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Modelling and Adequacy of Vector Autoregressive

G. O. Nwafor [1] E. O. Odok[2] J. U. Atsu [3] I. M .Esuabana [4]

[1, 2, 3] Department of Mathematics and Statistics
Cross River University of Technology Calabar, Nigeria

[4]Department of Mathematics,
University of Calabar, P.M.B 1115, Cross River State, Nigeria

Abstract

The use of VAR models can be justified in many ways. Here we employ the theory and evidence application of the method to explore the major building block and stepwise in determining the adequacy of vector autoregressive via specifying and estimating a model. The problem of the determination of lag order and adequacy of any stated model is overcome by the techniques employed in the study, which allows the estimated model an explicit economic interpretation. It is relevant to stress that, achieving stationarity is a precondition for the estimating of VAR model which the study explains. The class of tests to lag selection or model selection directly examines the properties of VAR residuals and AR characteristic polynomial with the inverse root of characteristic polynomial lies within the unit circle. The VAR residual portmanteau tests for autocorrelations checks the null hypothesis that all residual autocovariance are zero and thus achieves model adequacy.

Keywords: *Identification, specification, modelling, adequacy, vector autoregressive*

1. Introduction

The VAR model has been developed to address the fact that most questions of interest to empirical macro-economists involve several variables and thus must be addressed using multivariate methods. Adequate results from VAR models are superior to those from univariate models, quite flexible because they can be made conditional on the potential future paths of specified variables in the models. However, adding variables to the VAR creates complications, because the number of VAR parameters increases as the square of the number of variables, another limitation is that, without modification, standard VAR, miss nonlinearities.

[1] achieve a high and sustainable economic growth rate coupled with the economic indicators on the multivariate time series modeling of major economic indicators in Nigeria, aims at providing quantitative analysis of the dynamics on currency in circulation, exchange rate, external reserve, gross domestic product, money supply and price deflator. The study employed the newly developed multivariate time series estimation technique via Vector Autoregressive modeling to model the economic indicators in Nigeria.

The empirical result yields a stable and sustainable economic model for the six economic variables in the study.

The vector autoregressive (VAR) approach has become somewhat standard in modeling because compared to the structural approach; it avoids the need to provide a dynamic theory specifying the relationships among the jointly determined variables/observations. A VAR model in practice is the principle of straight forward algorithm.

[2] VAR methodology superficially resemble simultaneous equation modeling in that it consider several endogenous variables together. But each endogenous variable is explained by its lagged, or past, values and the lagged values of all other endogenous variables in the model, usually, there are no exogenous variables in the model.

The structural specification and estimation of reduced form VAR model Model checking, forecasting, Causality analysis, Structural specification and estimation, impulse response analysis and forecast error variance decomposition. To conduct structural analyses, one therefore starts from an unrestricted VAR(q) where all variables appear with the same lags in each equation, estimates the parameters of the VAR , imposes a minimal set of ”structural” restrictions, possibly consistent with a variety of behavioral theories, and constructs impulse responses, historical decomposition, etc. to structural shocks. In this sense, VARs are at the antipodes of maximum likelihood or generalized method of moment’s approaches: the majority of the theoretical restrictions are disregarded; there is no interest in estimating preference and technology parameters; and only a structural interpretation of the shocks is sought.

[3] standard practice in VAR analysis is to report results from Granger – causality test, impulse responses and forecast error variance decompositions.

2. Material and Methodology

The data type and source of this paper mainly, the secondary macroeconomic time series data in its analysis. All data used in the analysis was sourced from Central Bank of Nigeria Statistical Bulletin [4]. A number of alternative tests are available for testing whether a series is stationary or not, the Augmented Dickey – fuller (ADF) was applied.

[5] “if there is true simultaneity among a set of variable, they should all be treated on an equal footing, there should not be any a priori distinction between endogenous and exogenous variables”. It is in this spirit that [5] developed his VAR model.

The stochastic part y_t is a assumed to be generated by a VAR process of order p (VAR (p)) of the form.

$$Y_t = A_{1y_{t-1}} + A_{2y_{t-2}} + \cdots + A_{py_{t-p}} + \mu_t + \varepsilon_t \qquad (1)$$

Where

$A_i \quad \forall i = 1,2,\dots p$ are (kxk) parameter matrices.

The error process $\mu_t = (\mu_{1t}\mu_{2t},\dots\mu_{kt})'$ is a k – dimensional zero mean white noise process with covariance matrix:

$$E(\mu_t, \mu_t') = \varepsilon_\mu$$

In matrix notations the m time series

$$y_{it}, \quad i = 1,2,\dots m,$$

$$\text{and} \quad t = 1,\dots T$$

Where, t is the common length of the time series.

2.1 VAR Algorithm

The main steps of a VAR analysis,

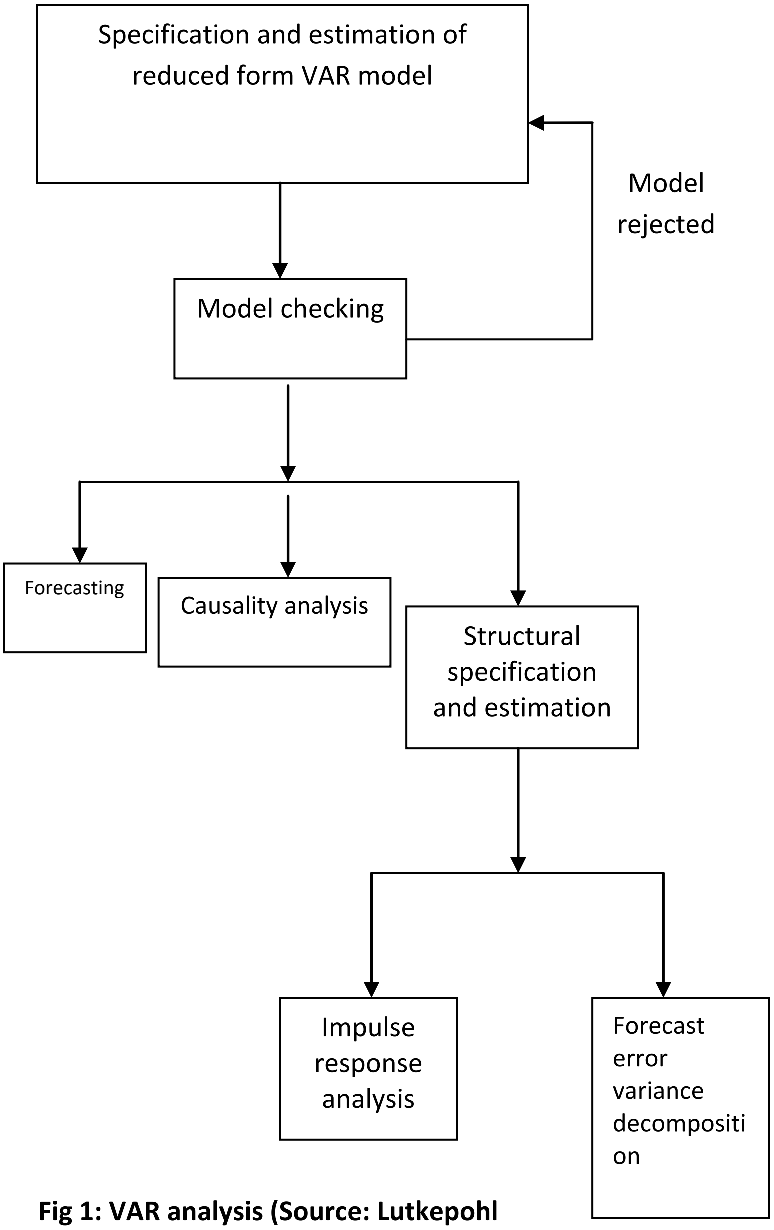

Fig 1: VAR analysis (Source: Lutkepohl (2007))

2.1.1 Model Specification

[1] states that model specification in the present context involves selection of the VAR order and in VECM also choosing the cointegration rank. [6] because the number of parameters in these models increases with the square of the number of variables it is also often desirable to impose zero restrictions on the parameter matrices and thereby eliminates some lagged variables from some of the equations of the systems.

2.1.2 Lag Length Selection/Choosing the Lag Order

There are several methods to select the lag length of a VAR. The simplest is based on a likelihood ratio (LR) test. . Here the model with a smaller number of lags is treated as an restricted version of a larger dimensional model. Since the two models are nested, under the null that the restricted model is correct, differences in the likelihoods should be small. The most common procedures for VAR order selected are sequential testing procedure and application of model selection criteria. For the purpose of any research study the model selection criteria will be applied.

2.1.3 Model Selection Criteria

The standard model selection criteria which are used in this context chosen the VAR order which minimizes them over a set of possible orders $m = 0, 1\ 2 \ldots\ldots\ Pmax$

The general form of a set of such criteria is

$$C(m) = \log\det(\Sigma_m^{\wedge}) + CT\varphi(m) \qquad (2)$$

Where $\Sigma_m^{\wedge} = T^{-1}\sum_{t=1}^{T}\hat{u}_t\hat{u}_t'$ is the residual covariance matrix estimator for a model of order m. φ(m) Is a function of the order m which penalizes large VAR orders. CT is a sequence which may depend on the sample size and identifies the specific criterion. The term $logdet(\hat{\epsilon}_m)$is a monincreasing function of the order m which $\varphi(m)$ increases with m. The lag order is chosen which optimally balances these two forces. [5], to estimate the system, the order p i.e. the maximal lag of the system has to be determined. The multivariate case with k variables, T observations, a constant term and a maximal lag of p, these criteria are as follows:

Final prediction error (FPE)

$$FPE(P) = \left[\frac{T+kp+1}{T-kp-1}\right]^k / \sum \hat{u}\hat{u}(p) / \qquad (3)$$

Akaike information criterion (AIC)

$$AIC(p) = In / \sum \hat{u}\hat{u}(p) / + (k + pk^2)\frac{2}{T} \qquad (4)$$

Hannan – Quinn criterion (HQ)

$$HQ(p) = In / \sum \hat{u}\hat{u}(p) / + (k + pk^2)\frac{2In(In(T))}{T} \qquad (5)$$

Shwarz criterion (SC)

$$SC(p) = In / \sum \hat{u}\hat{u}(p) / + (k + pk^2)\frac{In(T)}{T} \qquad (6)$$

$/ \sum \hat{u}\hat{u}(p) /$ is the determinant of the variance covariance matrix of the estimated residuals. [6], AIC suggests the largest order, SC chooses the smallest order and HQ is in between. The choice of the best model is based on the model selection criteria, the minimum model selection criteria compared to others. [2] VAR methodology superficially resembles simultaneous equation modeling in that it considers several endogenous variables together. But each endogenous variable is explained by its lagged, or past, values and the lagged values of all other endogenous variables in the model, usually, there are no exogenous variables in the model.

2.1.4 Identification

So far in this paper, economic theory on projections methods are used to derive the wold theorem; statistical and numerical analysis are used to estimate the parameters and the distributions of interesting functions of the parameters. Since VARs are reduced form models it is impossible to structurally interpret the dynamics induced by their disturbances unless economic theory comes into play.

The reduced form parameters are complicated functions of the structural ones and the resulting set of extensive cross equations restrictions could be used to disentangle the structural model if one is willing to take the model seriously as the process generating the data. When doubts about the quality of the model exist, one can still conduct inference as long as a subset of the model restrictions is credible or uncontroversial. Typical evaluations employed in the [1] on the VAR(4) estimation of economic indications upon which identification, specification, estimation of a reduced form VAR was done.

[8], modeling multivariate time series (MTS) data effectively is important for many decision making activities. Macro-economic practitioners frequently work with multivariate time series models such as Vector autoregressive (VAR), Vector error correction model (VECM), and factor augmented Autoregressive (ARs) as well as time-varying parameter.

2.1.5 The VAR Model

Then a Vector Autoregressive Model is defined as

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \\ \vdots \\ Y_{mt} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ . \\ \mu_m \end{pmatrix} + \begin{pmatrix} A_{11}^{(1)} & A_{12}^{(1)} & A_{1m}^{(1)} \\ A_{21}^{(1)} & A_{22}^{(1)} & A_{2m}^{(1)} \\ \vdots & \vdots & \vdots \\ A_{m1}^{(1)} & A_{m1}^{(1)} & A_{mm}^{(1)} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{m,t-1} \end{pmatrix}$$

$$+\cdots+\begin{pmatrix} A_{11}^{(p)} & A_{12}^{(p)} & A_{1m}^{(p)} \\ A_{21}^{(1)} & A_{22}^{(1)} & A_{2m}^{(p)} \\ \vdots & \vdots & \vdots \\ A_{m1}^{(p)} & A_{m1}^{(p)} & A_{mm}^{(p)} \end{pmatrix}\begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{m,t-p} \end{pmatrix}+\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{mt} \end{pmatrix} \quad (7)$$

Where

$Y_t = (y_{1t}, y_{2t}, \dots y_{mt})'$ denote $(nx1)$ vector of time series variables

A_i are (nxn) coefficient matrices

ε_t is an $(nx\,1)$ unobservable zero mean white noise vector process.

2.1.6 Tests for Residual Autocorrelation/Portmanteau Test

It is helpful to note that estimated residuals from VAR models are not white noise, while errors from the true VAR are white noise. The portmanteau test for residual autocorrelation checks the null hypothesis that all residual autocovariances are zero, (lutkepohl, 2007) that is

$$H_o: \epsilon(u_t, u_{t-1}') = 0 \quad i = 1\ 2 \dots ..$$

$$H_1: \epsilon(u_t, u_{t-1}') \neq 0 \quad i = 1\ 2 \dots ..$$

The test statistic is based on the residual autocovariances and has the form

$$Q_h = T\sum_{j=1}^{h} t_r\left(\hat{C}_j'\hat{C}_o^{-1}\hat{C}_j\hat{C}_0^{-1}\right) \quad (8)$$

Where

$$\hat{C}_j = T^{-1}\sum_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j}' \quad (9)$$

$\hat{u}_t'$ are the estimated residuals.For an unrestricted stationary VAR (p) process the null distribution of Q_h can be approximated by a $k^2(k^2(h-p))$ Distribution if T and h approach infinity such that h/T → 0. If there are parameters restriction the degrees of freedom of the approximated x^2 distribution are obtained as the differences between the number of (non-instantaneous) auto-covariance included in the statistics (k^2h) and the number of estimated VAR parameters.

2.1.7 Stability

The system is stable at the model (q) of all included variables if the empirical result indicates that the eigen values of the system obtainable in modulus, lies within the unit circle or, equivalently, if the eigen values of the companion matrix have modulus less than one. AR root table reports the inverse roots of the characteristic AR polynomial, lutkepohl (1991). The estimated VAR is stable (stationary) if all roots have modulus less than one and lies inside the unit circle. The roots of characteristics polynomial does not lies outside the unit circle. The VAR (q) is stable if the roots of

$$det\ \left(I_n = \pi_{12-\dots-=\pi r^p}\right) = 0 \quad (10)$$

2.1..8 Forecasting

Forecasting from a VAR (p) is a straight forward extension of forecasting from an Autoregressive (p). The multivariate wold form is [9]

$$Y_t = \mu + \varepsilon_t + \Psi_1\varepsilon_{t-1} + \Psi_2\varepsilon_{t-2} + \cdots \quad (11)$$

$$Y_t = \mu + \varepsilon_t + \Psi_1\varepsilon_{t+h-1} + \cdots + \Psi_{h-1}\varepsilon_{t+1}, \dots \quad (11.1)$$

Noted that

$$E(Y_t) = \mu$$

$$Var(Y_t) = E[(Y_t - \mu)(Y_t - \mu)']$$

$$= E\left[\left(\sum_{k=1}^{\infty} \Psi_k \varepsilon_{t-k}\right)\left(\sum_{k=1}^{\infty} \Psi_k \varepsilon_{t-k}\right)^{/}\right]$$

$$= \sum_{k=1}^{\infty} \Psi_k \sum \Psi_k' \qquad (11.2)$$

The minimum MSE linear forecast of Y_{t+h} based on it is

$$Y_{t+h}/t = \mu + \Psi_h \varepsilon_t + \Psi_{h+1}\varepsilon_{t-1} + \cdots \quad (11.3)$$

The forecast error is

$$\varepsilon_{t+h/t} = Y_{t+h} - Y_{t+h/t}, \qquad (11.4)$$

The forecast error MSE is

$$MSE\left(\varepsilon_{t+\frac{h}{t}}\right) = E\left[\varepsilon_{t+h/t}\varepsilon_{t+h/t}'\right] \qquad (11.5)$$

$$= \sum + \Psi_1 \sum \Psi_1' + \cdots + \Psi_{h-1} \sum \Psi_{h-1}'$$

$$= \sum_{s=1}^{h-1} \Psi_s \sum \Psi_s'$$

3. ESTIMATION RESULTS

An informal inspection of the trends in the logarithmic levels and differences was done, to avert false result; the order of integration of each of the variable is inspected using the [10] Augmented Dickey-Fuller (ADF) unit roots tests. The plot of each of the original series indicates that the variables are non stationary at their levels. However, the first difference indicates that the variables are stationary and provides some insights into the order of integration of the variables.

Table 1 Augmented Dickey-Fuller Unit Root Test

first difference		
variable	constant no trend	Constant Trend
ΔIn ClC	0.051130*	-2.63E-05***
ΔIn Exchange Rate	-0.0576***	0.00009***
ΔInExternal Reverve	0.04141***	-0.00010***
ΔIn GDP	0.00396**	0.00019***
ΔIn money supply	0.05117*	0.00018***
ΔIn price Deflator	0.05991*	-0.00032***

The Null hypothesis is that the series is non-stationary, or contain a unit root. The rejection of the null hypothesis based on the mackinnon critical values. *,** and *** indicates the rejection of the null hypothesis at 10%, 5% and 1% significance level respectively.

Table 2: VAR Lag Order Selection Criteria

Lag	LR	FPE	AIC	SC	HQ
0	NA	1.77E-10	-5.428844	-5.282383*	-5.369429
1	66.13999	1.79E-10	-5.416157	-4.390929	-5.000252
2	118.1071	1.03E-10	-5.972683	-4.068688	-5.200288
3	87.88225	7.71E-11	-6.279276	-3.496515	-5.150392
4	129.3806*	3.36e-11*	-7.13505*	-3.473526	-5.64968*
5	27.06777	4.80E-11	-6.824752	-2.284458	-4.982889
6	22.91169	7.20E-11	-6.48572	-1.06666	-4.287368
7	36.04466	8.94E-11	-6.36714	-0.069313	-3.812298
8	42.98844	9.78E-11	-6.411854	0.76474	-3.500522

* indicates lag order selected by the criterion ,LR: sequential modified LR test statistic (each test at 5%level), FPE: Final prediction error, AIC: Akaike information criterion, SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion. Based on the Table 2 of the model selection criteria it computes various criteria to select the lag order of an unrestricted VAR on various information criteria for all lags up to the specified maximum (If there are no exogenous variables in the VAR, the lag starts at 1; otherwise the lag starts at 0.) The table indicates the selected lag from each column criterion by an asterisk "*". For columns 0–8, these are the lags with the smallest value of the criterion. The chosen Akaike information criterion (AlC) has the least value and was identified to be application and best for the study. Thus, model 4 is the chosen model for this study since it has the minimum AlC compared to the others.

Table 3 VAR Residual Portmanteau Tests for Autocorrelations

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	3.994599	NA*	4.029639	NA*	NA*
2	14.16872	NA*	14.38383	NA*	NA*
3	27.25786	NA*	27.82357	NA*	NA*
4	52.56247	NA*	54.04006	NA*	NA*
5	82.56758	0.0952	85.40904	0.0642	67
6	110.2313	0.295	114.5956	0.2045	103
7	142.8224	0.3947	149.299	0.2602	139
8	202.8314	0.0734	213.7947	0.0243	175
9	226.6797	0.2185	239.6678	0.0855	211
10	252.6127	0.3895	268.0706	0.1705	247
11	273.4452	0.6473	291.1066	0.3574	283
12	305.3112	0.6997	326.6852	0.3714	319

Null Hypothesis: no residual autocorrelations up to lag h. *the test is valid only for lags larger than the VAR lag order. D.F is degrees of freedom for (approximate) chi-square distribution.

The table 3 indicates that no root lies outside the unit circle, VAR satisfies the stability condition.

3.1.2 Roots of Characteristics Polynomial

Table 4 Roots of Characteristic Polynomial

Root	Modulus
0.018593 - 0.999676i	0.999849
0.018593 + 0.999676i	0.999849
-0.940879	0.940879
-0.013457 + 0.868429i	0.868534
-0.013457 - 0.868429i	0.868534
-0.798217 - 0.049903i	0.799775
-0.798217 + 0.049903i	0.799775
0.774217	0.774217
0.608286 - 0.363135i	0.708434
0.608286 + 0.363135i	0.708434
0.699907	0.699907
-0.548645 + 0.382819i	0.669001
-0.548645 - 0.382819i	0.669001
-0.336111 + 0.569123i	0.660962
-0.336111 - 0.569123i	0.660962
0.345912 - 0.468940i	0.582717
0.345912 + 0.468940i	0.582717
0.067321 + 0.558584i	0.562626
0.067321 - 0.558584i	0.562626
-0.552904	0.552904
-0.110531 + 0.537035i	0.548292
-0.110531 - 0.537035i	0.548292
0.493137	0.493137
0.034139	0.034139

The above result (table 4) indicates that the eigen values of the system obtainable in modulus lies within the unit circle. The roots of characteristics polynomial does not lies outside the unit circle: Hence from the analysis VAR satisfies the stability condition.

4. Conclusion

To summarize, any vector of time series can be represented with a constant coefficient VAR(∞) under linearity, stationarity and invertibility. Hence, one can interchangeably think of data or the VAR for the data. Also, with a finite stretch of data only a VAR (q), q finite, can be used. The Wold theorem implies, among other things, that VAR residuals must be white noise. A LR test can therefore be interpreted as a diagnostic to check whether residuals satisfy this property. Similarly, AIC, HQ and SC can be seen as trading-off the white noise assumption on the residuals with the best possible out-of-sample forecasting performance. Another class of tests to lag selection directly examines the properties of VAR residuals and AR characteristic polynomial (table 4) with the inverse root of Characteristic Polynomial lies within the unit circle. The specification and estimating a model and the checking its adequacy is of great interest. The VAR residual portmanteau tests for autocorrelations checks the null hypothesis that all residual autocovariance are zero. The Portmanteau Autocorrelation test computes the multivariate Box-Pierce/Ljung-Box *Q*-statistics for residual serial correlation up to the specified order (table 3). Under the null hypothesis of no serial correlation up to lag, both statistics are approximately distributed with degrees $k^2\ (h - p)$ of freedom where p is the VAR lag order.

The choice of this model is also consistent as all the variables appear to be an important explanatory variable in the study. It must be emphasized, however that the estimations in table 2 are to be given absolute relevance since the variables are potentially stationary (table 1).

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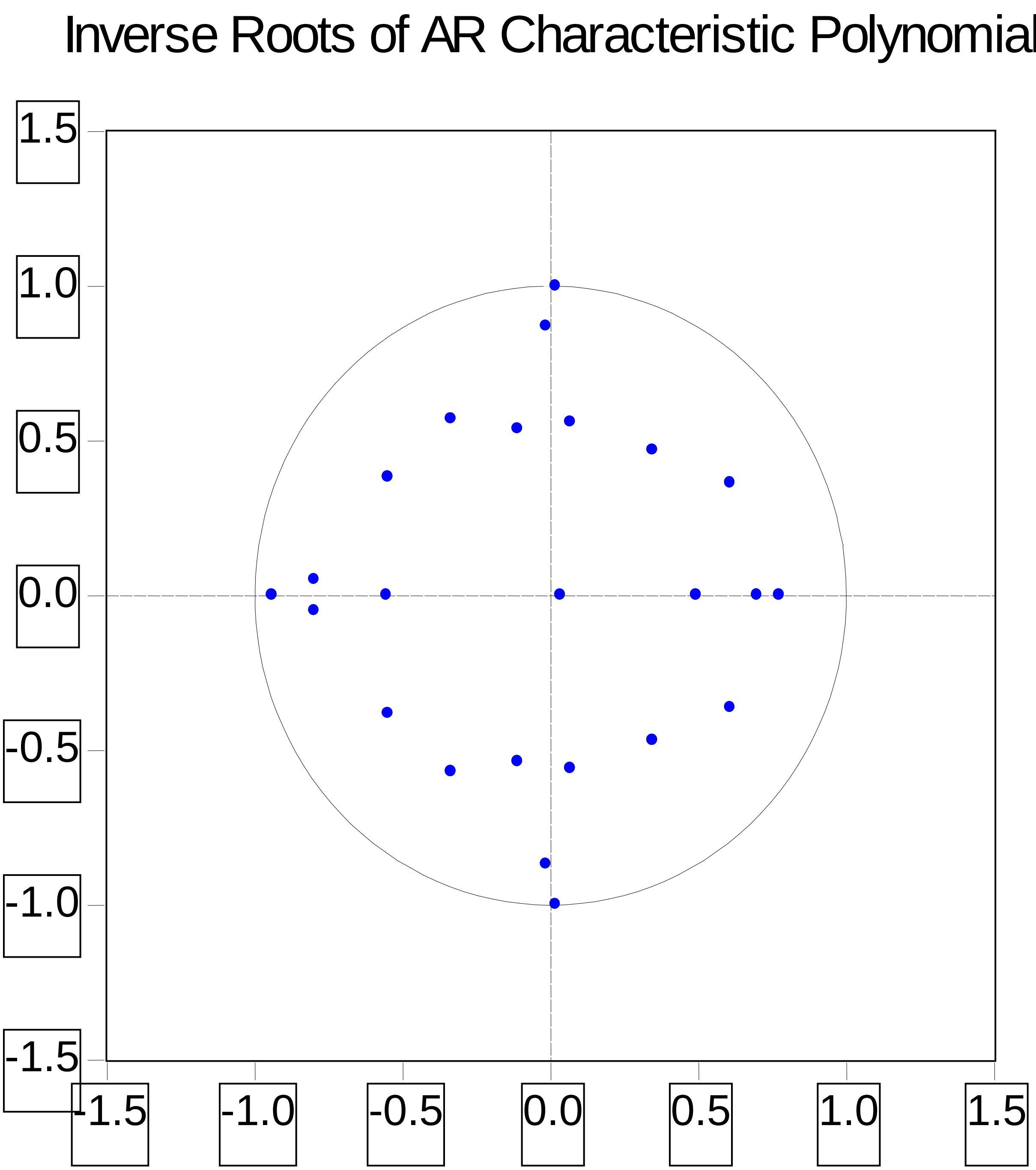
Fig. 1 AR Characteristic Polynomial of the endogenous graph of the all the economic variable.

Appendix



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