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g_u-Semi closed sets in generalized topological spaces

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Abstract: In this paper, we have introduced a new class of sets in generalized topological spaces called g_u-semi closed sets. Also we have investigated some of their basic properties.

Keywords: *Generalized topological spaces, g-semi closed sets, g_u-semi-closed sets.*

1. Introduction: The concept of generalized topological spaces was introduced and investigated by A. Csaszar[1]. Many g-closed sets like g-pre closed set, g-semi closed set etc., in generalized topological spaces are introduced by him. In this paper, we have introduced a new class of sets in generalized topological spaces called g_u-semi closed sets. Also we have investigated some of their basic properties.

2. Preliminaries

Definition 2.1: [1] Let X be a non empty set and let g be a collection of subsets of X. Then g is called a generalized topology (GT for short) on X if and only if $\emptyset \in g$ and $G_i \in g$ for $i \in I \neq \emptyset$ implies $G = \bigcup_{i\in I} G_i \in g$. The pair (X, g) is called as a generalized topological space (GTS for short) on X. The elements of g are called g-open sets and their complements are called g-closed sets.

We denote the family of all g-closed sets in X by g(X). The generalized closure of a subset S of X, denoted by $c_g(S)$, is the intersection of generalized closed sets including S. And the interior of S, denoted by $i_g(S)$, is the union of generalized open sets contained in S.

Note that $c_g(S) = X- i_g(X-S)$ and $i_g(S) = X- c_g(X-S)$.

Definition 2.2: [2] Let (X, g) be a generalized topological space and $A \subseteq X$. Then A is said to be

(i) g-semi closed if $i_g(c_g(A)) \subseteq A$
(ii) g-pre closed if $c_g(i_g(A)) \subseteq A$
(iii) g-α closed if $c_g(i_g(c_g(A))) \subseteq A$
(iv) g-β closed if $i_g(c_g(i_g(A))) \subseteq A$
(v) g- regular closed if $c_g(i_g(A)) = A$

The complement of g-semi closed (resp., g-pre closed, g-α closed, g-β closed, g-regular closed) is said to be g-semi open (resp., g- pre open, g-α open, g-β open, g-regular open).

3. g_u-semi closed sets in generalized topological space

In this section we introduced g_u-semi closed sets in generalized topological spaces and studied some of their basic properties.

Definition 3.1: Let (X, g) be a generalized topological space. Then a non empty subset A is said to be g_u-semi closed if $sc_g(A) \subseteq i_g(c_g(U))$ whenever $A \subseteq U$ and U is g-open.

Example 3.2: Let X = {a, b, c} and let g = {∅, {a}, {b, c}, X} then (X, g) is a generalized topological space. Now let A = {c} and U = {b, c}. Then $A \subseteq U$ and $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(\{c\})) = \{b, c\}$ and $i_g(c_g(U)) = i_g(c_g(\{b, c\})) = \{b, c\}$. Therefore $sc_g(A) \subseteq i_g(c_g(U))$. Therefore A is g_u-semi closed set in (X, g).

Definition 3.3: Let A be a non empty set in a generalized topological spaces in (X, g). Then the g-semi interior and g-semi closure of A are defined as

$si_g(A) = \cup\{G/G$ is a g-semi open set and $G \subseteq A\}$ and

$sc_g(A) = \cap\{G/G$ is a g-semi closed set and $G \supseteq A\}$.

It is to be noted that for any set A in (X, g), we have $(sc_g(A))^c = si_g(A^c)$ and $(si_g(A))^c = sc_g(A^c)$

Theorem 3.4: Every g-closed set in (X, g) is a g_u- semi closed sets in (X, g) but not conversely.

Proof: Let A be g-closed in (X, g) then $c_g(A) = A$. Now let $A \subseteq U$ and U is g-open. Then $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(A) \subseteq U = i_g(U) \subseteq i_g(c_g(U))$. Therefore by hypothesis $sc_g(A) \subseteq i_g(c_g(U))$. This implies A is g_u-semi closed set.

Example 3.5: Let X = {a, b, c} and let g = {∅, {a}, {b, c}, X} then (X, g) is a generalized topological space. Now let A = {c} and U = {b, c}. Then $A \subseteq U$ and $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(\{c\})) = \{b, c\}$ and $i_g(c_g(U)) = i_g(c_g(\{b, c\})) = \{b, c\}$. Therefore $sc_g(A) \subseteq i_g(c_g(U))$. Therefore A is g_u-semi closed set in (X, g). But $c_g(A) = c_g(\{c\}) = \{b, c\} \neq A$. Hence A is not g-closed in (X, g).

Remark 3.6: Not every g_u- semi closed set in (X, g) is g-pre closed in (X, g).

Example 3.7: Let X = {a, b, c} and let g = {∅, {a, b},{b, c}, X }.Then (X, g) is a generalized topological space. Now let A = {a, b} and U = {a, b}. Then $A \subseteq U$ and $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(\{a, b\})) = \{a, b\}$ and $i_g(c_g(U)) = i_g(c_g(\{a , b\})) = \{a, b\}$. Therefore $sc_g(A) \subseteq i_g(c_g(U))$. Therefore A is g_u- semi closed set in (X, g). But $c_g(i_g(A)) = c_g(i_g(\{a\})) = \{a, b\} \nsubseteq A$. Therefore A is not a g-pre closed set in (X, g).

Remark 3.8: Not every g_u-semi closed set in (X, g) is g-β closed set in (X, g).

Example 3.9: Let X = {a, b, c, d} and let g = {∅, {c}, {a, b}, {a, b, c},{a, b, d}, X}.Then (X, g) is a generalized topological space. Now let A = {a, b} and U = {a, b, d}. Then $A \subseteq U$ and $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(\{a, b\})) = \{a, b, d\}$ and $i_g(c_g(U)) = i_g(c_g(\{a ,b, d\})) = \{a, b, d\}$. Therefore $sc_g(A) \subseteq i_g(c_g(U))$. Therefore A is g_u- semi closed set in (X, g). But $i_g(c_g(i_g(A))) = i_g(c_g(i_g(\{a, b\}))) = \{a, b, d\} \nsubseteq A$. Hence A is not a g-β closed set in (X, g).

Theorem 3.10: Every g-semi closed set in (X,g) is a g_u- semi closed set in (X, g) but not conversely.

Proof: Let (X, g) be a generalized topological space. Let A be g-semi closed. Then $i_g(c_g(A)) \subseteq A$. Now let $A \subseteq U$ and U be g-open. Then $sc_g(A) = A \cup i_g(c_g(A)) = A \cup A = A \subseteq U = i_g(U) \subseteq i_g(c_g(U))$. Therefore A is g_u-semi closed set.

Example 3.11: Let X = {a, b, c} and let g = {∅, {a, b}, {b, c}, X}. Then (X, g) is a generalized topological space. Now let A = {b} and U = {a, b}. Then $A \subseteq U$ and $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(\{b\})) = X \subseteq X$ and $i_g(c_g(U)) = i_g(c_g(\{a, b\})) = X$. Therefore $sc_g(A) \subseteq i_g(c_g(U))$. Therefore A is a g_u- semi closed set in (X, g). But $i_g(c_g(A)) = i_g(c_g(\{b\})) = X \nsubseteq A$. Therefore A is not a g-semi closed set in (X, g).

Theorem 3.12: Every g- α closed set in (X, g) is a g_u- semi closed set in (X, g) but not conversely.

Proof: Let (X, g) be a generalized topological space. Let A be g- α closed. Then $c_g(i_g(c_g(A))) \subseteq A$. Now let $A \subseteq U$ and U be g-open. Then $sc_g(A) = A \cup i_g(c_g(A)) \subseteq A \cup c_g(i_g(c_g(A))) \subseteq A \cup A \subseteq A \subseteq U = i_g(U) \subseteq i_g(c_g(U))$. Therefore A is g_u-semi closed set.

Example 3.13: Let X = {a, b, c} and let g = {∅, {a, b}, {b, c}, X}. Then (X g) is a generalized topological space. Now let A = {b} and U = {a, b}. Then $A \subseteq U$ and $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(\{b\})) = X \subseteq X$ and $i_g(c_g(U)) = i_g(c_g(a, b)) = X$. Therefore $sc_g(A) \subseteq i_g(c_g(U))$. Therefore A is g_u- semi closed set (X, g). But $c_g(i_g(c_g(A))) = c_g(i_g(c_g(\{b\}))) = X \nsubseteq A$. Therefore A is not g-$\alpha$ closed set in (X, g).

Theorem 3.14: Every g-regular closed set in (X, g) is a g_u- semi closed set in (X, g) but not conversely.

Proof: Let A be g -regular closed set. Then $c_g(i_g(A)) = A$. Now let $A \subseteq U$ and U be g-open. Then $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(c_g(i_g(A)))) = A \cup i_g(c_g(i_g(A))) = A \cup i_g(A) \subseteq A \subseteq U = i_g(U) \subseteq i_g(c_g(U))$. Therefore A is g_u-semi closed set.

Example 3.15: Let X = {a, b, c} and let g = {∅, {a, b}, {b, c}, X}. Then(X, g) is a generalized topological space. Now let $A = \{b\}$ and $U = \{a, b\}$. Then $A \subseteq U$ and $sc_g(A) = A \cup i_g(c_g(A)) = A \cup i_g(c_g(\{b\})) = X \subseteq X$ and $i_g(c_g(U)) = i_g(c_g(a, b)) = X$. Therefore $sc_g(A) \subseteq i_g(c_g(U))$. Therefore A is g_u- semi closed set in (X, g). But $c_g(i_g(A)) = c_g(i_g(\{b\})) = \emptyset \neq A$. Therefore A is not g-regular closed set in (X, g).

Theorem 3.16: If a set A is g_u-semi closed in (X, g) then $sc_g(A)$-A contains no non empty closed set.

Proof: Suppose that A is a g_u-semi closed set (X, g). Let F be a closed subset of $sc_g(A)$-A. Then $F \subseteq A^c$ and hence $A \subseteq F^c$. Since F^c is g-open and A is g_u-semi closed set, $sc_g(A) \subseteq F^c$. This implies $F \subseteq (sc_g(A))^c$. Then by Theorem 3.3 we have $F \subseteq sc_g(A)$. But $sc_g(A) \cap (sc_g(A))^c = \emptyset$. Therefore $F = \emptyset$. Hence the theorem is proved.

Theorem 3.17: In a generalized topological space X, for each $x \in X$, $\{x\}$ is g-closed or its complement X-$\{x\}$ is g_u-semi closed in (X, g).

Proof: Suppose that $\{x\}$ is not g-closed in (X, g). Then X-$\{x\}$ is not g-open and the only g-open set containing X-$\{x\}$ is X. Therefore $sc_g(X\text{-}\{x\}) \subseteq X$.Therefore X-$\{x\}$ is g_u-semi closed in (X, g).

Theorem 3.18: If A is g-open and g_u-semi closed then A is semi closed.

Proof: Since A is g-open and g_u-semi closed, we have $A \subseteq X$ and $sc_g(A) \subseteq i_g(c_g(U))$. But $i_g(c_g(A)) \subset sc_g(A)$ as $sc_g(A) = A \cup i_g(c_g(A))$. Then $A = sc_g(A)$. Thus A is semi g-closed.

Theorem 3.19: Then the following two conditions are equivalent in a generalized topological space (X, g);

(i) A is g-open and g_u-semi closed

(ii) A is g-regular open

Proof: (i) ⇒ (ii) By (i) As A is g-open set and g-semi closed, we have $A \subseteq A$ also $i_g(A) = A$. Now $i_g(c_g(A)) \subseteq A \cup i_g(c_g(A)) = sc_g(A) \subseteq A = i_g(A) \subseteq i_g(c_g(A))$.Therefore $i_g(c_g(A)) = A$. Therefore A is g-regular open.

(ii) ⇒ (i) Since every g-regular open set is g-open, A is g-open and $i_g(c_g(A)) = A$. As $A \subseteq A$, we have $sc_g(A) = A \cup i_g(c_g(A)) = A \cup A = A = i_g(A) \subseteq i_g(c_g(A))$. Thus A is g_u-semi closed in (X, g).

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