International Journal of Scientific Engineering and Applied Science (IJSEAS) – Volume-2, Issue-8, August 2016
ISSN: 2395-3470
www.ijseas.com

Existence and Uniqueness of Solution of a Mathematical Model of T cell Infection by HIV

Egbetade, S.A. and Salawu, I.A.

Department of Mathematics and Statistics,

The Polytechnic, Ibadan.

*Corresponding Author

Abstract

This paper examines existence and uniqueness of solutions of the model equations proposed in [3] for the dynamics of HIV infection in T cells. We formulate theorem on existence and uniqueness of solutions and establish the proof of the theorem.

1. Mathematical Formulation

Let w denote resting T helper cells, x denote uninfected activated T cells

while y denote infected activated T cells. Then, we have the following model:

$$w' = \varepsilon - fw - rwy \tag{1}$$

$$x' = rwy - dx - \beta xy \tag{2}$$

$$y' = \beta xy - \alpha y \tag{3}$$

where ε denotes production rate of resting T helper cells

r denotes activation rate of resting T helper cells

f denotes death rate of resting T helper cells

d denotes death rate of uninfected activated T cells

β denotes replication rate of HIV in T cells

α denotes death rate of infected activated T cells

The above is the model of HIV dynamics in T cells.

The rest of the assumptions are as given [3].

2. EXISTENCE AND UNIQUENESS OF SOLUTIONS

In this section, we formulate existence and uniqueness theorem for equation (1) - (3) and we establish the proof of the theorem. In order to state and prove our theorem, we consider the system of equations.

$$\left.\begin{array}{ll} x'_1 = f_1(t, x_1, x_2, \ldots\ldots, x_n), & x_1(t_0) = x_{10} \\ x'_2 = f_2(t, x_1, x_2, \ldots\ldots, x_n), & x_2(t_0) = x_{20} \\ . \quad . \quad . \quad . \quad . \quad . \quad . & \\ & \\ x'_n = f_n(t, x_1, x_2, \ldots\ldots, x_n), & x_n(t_0) = x_{n0} \end{array}\right\} \tag{2.1}$$

We may write equation (2.1) in the compact form as

$$x' = f(t, x), \quad x(t_0) = x_0 \tag{2.2}$$

Theorem 2.1 [2]

Let D denote the region

$$\left.\begin{array}{l} |t - t_0| \le a, \ \|x - x_0\| \le b, \ \ x = (x_1, x_2, \ldots, x_n) \\ \qquad\qquad x_0 = (x_{10}, x_{20}, \ldots x_{n0}) \end{array}\right\} \tag{2.3}$$

and suppose that f (t, x) satisfies the Lipschitz condition

$$\| f(t, x_1) - f(t, x_2) \| \le K \|x_1 - x_2\| \tag{2.4}$$

whenever the pairs (t, x_1) and (t, x_2) belong to D, where K is a positive constant.

Then there exists a constant δ > 0 such that there exists a unique continuous vector solution $x(t)$ of the system (2.2) in the interval $|\, t-t_0 \,| < \delta$.

Remark: It is important to note that condition (2.4) is satisfied by the requirement that $\frac{\partial f_i}{\partial x_j}$, $i, j = 1,\ 2,\ \ldots, n$ is continuous and bounded in D.

Proof. From (2.3),

$$\left.\begin{aligned} &D = (w, x, y, t) \\ &\text{Where} \\ &|w-w_0| \le b_1,\ \ |x-x_0| \le b_2,\ \ |y-y_0| \le b_3,\ \ |t-t_0| \le b_4 \end{aligned}\right\} \qquad (2.5)$$

From equations (1) - (3),

Let $f_1 = \varepsilon - fw - rwy$ (2.6)

$$f_2 = rwy - dx - \beta xy \qquad (2.7)$$

$$f_3 = \beta xy - ay \qquad (2.8)$$

By theorem (2.1), it suffices to show that $\left|\frac{\partial f_i}{\partial x_j}\right|$ are bounded to prove existence and uniqueness of solution of equations (1)- (3).

Consider the partial derivatives

$$\frac{\partial f_1}{\partial w} = -f - ry,\quad \frac{\partial f_1}{\partial x} = 0,\quad \frac{\partial f_1}{\partial y} = -ry \qquad (2.9)$$

$$\frac{\partial f_2}{\partial w} = ry,\quad \frac{\partial f_2}{\partial x} = -d - \beta y, \frac{\partial f_2}{\partial y} = rw - \beta x \qquad (2.10)$$

$$\frac{\partial f_3}{\partial w} = -f - ry,\quad \frac{\partial f_3}{\partial x} = \beta y, \frac{\partial f_3}{\partial y} = \beta x - a \qquad (2.11)$$

Now, by substituting (2.5) into (2.9) - (2.11), we have

$$\left.\begin{aligned} &\left|\frac{\partial f_1}{\partial w}\right| = \left|-(f+ry)\right| = \left|f+ry\right| \le \left|f+r(y_0+b_3)\right| < \infty \\ &\left|\frac{\partial f_1}{\partial x}\right| = 0, \quad \left|\frac{\partial f_1}{\partial y}\right| = \left|-rw\right| = \left|rw\right| = \left|r(w_0+b_1\right| < \infty \end{aligned}\right\} \tag{2.12}$$

$$\left.\begin{aligned} &\left|\frac{\partial f_2}{\partial w}\right| = \left|ry\right| = \left|r(y_0+b_3\right| < \infty, \ \left|\frac{\partial f_2}{\partial w}\right| = \left|-(d+\beta y)\right| = \left|d+\beta y\right| \le \left|\mathrm{d}+\beta(y_0+b_3\right| < \infty \\ &\left|\frac{\partial f_2}{\partial y}\right| = \left|rw-\beta x\right| \le \left|r(w_0+b_1)-\beta(x_0+b_2\right| < \infty \end{aligned}\right\} \tag{2.13}$$

$$\left.\begin{aligned} &\left|\frac{\partial f_3}{\partial w}\right| = 0, \quad \left|\frac{\partial f_3}{\partial w}\right| = \left|\beta y\right| \le \left|\beta(y_0+b_3\right| < \infty \\ &\left|\frac{\partial f_3}{\partial y}\right| = \left|\beta x - a\right| \le \left|\beta(x_0+b_3)-a\right| < \infty \end{aligned}\right\} \tag{2.14}$$

Clearly equations (2.9) - (2.11) are bounded in D. Hence, by Theorem (2.1), there exists a unique solution of the system (1) - (3) i.e., there exists a unique solution of equation (2.9) - (2.11) which satisfies equation (2.5).

3. **CONCLUSION**

The existence and uniqueness results in the previous section show that solution of the model exists and is unique. This suggests the applicability of the model to the dynamics of HIV/AIDS disease in T cells.

4. REFERENCES

[1] Bukrinsky, M.I., Stanwick, T.L., Dempsey,M.P. and Stevenson, M. (1991), Quiscent lymphocytes - T as an inducible virus reservoir in HIV-1 infection. Science, 254, 423-427.

[2] William, R.D. and Stanley, I.G. (1976), Elementary differential equation with applications, Addison-Wesley Publishing Company Inc. Phillipines.

[3] Egbetade, S.A. (2002), A model of Macrophage and T cell infection by HIV. M.Sc. Thesis, Ladoke Akintola University of Technology, Ogbomoso (Unpublished).

[4] Egbetade, S.A. (2005), Criteria for existence and uniqueness of solutionof a mathematical model for macrophage infection by HIV. Bioscience 16(2), 1 - 6.

[5] Essunger, P. and Perelson, A.S. (1994). Modelling HIV-1 infection of CD4(+) T cell subopulation, J. theor. Biol. 170, 367-391.

[6] McLean, A.R. and Kirkwood, T.B.L. (1990). A model of human immune deficiency virus in T helper cell clones, J. theor Biol. 147, 177-203.