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A Note on Absolute Difference of Cubic and Square Sum Labeling of a Class of Trees

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Abstract

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In this paper, we introduce the new concept, an absolute difference of cubic and square sum labeling of a graph. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize few tree graphs for cubic and square sum labeling.

Keywords: *Graph labeling, sum square graph, square sum graphs, cubic graphs, tree graphs.*

1. Introduction

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q) graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2] and [3]. Some basic concepts are taken from Frank Harary [1]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph and we investigated few tree graphs for an absolute difference of cubic and square sum labeling.

Definition: 1.1

Let G = (V(G), E(G)) be a graph. A graph G is said to be absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection f : V(G) → {1,2,----------------,p} such that the induced function f^*_{adcss} : E(G) → multiples of 2 is given by $f^*_{adcss}(uv)$ = $f(u)^3 + f(v)^3 - (f(u)^2 + f(v)^2)$ is injective.

Definition: 1.2

A graph in which every edge associates distinct values with multiples of 2 is called the sum of the cubes of the vertices and the sum of the squares of the vertices. Such a labeling is called an absolute difference of cubic and square sum labeling or an absolute difference of css-labeling.

Main Results

Definition: 2.1

An (n,k) – banana tree, is a graph obtained by connecting one leaf of each of n copies of a k-star graph with a single root vertex that is distinct from all stars.

Theorem: 2.1

The banana tree B(n,k) is the absolute difference of the css-labeling.

Proof:

Let G = B(n,k) and let v_1, v_2,----------------,v_{nk+1} are the vertices of G.

Here |V(G)| = nk+1 and |E(G)| = nk

Define a function f : V → {1,2,3,--------,nk+1} by $f(v_i) = i$, i = 1,2,------,nk+1.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}\ [v_{(j-1)k+1} v_{i+(j-1)k+1}]$ = {(j-1)k+1}^{2} {(j-1)k} + {i+(j-1)k+1}^{2}{i+(j-1)k+1},

j = 1,2,-----,n

i = 2,4,------(k-1)

$f^*_{adcss}\ [v_{(i-1)k+2}\ v_{nk+1}]$ = {(i-1)k+2}^{2} {(i-1)k+1} + (nk+1)^2(nk)

i = 1,2,-----,k

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence B(n,k) admits absolute difference of css-labeling.

Definition 2.2

Consider k – copies of path p_n of length (n-1) and star S_m with m pendant vertices. Identify one of the two pendant vertices of the j^{th} path with the center of the j^{th} star. Identify the other pendant vertex of each path with a single vertex u_0 (u_0 is not in any of the star and path). The graph obtained is a regular bamboo tree B(k,n,m)

Theorem: 2.2

Bamboo tree B(k,n,m) is the absolute difference of the css-labeling.

Proof:

Let G = B(k,n,m) and let v_1, v_2,----------------,$v_{k(n-1)+km+1}$ are the vertices of G.

Here $|V(G)|$ = k(n-1)+km+1 and $|E(G)|$ = k(n-1)+km.

Define a function f : V → {1,2,3,--------, k(n-1)+km+1} by

$f(v_i)$ = i , i = 1,2,------, k(n-1)+km+1.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\ v_{i+1})$ = (i+1)^{2}i+i^2(i-1) ,

i = 1, 2, ---------------,n-2,
= n,n+1, -------------,2n-3
= 2n-1, 2n, ----------, 3n-4

= (k-1)n-k+2, ------------, kn-(k+1)

$f^*_{adcss}(v_{j(n-1)}\ v_{k(n-1)+(j-1)m+i})$ = {j(n-1)}^{2}{j(n-1)-1} + {k(n-1)+(j-1)m+i}^{2}{ k(n-1)+(j-1)m+i-1},

j = 1,2,-------------,k
i = 1,2, --------------,m.

$f^*_{adcss}(v_{1+(i-1)(n-1)}\ v_{k(n-1)+km+1})$ = {1+(i-1)(n-1)}^{2}{(i-1)(n-1)} + {k(n-1)+km+1}^{2}{ k(n-1)+km} ,

i = 2, ---------------,k.

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence B(k,n,m) admits absolute difference of css-labeling.

Definition 2.3

A coconut tree graph CT(m,n) is the graph obtained from the path P_n by appending m new pendant edges at an end vertex of P_n.

Theorem: 2.3

The graph CT(m,n) is the absolute difference of the css-labeling.

Proof :

Let G = CT(m,n) and let v_1, v_2,----------------,v_{n+m} are the vertices of G.

Here $|V(G)|$ = n+m and $|E(G)|$ = n+m-1 .

Define a function f : V → {1,2,3,--------, n+m} by

$f(v_i)$ = i , i = 1,2,------,n+m.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\ v_{i+1})$ = (i+1)^{2}i+i^2(i-1)

i = 1,2,3,--------------------,n-1

$f^*_{adcss}(v_n\ v_{n+i})$ = n^2(n-1) +(n+i)^2(n+i-1) ,

i = 1,2,-----------,m

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence CT(m,n) admits absolute difference of css-labeling.

Definition 2.4

The (n,2) - centipede tree, $C_{n,2}$, is the graph with $V(C_{n,2})$ = { v_1, v_2,--------------------v_{3n}},
and $E(C_{n,2})$ = {$v_{3k-1}\ v_{3k-2}$, $v_{3k-1}\ v_{3k}$, k = 1,2,----------n} ∪ {$v_{3k-1}\ v_{3k+2}$, k = 1,2,----------n-1}

Theorem: 2.4

The graph $C_{n,2}$ is the absolute difference of the css-labeling.

Proof :

Let G = $C_{n,2}$ and let v_1, v_2,----------------,v_{3n} are the vertices of G.

Here $|V(G)|$ = 3n and $|E(G)|$ = 3n-1

Define a function f : V → {1,2,3,--------, 3n} by

$f(v_i)$ = i , i = 1,2,------,3n.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) =$ (i+1)^{2}i+i^2(i-1)

i = 1,2,3,---------,n-1

$f^*_{adcss}(v_i\, v_{n+i}) =$ i^2(i-1) +(n+i)^2(n+i-1) ,

i = 1,2,-----------,n

$f^*_{adcss}(v_i\, v_{2n+i}) =$ i^2(i-1) +(2n+i)^2(2n+i-1) ,

i = 1,2,-----------,n

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $C_{n,2}$ admits absolute difference of css-labeling.

Definition 2.5

In the (n;k;m)-double star tree we have a path of length n whose end vertices v1 and vn are the central vertices for stars on k and m vertices respectively (not including the other vertices on the path).

Theorem: 2.5

The (n; k;m)-double star tree is the absolute difference of the css-labeling.

Proof :

Let G be the (n; k;m)-double star tree and let v_1,v_2,----------------,$v_{n+k+m-2}$ are the vertices of G.

Here $|V(G)|$ = n+k+m-2 and $|E(G)|$ = n+k+m-3.

Define a function f : V → {1,2,3,--------, n+k+m-2} by

f(v_i) = i , i = 1,2,------,n+k+m-2.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) =$ (i+1)^{2}i+i^2(i-1)

i = 1,2,3,-----------,n-1

$f^*_{adcss}(v_1\, v_{n+i}) =$ (n+i)^2(n+i-1) ,

i = 1, 2,-----------,k-1

$f^*_{adcss}(v_n\, v_{n+k+i-1}) =$ n^2(n-1) + (n+k+i-1)^2(n+k+i-2) ,

i = 1,2,-----------,m-1

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence (n;k;m)-double star tree admits absolute difference of css-labeling.

Theorem: 2.6

The Twig Graph is the absolute difference of the css-labeling.

Proof :

Let G be the twig graph and let v_1,v_2,--------------,v_{3n-4} are the vertices of G.

Here $|V(G)|$ = 3n-4 and $|E(G)|$ = 3n-5.

Define a function f : V → {1,2,3,--------, 3n-4} by

f(v_i) = i , i = 1,2,------,3n-4.

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) =$ (i+1)^{2}i+i^2(i-1)

i = 1,2,3,--------------------,n-1

$f^*_{adcss}(v_i\, v_{n+i-1}) =$ i^2(i-1) +(n+i)^2(n+i-1) ,

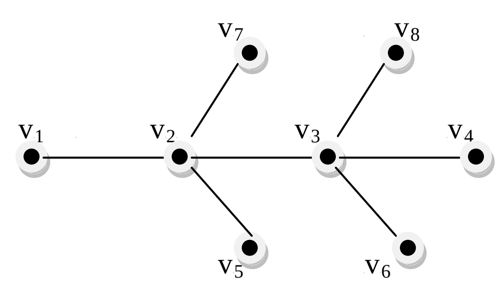
i = 2,-----------,n-1

$f^*_{adcss}(v_i\, v_{2n+i-3}) =$ i^2(i-1) +(2n+i)^2(2n+i-1) ,

i = 2,-----------,n-1

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence Twig graph admits absolute difference of css-labeling.

Example 2.1



Definition 2.6 An (n,k) -firecracker is a graph obtained by the concatenation of n k-stars by linking one leaf from each

Theorem: 2.7

Fire cracker graph $F_{n,k}$ is the absolute difference of the css-labeling.

Proof:

Let $G = F_{n,k}$ and let v_1, v_2,----------------,v_{nk} are the vertices of G.

Here $|V(G)| = nk$ and $|E(G)| = nk-1$.

Define a function $f : V \rightarrow \{1,2,3,--------,nk\}$ by

$$f(v_i) = i\ ,\ i = 1,2,------,nk.$$

For the vertex labeling f, the induced edge labeling f^*_{adcss} is defined as follows

$f^*_{adcss}(v_{jk+1}\ v_{jk+i+1}) = (jk+1)^2(jk) + (jk+i+1)^2(jk+i).$

j= 0,1,2,3------------------------,n-1
i = 1,2,3,-----------------------,k-1

$f^*_{adcss}(v_{ik}\ v_{(i+1)k} = (ik)^2(ik-1) + \{(i+1)k\}^2\{(i+1)k-1\}.$

i = 1,2,-----------,n-1

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $F_{n,k}$ admits absolute difference of css-labeling.

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