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A NOTE ON THE MATHEMATICAL MODEL FOR THE DYNAMICS OF BIRD FLU DISEASE

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Abstract

In a recent paper [1], a model for the dynamics of bird flu disease was studied and it was shown that the disease free equilibrium of the model is stable. We show in this paper, using a eliable Jacobian matrix and model parameter values taken from [1] that one endemic equilibrium is unstable.

INTRODUCTION

Recently, Aderinto [1] proposed a mathematical model for the dynamics of bird u disease. The author aimed at using the model to examine human populaiton with respect to the disease and its transmission. In particular, he showed that the disease free equilibrium is stable. In the present paper, we extend his results to endemic equilibrium.

2. MATHEMATICAL FORMULATION

The model is [1]

$$S'(t) = \beta N - \frac{\alpha S(t)A(t)I(t)}{N} - (\delta + \mathrm{d})\mathrm{S(t)} + \varphi_1 A(t) + \varphi_2 I(t) \quad (1)$$

$$A'(t) = \frac{\alpha S(t)A(t)}{N} + (1-r)\mu A(t) - \varphi_1 A(t) \quad (2)$$

$$I'(t) = \frac{\alpha S(t)A(t)}{N} - (\delta + d + \varphi_2)I(t) + r\mu A(t) - \varphi_1 A(t) \tag{3}$$

Satisfying $S(0) = S_0 > 0$, $I(0) = I_0 > 0$, $A(0) = A_0 > 0$, $0 < \alpha < 1$, $0 < \delta < 1$, $0 < r < 1$

where

S(t) = number of susceptible individuals at time t

A(t) = number ofasymptomatic infective persons (without symptoms) at time t

I(t) = number of infective persons with unset of symptoms at time t

N = total number of birds in the locality considered

α = transmission rate of infection from birds to human

β = average birth rate

δ = disease induced death rate

d = natural death rate

r = fraction of asymptotic individual progressing to the class I(t)

μ = rate of exposed to the disease

φ_1 = recovered rate of the class A(t)

φ_2 = recovered rate of the class I(t)

3. METHOD OF SOLUTION

For the endemic equilibrium (0, 0, 1), the Jacobian matrix is

$$J = \begin{pmatrix} -(\delta + d) & \varphi_1 & \varphi_2 \\ 0 & (1-r)\mu - 1 & 0 \\ \alpha\beta & r\mu - 1 & -(\delta + d) + \varphi_2 \end{pmatrix} \tag{4}$$

and the corresponding characteristics equation is given by

$$\lambda^3 + r_1\lambda^2 - r_2\lambda - r_3 = 0 \qquad (5)$$

where

$$r_1 = 2(\delta + d) + \mu(1 - r) + \varphi_1 + \varphi_2 \qquad (6)$$

$$r_2 = 2\delta[\mu(1 + r) - d - \varphi_2] - d[2(r + \varphi_1) + d + \varphi_2] - \delta(\delta + \varphi^2{}_2) + \varphi_2(r\mu + \varphi_1) + \alpha\beta \qquad (7)$$

$$r_3 = \delta[\mu(\delta^2 - \varphi_2 - r\delta - r\varphi_2 - 2rd) - \varphi_1(\delta + 2d - \varphi_2)] - \mu d[d + \varphi_2 + r(d - \varphi_2)] - \varphi_1 d(d - \varphi_2)$$
$$+ \quad \alpha\beta[\mu(1 - r) - \varphi_1] \qquad (8)$$

Theorem: If $r_1 \geq 0,\ r_2 \geq 0,\ r_3 \geq 0$, then equilibrium (0, 0, 1) is unstable.

Proof: For $r_1 \geq 0,\ r_2 \geq 0,\ r_3 \geq 0$, equation (5) has one positive root.

Hence the result.

CONCLUSION

Using the data in [1], $r_1 = 1.92$, $r_2 = 0.18$, $r_3 = 0.91$, it shows that J is unstable.

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