_0 r}{r} = -c_0 c_1 = -c_0 GM \quad (8)$$

If one assumes that for$r \to 0$ energy is minimum, i.e.

$$\frac{dE}{d\varphi} = 0 \quad (9)$$

It follows that

$$\frac{dE}{d\varphi} = \frac{\left(\frac{2}{c^2}\right) m_0 c^2}{\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{1/2}} - \frac{\frac{1}{2} \times \frac{2}{c^2}\left(1 + \frac{2\varphi}{c^2}\right) m_0 c^2}{\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{3/2}} = 0$$

$$\frac{m_0\left[2 + \frac{4\varphi}{c^2} - \frac{2v^2}{c^2} - 1 - \frac{2\varphi}{c^2}\right]}{\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{3/2}} = 0$$

$$\frac{2\varphi}{c^2} - \frac{2v^2}{c^2} + 1 = 0$$

$$\varphi = v^2 - \frac{c^2}{2} \quad (10)$$

For particle at rest

$v = 0$

$$\varphi = -\frac{c^2}{2} \quad (11)$$

For photon
$v = c$

$$\varphi = \frac{c^2}{2} \quad (12)$$

Since the star is a particle at rest thus (see equation (8))

$$\varphi = -c_0 GM = -\frac{c^2}{2} \quad (13)$$

Thus

$$c_0 = \frac{c^2}{2GM} \quad (14)$$

$$\varphi_s = \frac{GM}{r} e^{-\frac{c^2}{2GM} r} \quad (15)$$

It is very interesting to note that for large star having very large mass, such that

$$\frac{c^2}{2GM} r < 1 \quad (16)$$

$$r < \frac{2GM}{c^2} \quad (17)$$

Equation (15) yields

$$\varphi_s = \frac{GM}{r}\left(1 - \frac{c^2}{2GM} r\right) = \frac{GM}{r} - \frac{c^2}{2} \quad (18)$$

If

$$\varphi = -\varphi_s = \frac{c^2}{2} - \frac{GM}{r} \quad (19)$$

Thus the short range gravity potential reduces numerically to long range gravity potential beside zero point gravity potential corresponding to rest mass energy

$$E_0 = m|\varphi| = \frac{1}{2} mc^2 \quad (20)$$

It is very interesting to note that this is consistent with the zero point energy of harmonic oscillator

$$E_0 = \frac{1}{2} \hbar\omega \quad (21)$$

Since it means that

$$mc^2 = \hbar\omega \quad (22)$$

Which conforms with the Einstein and Plank expressions of energy. It also conforms with De Broglie hypothesis that

$$p = mc = \frac{mc^2}{c} = \frac{\hbar\omega}{c} = \frac{h}{\lambda} = \hbar k \quad (23)$$

This also agrees with quantum hypothesis where

$$\Psi = Ae^{i(kx-\omega t)} = Ae^{\frac{i}{\hbar}(px-Et)} \quad (24)$$

This means that at early stage of star evolution when

$$r \to 0 \tag{25}$$

The particles constituting the star beas a string.

It is also very interesting to note that for ordinary classical particle which is at rest equations (11) and (4) yields

$$E = \frac{(1 + 2\varphi/c^2)m_0c^2}{\sqrt{(1 + 2\varphi/c^2 - v^2/c^2)}} = \frac{(1-1)m_0c^2}{\sqrt{1 - 1 - v^2/c^2}} = \frac{0}{0}m_0c^2 \tag{26}$$

While for a photon which obeys quantum laws equations (12) and (4) gives

$$E = \frac{2m_0c^2}{\sqrt{2-1}} = 2m_0c^2 \tag{27}$$

This conforms to the fact that photons can produce particle pairs such that one can consider a star as consisting of photons gas. When minimizing Ew.r.tr

$$E = m_0c^2\left(1 - \frac{2MG}{rc^2}\right)\left(1 - \frac{2MG}{rc^2} - \frac{v^2}{c^2}\right)^{-1/2} \tag{28}$$

When the stars particles speed are small compared to speed of light

$$\frac{v^2}{c^2} \ll 1$$

Thus

$$E = m_0c^2\left(1 - \frac{2MG}{rc^2}\right)^{1/2} \tag{29}$$

$$\frac{dE}{dr} = m_0c^2\left(\frac{2MG}{r^2c^2}\right)\left(\frac{1}{2}\right)\left(1 - \frac{2MG}{rc^2}\right)^{-1/2}$$

$$= \frac{m_0c^2\left(\frac{MG}{r^2c^2}\right)\left(1 - \frac{2MG}{rc^2}\right)}{\left(1 - \frac{2MG}{rc^2}\right)^{3/2}} = 0$$

Thus the radius which makes E minimum is given by

$$1 - \frac{2MG}{rc^2} = 0$$

$$\frac{2MG}{rc^2} = 1$$

$$r = \frac{2MG}{c^2} \tag{30}$$

This is the black hole radius.

Thus the potential is given by

$$\varphi = -\frac{MG}{r} = -\frac{MGc^2}{2MG} = -\frac{c^2}{2} \tag{31}$$

It is very striking to note that this value is typical to the value of φ for

$$r \to 0 \tag{32}$$

Where in classical Limit equation (30) gives

$$c \to \infty$$

$$r = \frac{2MG}{c^2} \to 0 \quad (33)$$

Also when energy is minimum

$$E = Mc^2 \to 0 \quad (34)$$

$$M \to 0$$

$$r = \frac{2MG}{c^2} \to 0 \quad (35)$$

The stability condition requires also minimization of E w.r.t to star mass. Consider the star as a gas consisting of particles with rest mass m_0. By assuming that each particle is subjected to the effect of attractive gravitational potential

$$\varphi = -\frac{MG}{R} \quad (36)$$

The energy of m_0 is given by

$$E = m_0c^2\left(1+\frac{2\varphi}{c^2}\right)\left(1+\frac{2\varphi}{c^2}-\frac{v^2}{c^2}\right)^{-1/2}$$

$$E = m_0c^2\left(1-\frac{2MG}{Rc^2}\right)\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{-1/2} \quad (37)$$

minimizing E w.r.t M yields

$$\frac{dE}{dM} = m_0c^2\left[\frac{-\frac{2G}{Rc^2}}{\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{1/2}} + \frac{\left(1-\frac{2MG}{Rc^2}\right)\left(-\frac{1}{2}\times\frac{-2G}{Rc^2}\right)}{\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{3/2}}\right] = 0$$

Thus

$$\frac{-\frac{2G}{Rc^2}\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)+\frac{G}{Rc^2}\left(1-\frac{2MG}{Rc^2}\right)}{\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)^{3/2}} = 0$$

If one consider

$$v^2 \ll c^2$$

$$-\frac{2G}{Rc^2}\left(1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}\right)+\frac{G}{Rc^2}\left(1-\frac{2MG}{Rc^2}\right) = 0$$

$$-\frac{G}{Rc^2}\left(1-\frac{2MG}{Rc^2}\right) = 0$$

This requires

$$\frac{2MG}{Rc^2} = 1 \quad (38)$$

Thus the mass which makes E minimum is

$$M = \frac{Rc^2}{2G} \quad (39)$$

The gravitational energy takes the form

$$\varphi = -\frac{MG}{R} = -\frac{c^2}{2} \qquad (40)$$

According to General Relativity and The standard big bang model the universe expand or contract when

$$E = + \qquad \text{Expansion}$$
$$E = - \qquad \text{Contraction}$$

One can use the same argument for the star, on the basis of Generalized Special Relativity(GSR) expression for energy E , which is given by

$$E^2 = g_{00}^{-1}\, p^2c^2 + g_{00}m_0^2c^4 \qquad (41)$$

The conditions of star evolution can be started by adopting classical Limit, where

$$\frac{\varphi}{c^2} \ll 1 \quad ; \quad \frac{v^2}{c^2} \ll 1$$

To get

$$E = m_0c^2\left(1 + \frac{2\varphi}{c^2}\right)\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{-1/2}$$

$$E = m_0c^2\left(1 + \frac{2\varphi}{c^2}\right)\left(1 - \frac{\varphi}{c^2} + \frac{v^2}{2c^2}\right)$$

Neglecting higher order terms, yields

$$E = m_0c^2\left(1 - \frac{\varphi}{c^2} + \frac{v^2}{2c^2} + \frac{2\varphi}{c^2} - \frac{2\varphi^2}{c^4} + \frac{\varphi v^2}{c^4}\right)$$

$$E = m_0c^2 + m_0\varphi + \frac{1}{2}m_0v^2$$

$$= m_0c^2 + V + T_K \qquad (42)$$

Assuming the kinetic energy is due to thermal motion

$$U = \frac{3}{2}KT = \frac{1}{2}m_0v^2 \qquad (43)$$

Assuming also the potential energy of mass m_0 to be

$$V = -\frac{Gm_0M}{R} \qquad (44)$$

Thus the energy E become

$$E = m_0c^2 + \frac{3}{2}KT - \frac{Gm_0M}{R} \qquad (45)$$

The star explodes and expands when

$$E = m_0c^2 + \frac{3}{2}KT - \frac{Gm_0M}{R} > 0 \qquad (46)$$

i.e

$$m_0c^2 + \frac{3}{2}KT > \frac{Gm_0M}{R} \qquad (47)$$

This is quite obvious from the point of view of common since this equation indicates that expansion happen when thermal and rest mass energies exceeds attractive gravity energy.However it collapse and contract when

$$m_0c^2 + \frac{3}{2}KT < \frac{Gm_0M}{r} \quad (48)$$

$$E = g_{00}^{1/2}[g_{00}^{-2}\, p^2c^2 + m_0^2c^4]^{1/2} \quad (49)$$

consider the case

$$g_{00}^{-2}p^2c^2 \gg m_0^2c^4 \quad (50)$$

$$E = g_{00}^{1/2}\, g_{00}^{-1}\, pc\left[1 + g_{00}^2\frac{m_0^2c^4}{p^2c^2}\right]^{1/2}$$

$$E = g_{00}^{-1/2}\, pc\left[1 + \frac{1}{2}g_{00}^2\frac{m_0^2c^4}{p^2c^2}\right]$$

$$E = g_{00}^{-1/2}\, pc\left[\frac{2p^2c^2 + g_{00}^2m_0^2c^4}{2p^2c^2}\right]$$

But

$$\frac{3}{2}KT = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

Thus the momentum takes the form

$$p^2 = 3mKT \quad (51)$$

Therefore

$$E = g_{00}^{-1/2}\left[\frac{6mc^2KT + \left(1 - \frac{2MG}{Rc^2}\right)^2 m_0^2c^4}{2pc}\right] \quad (52)$$

If

$$\varphi = \frac{MG}{Rc^2} < 1 \quad (53)$$

$$\left(1 - \frac{2MG}{Rc^2}\right)^2 = 1 - \frac{4MG}{Rc^2} + \frac{4M^2G^2}{R^2c^4} \approx 1 - \frac{4MG}{Rc^2} \quad (54)$$

$$E = g_{00}^{-1/2}\left[\frac{6mc^2KT + m_0{}^2c^4 - \frac{4MGm_0^2c^4}{Rc^2}}{2pc}\right] \quad (55)$$

Therefore the star explodes when

$$\frac{1}{3}m_0^2c^4 + 2mc^2KT - \frac{4}{3}\frac{MG}{R}m_0^2c^2 > 0 \quad (56)$$

When one assumes

$$m \approx m_0$$

$$\frac{1}{3}m_0c^2 + 2KT > \frac{4}{3}\frac{MGm_0}{R} \quad (57)$$

But the star collapse when

$$\frac{1}{3}m_0c^2 + 2KT - \frac{4}{3}\frac{MGm_0^2c^2}{R} < 0 \qquad (58)$$

i.e when

$$\frac{1}{3}m_0c^2 + 2KT < \frac{4}{3}\frac{MGm_0}{R} \qquad (59)$$

When

$$g_{00}^{-2}\ p^2c^2 \ll m_0^2c^4 \qquad (60)$$

$$E = g_{00}^{1/2}m_0c^2\left[1 + \frac{g_{00}^{-2}p^2c^2}{m_0^2c^4}\right]^{1/2}$$

$$E = g_{00}^{1/2}\ m_0c^2\left[1 + \frac{g_{00}^{-2}p^2c^2}{2m_0^2c^4}\right]$$

$$E = g_{00}^{1/2}m_0c^2\left[\frac{2m_0^2c^4g_{00}^2 + p^2c^2}{2m_0^2c^4}\right]g_{00}^{-2} \qquad (61)$$

Using equation (51) gives

$$E = g_{00}^{-3/2}\left[\frac{2m_0^2c^4\left(1 - \frac{2MG}{Rc^2}\right)^2 + 3mc^2KT}{2m_0c^2}\right] \qquad (62)$$

With the aid of the approximation in equation (54) , one gets

$$E = \frac{g_{00}^{-3/2}}{2m_0c^2}\left[2m_0^2c^4 - \frac{8GM}{Rc^2}m_0^2c^4 + 3mc^2KT\right] \qquad (63)$$

$$E = \frac{g_{00}^{-3/2}}{2m_0c^2}\left[2m_0^2c^4 + 3mc^2KT - \frac{8GM}{R}m_0^2c^2\right] \qquad (64)$$

Thus explosion is expected when

$$E > 0 \qquad (65)$$

i.e

$$2m_0^2c^4 + 3mc^2KT - \frac{8GMm_0^2c^2}{R} > 0 \qquad (66)$$

$$\frac{2}{3}m_0c^2 + KT > \frac{8GMm_0}{R} \qquad (67)$$

While contraction takes place when

$$E < 0 \qquad (68)$$

$$\frac{2}{3}m_0c^2 + KT < \frac{8GMm_0}{R} \qquad (69)$$

3. Discussion:

According to Generalized General Relativity theory there is a short range repulsive gravitational force given by equation (1). In this work one assumes the ordinary. Attractive gravity force beside the repulsive force (see equation (3)). To find mass self energy, by assuming it resulting from gravity potential energy only, one considers the potential behavior when r approaches zero. The finiteness of φ requires the constant parameter $c_1$to be given by (7). This makes φfinite and constant as shown by equation (8). Since the matter self energyis the minimum energy, thus one needs minimization of E. Minimizing E, by using Generalized Special Relativity expression in (9) required φ to be related to the speed of light in vacuum. If one believes in Einstein energy – mass relation and Plank energy expressions for photon, equation (20). Shows that mass self energy expression resembles the zero point energy of harmonic oscillator (see equation (21)). This indicates that the matter building blocks are strings. It is also very interesting to note that the physical restrictions imposed on $c_0$and c_1 , indicates, according to equation (15), that the field becomes string when the mass increases and the distance decreases. This conforms with common sense and physical intuition.Surprisingly the minimization of energy w.r.t radius and mass lead to the same radius of black hole and string nature of building blocks. This means that black holes are the state of matter where energy is minimum w.r.t to potential, mass and radius (see equations (29) – (40))The star evolution to be came black hole or supernova is also tackled for slow speed and weak potential. Using different approximations. For weak field and slow speed (see (42)), high speed and large momentum compared to the rest mass (see (50)) and small momentum compared to the rest mass (see (60)), the star evolution gives the same scenario. For all approximation supernova is observed when rest mass energy and thermal energy exceeds attractive gravity energy as shown by equations (47), (57) and (67). In contrary the star became a black hole when thermal and mass energy exceeds attractive gravity force. This result conforms with common sense and General Relativity approach. The incorporation of rest mass beside thermal energy may be related to the fact that in the vicinity of Centre of mass repulsive gravity acts as a repulsive force against attractive force.

4. Conclusion:

Generalized Special Relativity model is successful in describing the star evolution. It shows that the mass building blocks are strings. It also shows that black holes have minimum no zero radius. It also shows that stars became black holes when attractive force dominates, while it became supernova when thermal energy dominates. It also shows that the elementary particles have finite self energy when the radius becomes vanishingly small.

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