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Three-dimensional Guidance Law with Finite-time Convergence for Guided Bomb

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Abstract
Many existing guidance laws were designed by decoupling analysis on the relative motion model of the bomb and target. The ignorance of the cross coupling between the pitch and yaw channels would impact the guidance performance in practical applications. Therefore, based on the finite-time control theory, a coupled three-dimensional guidance law using sliding mode control was proposed in this paper. This guidance law can make the guided bomb converge in finite time, and hit the target with a desired impact angle. Simulink results have verified that comparing to the proportion guidance law, this method has more efficient guidance performance.
Keywords: *Guided Bomb, Three-dimensional Guidance Law, Sliding Mode Control, Impact Angular Constraint, Finite-time Convergence*

1. Introduction

As an important member of the precision guided weapons, guided bombs have been widely used in modern battlefields. And the design of the guidance law plays an important role in the study of guided bombs. At present, the most popular guidance law in practical engineering is proportional navigation guidance (PNG) and its variations, which are technically easy to implement and ensure precise strike. However, guided bombs are also expected that the impact angle can converge to a subvertical direction to improve the damage capacity. And the impact angle is hoped to converge in advance to reduce the kinetic energy loss of the guidance system. PNG is difficult to realize these guidance constraints. Therefore, it is necessary to introduce modern control theories to design new guidance laws with more constraints.

Sliding mode control (SMC), as one of the modern theories, is widely applied in complicated control systems. Considering that the systems with SMC are insensitive to external disturbance and parameter variation, SMC has provided an effective option in the guidance law design. In 1990, SMC was applied in guidance law for the first time [1]. Combined with practical demands, the impact angular constraint was considered in the guidance law design, and a number of remarkable results have been achieved [2-4]. Then, the finite-time convergence stability theory was also introduced to further the theoretical research of guidance law [5-7]. The above methods were all given in the longitudinal plane, which ignored the cross coupling between the pitch and yaw channels. Several three-dimensional coupled guidance laws were proposed in [8-10]. However, the angle of light-of-sight (LOS) didn't get convergence in finite time in [8]. The desired impact angle were not achieved in [9]. And the overload command was too heavy for the guided bomb in [10].

As described above, it can be seen that the research of the three-dimensional guidance law with various constraints needs to be furthered. Based on Lyapunov stability theory and the finite-time convergence stability theory, a three-dimensional sliding mode guidance law is proposed in this paper, and then the finite-time stability of the guidance law is proved. Theoretical analysis and simulation results show that this guidance law can make the bomb converge in finite time and attack the target with a desired subvertical angle, which demonstrates the effectiveness of the proposed guidance law.

2. The Problem Formulation

2.1 Relative Motion Model of the Bomb and Target

Before the design of the guidance law, we should firstly establish the relative motion model of the bomb and target. Since guided bombs are commonly used to attack the stationary or small maneuvering target, it can be supposed that the bomb and target are two particles in the three-dimensional space, and the target is stationary. Their relative motion is shown as Fig.1.

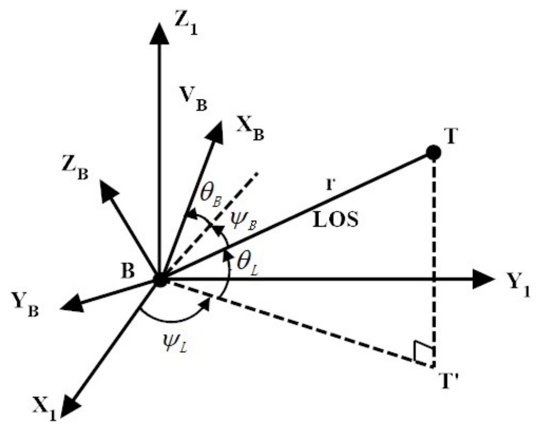

Fig. 1 The three-dimensional engagement geometry.

In Fig.1, $X_1Y_1Z_1$ and $X_LY_LZ_L$ are the inertial coordinate system and the LOS coordinate system, respectively. The bomb and the target are represented by the symbol B and T, their relative distance is defined as r, θ_L and ψ_L indicate the LOS incidence angle and LOS azimuthal angle. The velocity of the guided bomb is V_B, and its flying direction is determined by the incidence angle θ_B and the azimuthal angle ψ_B based on the LOS coordinate system $X_LY_LZ_L$. The longitudinal and lateral normal acceleration is represented as A_{yB} and A_{zB}, respectively. Therefore, the equations of the relative motion between the bomb and target are given as Eqs.(1-5)[3]:

$$\dot{r} = -V_B \cos\theta_B \cos\psi_B \tag{1}$$

$$r\dot{\theta}_L = -V_B \sin\theta_B \tag{2}$$

$$r\dot{\psi}_L \cos\theta_L = -V_B \cos\theta_B \sin\psi_B \tag{3}$$

$$\dot{\theta}_B = \frac{A_{zB}}{V_B} - \dot{\psi}_L \sin\theta_L \sin\psi_B - \dot{\theta}_L \cos\psi_B \tag{4}$$

$$\dot{\psi}_B = \frac{A_{yB}}{V_B \cos\theta_B} + \dot{\psi}_L \tan\theta_B \cos\psi_B \sin\theta_L - \dot{\theta}_L \tan\theta_B \sin\psi_B - \dot{\psi}_L \cos\theta_L \tag{5}$$

In order to make the bomb hit the target with desired impact angle, some mathematical analysis needs to be conducted. Suppose that $\dot{\theta}_L = 0$, $\dot{\psi}_L = 0$, $\theta_L = \theta_{Ld}$, $\psi_L = \psi_{Ld}$ .it can be concluded that $\sin\theta_B = 0$ from Eq.(2) and $\sin\psi_B = 0$ ($\theta_B \neq \pm\pi/2$) from Eq.(3), which means $\theta_B = 0$, and $\psi_B = 0$. At this moment, the direction of the velocity is coincident with the LOS. Considering that $\theta_L = \theta_{Ld}$, $\psi_L = \psi_{Ld}$, the incidence angle and azimuthal angle of velocity can be described as θ_{Ld} and ψ_{Ld} in the inertial coordinate system $X_1Y_1Z_1$, separately. Hence, if the following conditions as Eq.(6) is meet at the time of interception, the guided bomb can hit the target with the desired impact angle.

$$\dot{\theta}_L = 0,\ \theta_L = \theta_{Ld},\ \dot{\psi}_L = 0,\ \psi_L = \psi_{Ld} \tag{6}$$

2.2 Finite-time Stability of Nonlinear Systems

To satisfy the guidance constraints of finite-time convergence, Lemma 1 is introduced to carry out the research of the finite-time stability [11].

Lemma 1: Consider the nonlinear system (7),

$$\dot{x} = f(x,t),\ f(0,t) = 0,\ x \in R^n \tag{7}$$

where $f: U_0 \times R \to R^n$ is continuous on $U_0 \times R$, and U_0 is an open neighborhood of the origin $x = 0$. Suppose that there is a continuously differentiable function $V(x,t)$ defined in a neighborhood $\hat{U} \subset R^n$ of the origin, and that there are real numbers $c > 0$ and $0 < \alpha < 1$, such that $V(x,t)$ is positive definite on $\hat{U}$ and that $\dot{V}(x,t) + cV^{\alpha}(x,t) \le 0$ on $\hat{U}$. Then, the zero solution of system (7) is finite-time stable.
The proof of Lemma 1 can refer to Reference [11].

Remark 1: Note that if $\hat{U} = R^n$ and $V(x,t)$ is radially unbounded, then the origin is globally finite time stable.

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3. Guidance Law with Finite-time Convergence

Based on the equations of the relative motion between the bomb and target Eqs.(1-5), a new three-dimensional guidance law with terminal angular constraint and finite-time convergence is proposed in this section. Meanwhile, according to the finite-time stability of nonlinear control systems, finite-time stability analysis of guidance system is conducted.

3.1 Guidance Law Design

First of all, the relation motion equations need some deduction. Differentiate Eq.(2) with respect to time yields and substitute into Eqs.(1,2,4), Eq.(8) is derived as:

$$\ddot{\theta}_L = -\frac{2\dot{r}\theta_L}{r} - \dot{\psi}_L^2 \sin\theta_L \cos\theta_L - \frac{\cos\theta_B}{r} A_{zB} \quad (8)$$

Differentiate Eq.(3) and substitute into Eqs.(1,2,4,5), Eq.(9) is obtained as:

$$\ddot{\psi}_L = -\frac{2\dot{r}\psi_L}{r} + 2\dot{\psi}_L\dot{\theta}_L \tan\theta_L + \frac{\sin\theta_B \sin\psi_B}{r\cos\theta_L} A_{zB} - \frac{\cos\psi_B}{r\cos\theta_L} A_{yB} \quad (9)$$

From Eq.(8) and Eq.(9), it can be seen that the lateral normal acceleration A_{zB} influences the LOS incidence angle θ_L and LOS azimuthal angle ψ_L at the same time, which means that the longitudinal and lateral dynamic of guided bomb has couple effect. Once the dynamic is decoupled, it will cause deviation from actual situation. Hence, a coupled three-dimensional guidance law is proposed in this section to decrease this deviation.

Transform Eq.(8) and Eq.(9), it can be derived as:

$$\begin{bmatrix} \ddot{\theta}_L \\ \ddot{\psi}_L \end{bmatrix} = C + D \begin{bmatrix} A_{zB} \\ A_{yB} \end{bmatrix} \quad (10)$$

where $C = \begin{bmatrix} -\frac{2\dot{r}\theta_L}{r} - \dot{\psi}_L^2 \sin\theta_L \cos\theta_L \\ -\frac{2\dot{r}\psi_L}{r} + 2\dot{\psi}_L\dot{\theta}_L \tan\theta_L \end{bmatrix}$,

$$D = \begin{bmatrix} -\frac{\cos\theta_B}{r} & 0 \\ \frac{\sin\theta_B \sin\psi_B}{r\cos\theta_L} & -\frac{\cos\psi_B}{r\cos\theta_L} \end{bmatrix}.$$

Choose the state variables of guidance system as $x_1 = \theta_L - \theta_{Ld}$, $x_2 = \dot{\theta}_L$, $x_3 = \psi_L - \psi_{Ld}$, $x_4 = \dot{\psi}_L$. In order to satisfy the performance indicators of guided bomb, the switching function of the sliding mode surface is designed as:

$$S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} x_2 + k_1|x_1|^{\alpha_1}\operatorname{sgn}(x_1) \\ x_4 + k_2|x_3|^{\alpha_2}\operatorname{sgn}(x_3) \end{bmatrix} \quad (11)$$

where $0 < \alpha_1, \alpha_2 < 1$ and $k_1, k_2 > 0$, sgn() denotes the signum function. The former item of s_1 and s_2 can ensure a small miss-distance, and the latter can satisfy the requirement of impact angular constraint.

Then, Differentiating Eq.(11) with respect to time yields, Eq.(12) is derived as:

$$\dot{S} = \begin{bmatrix} \dot{x}_2 + k_1\alpha_1|x_1|^{\alpha_1-1}x_2 \\ \dot{x}_4 + k_2\alpha_2|x_3|^{\alpha_2-1}x_4 \end{bmatrix} \quad (12)$$

Consider a fast power reaching law to derivate the controller as Eq.(13):

$$\dot{S} = -\frac{1}{r}\begin{bmatrix} \varepsilon_1\operatorname{sgn}(s_1) \\ \varepsilon_2\operatorname{sgn}(s_2) \end{bmatrix} \quad (13)$$

where $\varepsilon_1, \varepsilon_2 > 0$. When the bomb is comparatively far from the target, this reaching law can properly slow down the rate of the LOS angular convergence to zero, so as to avoid a heavy initial overload command. Besides, the convergence rate increases as the bomb get closer, which can guarantee the convergence of θ_L , ψ_L and improve the attack accuracy.

Combining Eqs.(10,12,13), we have:

$$-\frac{1}{r}\begin{bmatrix} \varepsilon_1\operatorname{sgn}(s_1) \\ \varepsilon_2\operatorname{sgn}(s_2) \end{bmatrix} = \begin{bmatrix} \dot{x}_2 + k_1\alpha_1|x_1|^{\alpha_1-1}x_2 \\ \dot{x}_4 + k_2\alpha_2|x_3|^{\alpha_2-1}x_4 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_L \\ \ddot{\psi}_L \end{bmatrix} + \begin{bmatrix} k_1\alpha_1|x_1|^{\alpha_1-1}x_2 \\ k_2\alpha_2|x_3|^{\alpha_2-1}x_4 \end{bmatrix} \quad (14)$$

Transform Eq.(14), it can be derived as:

$$F = C + D\begin{bmatrix} A_{zB} \\ A_{yB} \end{bmatrix} + E \quad (15)$$

where $E = \begin{bmatrix} k_1\alpha_1|x_1|^{\alpha_1-1}x_2 \\ k_2\alpha_2|x_3|^{\alpha_2-1}x_4 \end{bmatrix}$, $F = -\frac{1}{r}\begin{bmatrix} \varepsilon_1\operatorname{sgn}(s_1) \\ \varepsilon_2\operatorname{sgn}(s_2) \end{bmatrix}$.

According to Eq.(15), the overload command of the guidance system is obtained as Eq.(16).

$$\begin{bmatrix} A_{zB} \\ A_{yB} \end{bmatrix} = D^{-1}(F-C-E) \tag{16}$$

where $D^{-1} = \begin{bmatrix} -\dfrac{r}{\cos\theta_B} & 0 \\ -r\tan\theta_B\tan\psi_B & -\dfrac{r\cos\theta_L}{\cos\psi_B} \end{bmatrix}$,

and $\cos\theta_B, \cos\psi_B \neq 0$, that is to say $\theta_B, \psi_B \neq \pm\pi/2$. Eq.(16) gives out the three-dimensional sliding mode guidance law with terminal angular constraint and finite-time convergence.

3.2 Finite-time convergence analysis

The analysis of the finite-time stability during the phase of reaching the sliding mode surface and moving along the surface is carried out as follows. Take the longitudinal guidance law as an example. Multiply s on both sides of $\dot{s} = -\varepsilon_2\operatorname{sgn}(s)/r$ in Eq.(13), we have:

$$s\dot{s} = -\frac{\varepsilon_2}{r}\operatorname{sgn}(s)s \leq 0 \tag{17}$$

which satisfies the condition of reaching the sliding surface for the system: $s\dot{s} \leq 0$.

During the sliding mode reaching phase, the Lyapunov function is chosen as:

$$V_1 = s^2 \tag{18}$$

Differentiating Eq.(18), Eq.(19) is obtained.

$$\dot{V}_1 = 2s\dot{s} = -\frac{2s}{r}\varepsilon_2\operatorname{sgn}(s) \leq 0 \tag{19}$$

Combining Eq.(18) and Eq.(19), Eq.(20) can be derived as:

$$\dot{V}_1 \leq -\frac{2\varepsilon_2}{r}V_1^{\frac{1}{2}} \tag{20}$$

Considering that r satisfies $\dot{r} < 0$, $0 \leq r \leq r_0$, $\forall r > 0$ during the guidance process, Eq.(21) can be got as:

$$\dot{V}_1 \leq -\frac{2\varepsilon_2}{r_0}V_1^{\frac{1}{2}} \tag{21}$$

According to *Lemma 1*, the guidance system can converge to the sliding mode surface $s = 0$ in a finite time.

After reaching the sliding mode surface, the guidance system begins to move along this surface. In the motion phase, the sliding mode surface s satisfies:

$$s = x_4 + k_2|x_3|^{\alpha_2}\operatorname{sgn}(x_3) = 0 \tag{22}$$

Substituting $\dot{x}_3 = x_4$ into Eq.(22),Eq.(23) is obtained.

$$\dot{x}_3 = -k_2|x_3|^{\alpha_2}\operatorname{sgn}(x_3) \tag{23}$$

During this phase, another Lyapunov function is chosen as:

$$V_2 = x_3^2 \tag{24}$$

Differentiating Eq.(24), Eq.(25) is derived as:

$$\dot{V}_2 = 2x_3\dot{x}_3 = -2k_2|x_3|^{\alpha_2+1} \leq 0 \tag{25}$$

Combining Eq.(24) and Eq.(25), Eq.(26) can be obtained as:

$$\dot{V}_2 = -2k_2V_2^{\frac{\alpha_2+1}{2}} \tag{26}$$

According to *Lemma 1*, the system can also get convergence in finite time during the second phase.

The finite-time stability analysis of the lateral guidance law is same as the longitudinal one. Through the analysis of the two phases as above mentioned, the guidance system is proofed to converge in finite time during the whole guiding process. Thus, it can be concluded that the formula of the switching function and the reaching law is reasonable and available.

4. Simulation Results

The simulation is carried out on a certain type of guided bomb in an air-to-surface engagement scenario. In the same simulation environment, the proportional guidance law is introduced as a contrast to verify the rationality and validity of the proposed guidance law. The simulation step is set as 0.01s, and the initial parameters of the experiment is given in Table 1.

Table 1 Initial parameters of the simulation

Parameters	Initial Values

Target position(km)	[6000 8000 0]
Bomb launch position(km)	[0 0 5000]
Velocity of bomb(m/s)	350
Velocity angle of bomb(deg)	15, 15
Desired terminal angle(deg)	-70, 30
Available overload(g)	[-5,5]

Because of the existence of $\mathrm{sgn}(s)$, it's easy to cause chattering after the guidance system reached the sliding mode surface. Therefore, the saturation function $sat(s)$ is used to replace the switching function $\mathrm{sgn}(s)$, which can smooth the chattering phenomenon.

$$sat(s)=\begin{cases}1 & s>\Delta \\ \gamma s & |s|\le\Delta\ ,\ \gamma=1/\Delta \\ -1 & s<-\Delta\end{cases} \tag{26}$$

The guidance parameters of the simulation are given in Table 2.

Table 2 Guidance parameters of the simulation

Parameters	Values	Parameters	Values
k_1	0.56	k_2	0.75
α_1	0.8	α_2	0.7
ε_1	105	ε_2	95
Δ	0.1		

The longitudinal and lateral navigation ratio of the proportional are both set as 3. The simulation results are given as Fig.2-Fig.5.

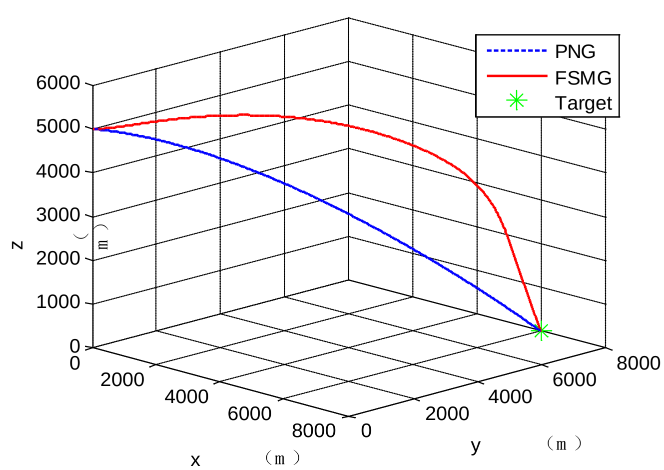

Fig.2 Comparison result of the trajectory

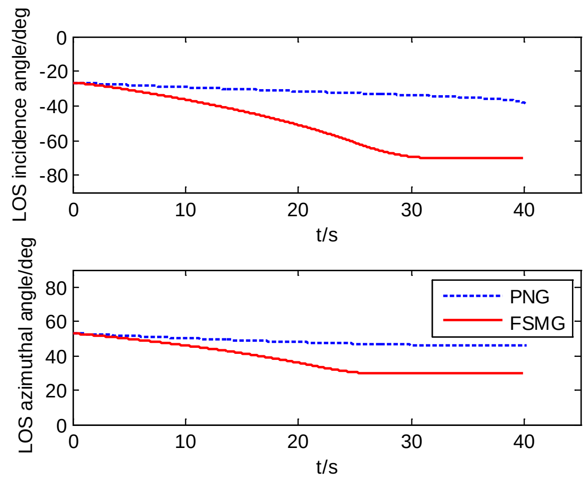

Fig.3 Comparison result of the light-of-sight (LOS) angle

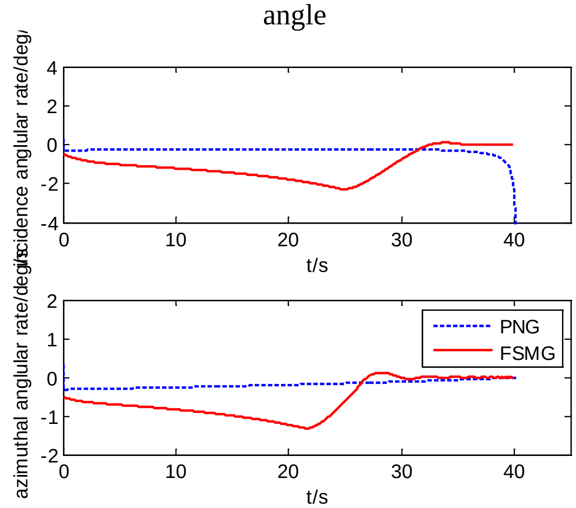

Fig.4 Comparison result of the angular rate of LOS

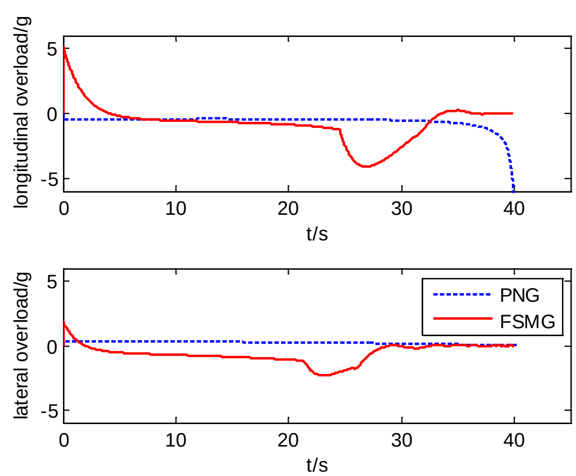

Fig.5 Comparison result of the overload

In the above figures, PNG means the proportional guidance law, and FSMG represents the finite-time converged sliding mode guidance law. It can be seen from Fig.2 to Fig.4 that the flying time of the guided bomb is about 40s. The bomb with PNG doesn't realize the impact angular constraint or finite-time convergence. However, the LOS incidence angle and azimuthal angle of the bomb with FSMG get convergence at the 26^{th} and 33^{rd} second, respectively. Thus, the bomb with FSMG can fly along a nearly straight trajectory with less kinetic energy loss during the last flying time, and hit the target with a desired impact angle. This will improve the precision and the strength of the attack effectively. The longitudinal and lateral overload of guided bomb is given in Fig.5. It shows that although there is slight vibration in the curve of FSMG, it still converges to zero in the later stage of guidance. Besides, the overload of FSMG keeps in the range of the available overload of bomb, which means the design of FSMG is reasonable in the practical engineering.

5. Conclusions

In this paper, a three-dimensional sliding mode guidance law is proposed. It is combined with the flight dynamics of the guided bomb, and the cross coupling between the pitch and yaw channel is considered. Simulation results have proved that the proposed guidance law can realize the impact angular constraint and the finite-time convergence, which can ensure a fine guidance performance of the guided bomb.

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