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Estimation of Variance of Time to Recruitment for a Two Grade Manpower System with Single Source of Depletion Using a Different Probabilistic Analysis

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Abstract

In this paper a marketing organization consisting of two grades with single source of depletion is considered. The problem of time to recruitment is analyzed for this system using a univariate MAX policy of recruitment based on shock model approach. Two mathematical models are constructed and the variance of time to recruitment is obtained for both the models when the loss of manpower form a sequence of independent and identically distributed exponential random variables and the threshold for the maximum loss of manpower in the organization is the combined constant thresholds for each grade. While in Model-I, the inter-decision times are independent and identically distributed random variables, in Model-II, they are exchangeable and constantly correlated exponential random variables. A different probabilistic analysis is used to derive the analytical results for both the models.

Keywords: *Combined Constant Threshold, Shock Model Approach, Two Grade, Univariate MAX Policy, Variance of Time to Recruitment.*

1. Introduction

In administrative as well as production-oriented organization it is a usual phenomenon that exit of personnel happens whenever policy decision regarding revision of wages, incentives and revised sales and targets are announced. This in turn leads to depletion of manpower, which can be conceptualized in terms of man hours. As the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitment and hence suitable recruitment planning has to be designed in order to offset the loss in manpower. In the design of the recruitment policies, several authors have used shock model approach in reliability theory. If the total loss or maximum loss of man hours due to the exit of personnel crosses a particular level, known as threshold for the organization, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is to be done at this point.

The problem of finding the time to recruitment for a two graded manpower system using shock model approach was initiated by the authors in [6]. They have studied this problem when the loss of manpower in the organization is maximum (minimum) of thresholds for the loss of manpower in the two grades. Later, several researchers [3], [4], [5], [7], [8] and [9] have studied the problem of time to recruitment for a single and multi grade manpower system under different conditions on the loss of manpower, inter-decision times and the threshold for the loss of manpower using univariate CUM policy of recruitment. In [10], the author has obtained the variance of time to recruitment using univariate MAX policy of recruitment when (i) the threshold for the organization is the sum of the constant thresholds for the loss of manpower in the two grades and (ii) the inter-decision times are independent and identically distributed exponential random variables, by Laplace transform Method. In [10], the author has also studied this problem when the inter-decision times are exchangeable and constantly correlated exponential random variables. Recently in [1], the authors have obtained the variance of time to recruitment for a single grade manpower system with two sources of depletion using univariate CUM policy of recruitment and Laplace transform in the analysis when the loss of man hours, inter-policy decision times and inter-transfer decision times are independent and non-identically distributed exponential random variables and the breakdown threshold has single component. The limitation of Laplace transform method is that it requires the probability density function of inter-decision times. The main objective of the present paper is to study both the above cited work in [10] by a different probabilistic analysis which requires only the first two moments of inter-decision times for the independent case. This method does not require the knowledge about the probability density function of the inter-decision times when they are independent. However, in the study of the problem for correlated inter-decision times using the different probabilistic analysis, the knowledge about the probability density function of the inter-decision times is required.

In this paper two mathematical models (Model-I and Model-II) are constructed and the variance of time to recruitment is obtained for a two grade manpower system using univariate MAX policy of recruitment. In Model-I, the analytical results are obtained by using a different probabilistic analysis, when the (i) loss of man hours process in each grade forms a sequence of independent and identically distributed exponential random variables, (ii) inter-decision times are independent and identically distributed random variables and (iii) the threshold for the organization is the sum of the constant thresholds for the maximum loss of manpower in the two grades. In Model-II, the results of Model-I have been analyzed when the inter-decision times are exchangeable and constantly correlated exponential random variables.

2. Model Description and analysis for Model-I

Consider an organization having two grades A and B taking policy decisions at random epochs in the interval $(0,\infty]$. At every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. For $i=1,2,3,...$, let X_{iA} (X_{iB}) be independent and identically distributed exponential random variable representing the amount of depletion of manpower (loss of man hours) due to i^{th} policy decision for grade A (grade B) with distribution $\phi_A(\cdot)$ ($\phi_B(\cdot)$). Let $Z_{kA}=\max\limits_{1\le i\le k} X_{iA}$ ($Z_{kB}=\max\limits_{1\le i\le k} X_{iB}$) be the maximum loss of man hours in grade A (grade B) in the first k decisions with distribution $\phi_{kA}(\cdot)$ ($\phi_{kB}(\cdot)$). We define $\overline{Z_k}=\max\left(Z_{kA},Z_{kB}\right)$, $k=1,2,....$ Note that $\overline{Z_k}$ is the maximum loss of man hours in the organization in the first k decisions. Let U_i be the time between $(i-1)^{th}$ and i^{th} policy decisions. It is assumed that the number of policy decisions announced in the two grades is governed by the same renewal process with independent and identically distributed inter-decision times with mean $E[U]$ and variance $Var(U)$. Let R_{k+1} be the waiting time upto $(k+1)^{th}$ decision. Let T ($T>0$) be the constant combined threshold for the two grades. Let W be the continuous random variable denoting the time to recruitment in the organization with mean $E[W]$ and variance $Var(W)$. It is assumed that the loss of man hour process and the process of inter-decision times are statistically independent. The recruitment policy employed in this chapter is the MAX policy of recruitment which states that, ***"Recruitment is done whenever the maximum of the maximum loss of man hours in grade A and the maximum loss of man hours in grade B due to the policy decisions exceeds the combined constant threshold level T."***

2.1 Main Results

By the recruitment policy, recruitment is done whenever the maximum of the maximum loss of man hours in grade A and the maximum loss of man hours in grade B due to the policy decisions exceeds the threshold level T. When the first decision is taken, recruitment would not have been done for U_1 units of time if $\overline{Z_1}\le T$. If the loss of manpower $\overline{Z_1}$ due to the first policy of decision is greater than T, then the recruitment is done and in this case $W=U_1=R_1$. However, if $\overline{Z_1}\le T$, the non-recruitment period will continue till the next policy decision is taken. If $\overline{Z_2}>T$, then recruitment is done and in this case $W=U_1+U_2=R_2$. If $\overline{Z_2}\le T$, then the non-recruitment period will continue till the next policy decision is taken and depending on $\overline{Z_3}>T$ or $\overline{Z_3}\le T$, recruitment is done or the non-recruitment period continues and so on. Hence

$$W=\sum_{k=0}^{\infty}R_{k+1}\ \chi\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right). \qquad (2.1)$$

By hypothesis, R_{k+1} and $\overline{Z_{k+1}}$ are statistically independent for $k=0,1,2,..$. Taking expectations on both sides of (2.1) and using the result $E\left[X\,\chi(A)\right]=E[X]\ P(A)$, when X and $\chi(A)$ are independent, in (2.1), we get

$$E[W]=\sum_{k=0}^{\infty}E\left[R_{k+1}\right]\ P\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right). \qquad (2.2)$$

Since $R_{k+1}=U_1+U_2+...+U_{k+1}$ and $E\left[R_{k+1}\right]=(k+1)E[U]$, from (2.2) we get

$$E[W]=E[U]\sum_{k=0}^{\infty}(k+1)\ P\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right). \qquad (2.3)$$

We now evaluate $P\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right)$ in (2.3).

Note that

$$P\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right)=P\left(\left\{\overline{Z_k}\le T\right\}\cap\begin{bmatrix}\left\{X_{k+1,A}>T\cap X_{k+1,B}\le T\right\}\cup\\ \left\{X_{k+1,A}\le T\cap X_{k+1,B}>T\right\}\cup\\ \left\{X_{k+1,A}>T\cap X_{k+1,B}>T\right\}\end{bmatrix}\right) \qquad (2.4)$$

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Define the events

$E=\{\overline{Z_k}\le T<\overline{Z_{k+1}}\}$; $E_1=\{\overline{Z_k}\le T\}$; $E_2=\{X_{k+1,A}>T\}$ and $E_3=\{X_{k+1,B}\le T\}$. (2.5)

Using (2.5) in (2.4), we get

$$P(E)=P(E_1)\left[P(E_2)P(E_3)+P\left(E_2^c\right)P\left(E_3^c\right)+P(E_2)P\left(E_3^c\right)\right]. \tag{2.6}$$

Now we evaluate the probabilities in the right side of (2.6).

By definition

$P(E_1)=[\phi_A(T)\phi_B(T)]^k$, $P(E_2)=1-\phi_A(T)$ and $P(E_3)=\phi_B(T)$ (2.7)

Therefore from (2.6) and (2.7), we get

$$P(E)=[\phi_A(T)\phi_B(T)]^k\begin{bmatrix}\{1-\phi_A(T)\}\phi_B(T)+\\ \phi_A(T)\{1-\phi_B(T)\}+\\ \{1-\phi_A(T)\}\{1-\phi_B(T)\}\end{bmatrix}$$

i.e. $P\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right)=[\phi_A(T)\phi_B(T)]^k\,[1-\phi_A(T)\phi_B(T)]$. (2.8)

Using (2.8) in (2.3), we get

$$E[W]=E[U][1-\phi_A(T)\phi_B(T)]\left\{\sum_{k=0}^{\infty}(k+1)\ [\phi_A(T)\phi_B(T)]^k\right\} \tag{2.9}$$

since

$$\sum_{k=0}^{\infty}(k+1)\ [\phi_A(T)\phi_B(T)]^k=[1-\phi_A(T)\phi_B(T)]^{-2},$$

from (2.9) we get

$$E[W]=\frac{E[U]}{[1-\phi_A(T)\phi_B(T)]} \tag{2.10}$$

Equation (2.10) gives the mean time to recruitment for Model-I.

Now we determine $E\left[W^2\right]$.

From (2.1)

$$W^2=\left[\sum_{k=0}^{\infty}R_{k+1}\ \chi\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right)\right]^2. \tag{2.11}$$

Since $\chi^2(A)=\chi(A)$ and

$$\left[X\chi(A)+Y\chi\left(A^c\right)\right]^2=X^2\chi(A)+Y^2\chi\left(A^c\right),$$

from (2.11) we get

$$W^2=\sum_{k=0}^{\infty}R_{k+1}{}^2\ \chi\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right) \tag{2.12}$$

and

$$E\left[W^2\right]=\sum_{k=0}^{\infty}E\left[R_{k+1}{}^2\right]\ P\left(\overline{Z_k}\le T<\overline{Z_{k+1}}\right) \tag{2.13}$$

Since $E\left[R_{k+1}{}^2\right]=Var(R_{k+1})+\left(E[R_{k+1}]\right)^2$ by definition, $Var(R_{k+1})=(k+1)Var(U)$ and $\mathrm{E}[R_{k+1}]=(k+1)E[U]$ by hypothesis, we get

$$E\left[R_{k+1}{}^2\right]=(k+1)\left[Var(U)+(k+1)\left(E[U]\right)^2\right]. \tag{2.14}$$

From (2.8), (3.2.13) and (3.2.14) we get

$$E\left[W^2\right]=[1-\phi_A(T)\phi_B(T)]\cdot\left\{\sum_{k=0}^{\infty}(k+1)\left[Var(U)+(k+1)\left(E[U]\right)^2\right]\ [\phi_A(T)\phi_B(T)]^k\right\} \tag{2.15}$$

Since $\sum_{k=0}^{\infty}(k+1)\ [\phi_A(T)\phi_B(T)]^k=[1-\phi_A(T)\phi_B(T)]^{-2}$

and

$$\sum_{k=0}^{\infty}k(k+1)[\phi_A(T)\phi_B(T)]^k=2\phi_A(T)\phi_B(T)[1-\phi_A(T)\phi_B(T)]^{-3}$$

from (2.15) we get

$$E\left[W^2\right]=\frac{1}{[1-\phi_A(T)\phi_B(T)]}\left\{Var(U)+\left(E[U]\right)^2\left[\frac{1+\phi_A(T)\phi_B(T)}{1-\phi_A(T)\phi_B(T)}\right]\right\} \tag{2.16}$$

Since $Var(W)=E\left[W^2\right]-\left(E[W]\right)^2$ from (2.10) and (2.16) we get

$$Var(W)=\frac{1}{[1-\phi_A(T)\phi_B(T)]^2}\left\{\begin{matrix}Var(U)[1-\phi_A(T)\phi_B(T)]\\ +\left(E[U]\right)^2\phi_A(T)\phi_B(T)\end{matrix}\right\} \tag{2.17}$$

Equation (2.17) gives the variance of time to recruitment for Mode-I.

Now we determine the mean and variance of time to recruitment in the closed form by considering specific distribution to loss of manpower.

2.2 Special Case

Assume that the distribution of loss of man hours in grade A is exponential with parameter α_A and the distribution of loss of man hours in grade B is exponential with parameter α_B.

In symbols we have

$\phi_A(T) = 1 - e^{-\alpha_A T}$ and $\phi_B(T) = 1 - e^{-\alpha_B T}$

Therefore

$$\phi_A(T)\phi_B(T) = 1 - e^{-\alpha_A T} - e^{-\alpha_B T} + e^{-(\alpha_A+\alpha_B)T} \qquad (2.18)$$

For the special case the mean and variance of time to recruitment for Model-I are given by (2.10) and (2.17) respectively where $\phi_A(T)\phi_B(T)$ is given by (2.18).

3. Model Description and Analysis for Model-II

Description of Model-II is similar to that of Model-I except the condition on inter-decision times. In Model-II we assume that, the inter-decision times U_i 's are exchangeable and constantly correlated exponential random variables. Let ρ be the correlation between U_i and U_j, $i \neq j$. Let $u\left(=\frac{v}{1-\rho}\right)$ be the mean of inter-decision times.

3.1 Main Results

Proceeding as in Model-I we find that

$$E[W] = \frac{u}{1-\phi_A(T)\phi_B(T)} \qquad (3.1)$$

where $u = \frac{v}{1-\rho}$.

Equation (3.1) gives the mean time to recruitment for Model-II.

Now we determine $E\left[W^2\right]$.

Proceeding as in Model-I, we find that

$$E\left[W^2\right] = \sum_{k=0}^{\infty} E\left[{R_{k+1}}^2\right] P\left(\overline{Z_k} \le T < \overline{Z_{k+1}}\right) \qquad (3.2)$$

From [2], it can be shown that

$$E\left[{R_{k+1}}^2\right] = u^2\left[\rho^2 k(k+1) + (k+1)(k+2)\right],$$

From (2.8) and (3.2) we get

$$E\left[W^2\right] = u^2\left[1-\phi_A(T)\phi_B(T)\right]\left\{\begin{array}{l}\rho^2\sum_{k=0}^{\infty} k(k+1)\left[\phi_A(T)\phi_B(T)\right]^k + \\ \sum_{k=0}^{\infty}(k+1)(k+2)\left[\phi_A(T)\phi_B(T)\right]^k\end{array}\right\} \qquad (3.3)$$

Since

$$\sum_{k=0}^{\infty} k(k+1)\left[\phi_A(T)\phi_B(T)\right]^k = 2\ \phi_A(T)\phi_B(T)\left[1-\phi_A(T)\phi_B(T)\right]^{-3}$$

and

$$\sum_{k=0}^{\infty}(k+1)(k+2)\ \left[\phi_A(T)\phi_B(T)\right]^k = 2\ \left[1-\phi_A(T)\phi_B(T)\right]^{-3}$$

from (3.3), we get

$$E\left[W^2\right] = 2u^2\frac{\left\{1+\rho^2\phi_A(T)\phi_B(T)\right\}}{\left[1-\phi_A(T)\phi_B(T)\right]^2} \qquad (3.4)$$

From (3.1) and (3.4), we get

$$Var(W) = \frac{u^2\left\{1+2\rho^2\phi_A(T)\phi_B(T)\right\}}{\left[1-\phi_A(T)\phi_B(T)\right]^2} \qquad (3.5)$$

where $u = \frac{v}{1-\rho}$.

Equation (3.5) gives the variance of time to recruitment for Model-II.

Now we determine the mean and variance of time to recruitment in the closed form by considering specific distribution to loss of manpower.

3.2 Special Case:

Assume that the distribution of loss of man hours in grade A is exponential with parameter α_A and the distribution of loss of man hours in grade B is exponential with parameter α_B. For the special case the mean and variance of time to recruitment for Model-II are given by (3.1) and (3.5) respectively where $\phi_A(T)\phi_B(T)$ is given by (2.18).

4. Conclusion

This paper studies the problem of time to recruitment when the inter-decision times are independent and dependent by using a different probabilistic analysis which requires only the first two moments of the inter-decision times in the independent case, unlike the conventional Laplace transform technique which can be used only when the distribution of the inter-decision times is assumed. There is a good scope for further work. The present work can be studied when the wastages are also dependent.

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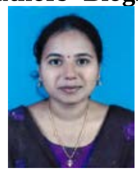
Authors' Biography



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