

Combined Effect of Hall Current and Rotation on Thermal Stability of Ferromagnetic Fluids saturating in a Porous Medium under Varying Gravity Field

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Abstract

This paper deals with the theoretical investigation of the effect of the Hall current and rotation on ferromagnetic fluids saturating in a porous medium under the varying gravity field. To find the exact solution for a ferromagnetic fluid layer contained between two free boundaries, we have used a linear stability analysis and normal mode analysis methods. A dispersion relation governing the effect of Hall current, rotation and magnetic field is derived. From the study, we have found that the Hall current has stabilizing effect on the system under the condition $T_{A_1} > \frac{\epsilon^2}{\rho^2} M$ and $\lambda > 0$. For $\lambda < 0$, Hall current has destabilizing effect on the system. Further, rotation is found to have stabilizing effect on the system for the case $\lambda > 0$ and destabilizing effect for $\lambda < 0$. The effect of magnetic field on the system is to stabilize the system under the condition $T_{A_1} < \frac{4M\epsilon^2}{\rho^2}$ and $\lambda > 0$ and to destabilize the system for $\lambda < 0$. The principle of exchange of stabilities is not satisfied for the present problem while in the absence of rotation and Hall current, it is found to be satisfied under certain condition.

Keywords: Hall current, Magnetic field, Ferromagnetic fluid, Rotation, Thermal stability

1. Introduction

Ferromagnetic fluid (also called ferrofluid or magnetic fluid) is electrically non-conducting colloidal suspensions of solid ferromagnetic particles in a non-electrically conducting carrier fluid like water, kerosene, hydrocarbon or organic solvent etc. These colloidal particles are coated with a stabilizing dispersing agent (surfactants) who prevents particle agglomeration even when a strong magnetic field gradient is applied to the ferromagnetic fluid. These suspensions are stable and maintain their properties at extreme temperatures and over a long period of time. Rosenweig (1985) has discussed, in detail, an authoritative introduction to this subject in his celebrated monograph. In this monograph, he reviews several applications of heat transfer through ferromagnetic fluids. Ferromagnetic fluids have very large potential applications in electronic devices, mechanical engineering, material science, analytical instrumentation, medicines, optics, arts etc. Owing the applications of the ferromagnetic fluid, its study is important to researchers. Ferrofluid technology is well established and capable of solving a wide variety of technical problems. There are many successful applications of this engineering material and there is an immense scope of further research. There are various stability problems on ferromagnetic fluids. The convective instability, also known as Bénard convection (Chandrasekhar, 1981), is one of the instability of ferromagnetic fluid. Finlayson (1970) have studied the convective instability of the ferromagnetic fluid for a

fluid layer heated from below in the presence of uniform vertical magnetic field and explained the concept of thermo-mechanical interaction in ferromagnetic fluids. L alas and Carmi (1971) have discussed the thermo-convective stability of ferromagnetic fluids without considering the buoyancy effect. Many authors (Siddheswar, 1993, 95; Venkatasubramaniam, et al., 1994; Sunil, et al., 2006, 07 and Aggarwal, et al., 2009) have considered the Bénard convection in ferromagnetic fluid in non-porous medium and many authors (Lapwood, 1948; Wooding, 1960 and Sunil, et al., 2008) have studied the stability of fluid flow through a porous medium. A porous medium is defined as a solid with holes in it. It is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location and shape. The flow of a fluid through isotropic and homogeneous porous medium is governed by Darcy's law. In 1982, Sharma and Sharma have been discussed the rotation and solute gradient on the thermal instability of fluids through a porous medium. Rotation also plays an important role in the thermal instability of fluid layer. In case of stationary convection, rotation stabilizes the fluid layer while magnetization parameters destabilize the fluid (Chand and Bala, 2013).

The Hall currents are also likely to be important in flows of laboratory plasma as well as in many geophysical and astrophysical situations. When a strong electric field is applied, the electric conductivity is affected by the magnetic field. As a result, the conductivity parallel to the electric field is reduced and hence, the current is reduced in the direction normal to both electric and magnetic field. This phenomenon is called Hall Effect and the current is known as Hall current. The effect of Hall current on thermal instability has also been discussed by several authors (Gupta, 1967; Raghavachar, et al., 1988; Sharma, et al., 1993; Sharma, et al., 2000; Sunil, et al., 2005 and Gupta et al., 2011, 12). Aggarwal and Makhija (2014) have studied the effect of Hall current on thermal stability of ferromagnetic fluids heated from below in porous medium in the presence of horizontal magnetic field. They found that Hall currents have a destabilizing effect while magnetization has a stabilizing effect. In all the above studies, the gravity field was assumed to be constant. However, the earth's gravity varies with height from its surface. But usually we neglect this variation of gravity for laboratory purposes and treat the field as a constant. This may not be the case for large scale flows in the ocean or the atmosphere. Considering the gravity as a quantity varying with distance from the centre can become imperative.

In the present study, we have studied the effect of Hall current and Rotation on thermal stability of ferromagnetic fluids saturating in a porous medium under varying gravity field. We have assumed that the gravity is varying as, $g = \lambda g_0$, where g_0 is the value of g at the Earth's surface, which is always positive and λ can be positive or negative as gravity increases or decreases upwards from its value g_0 .

2. Mathematical Formulation of the Problem

We consider an infinite, incompressible, electrically non-conducting and thin layer of ferromagnetic fluid which is bounded by the planes $z = 0$ and $z = d$, as shown in Fig 1. The fluid layer is heated from below so that a uniform temperature gradient $\beta = \left| \frac{dT}{dz} \right|$ is maintained within the fluid. The system is acted upon by a uniform vertical magnetic field $\vec{H} (0,0, H)$ and variable gravity field $\vec{g} (0,0, -g)$, where $g = \lambda g_0$, g_0 is the value of g at $z = 0$ which is always positive and λ can be positive or negative as gravity increases or decreases upwards from its value g_0 . The whole system is assumed to be rotating about z -axis with uniform angular velocity $\vec{\Omega} = (0,0, \Omega_0)$. The ferromagnetic fluid layer is assumed to be

flowing through an isotropic and homogeneous porous medium of porosity ε which is defined as the fraction of the total volume of the medium that is occupied by void space. Thus, the fraction $1 - \varepsilon$ is occupied by solid.

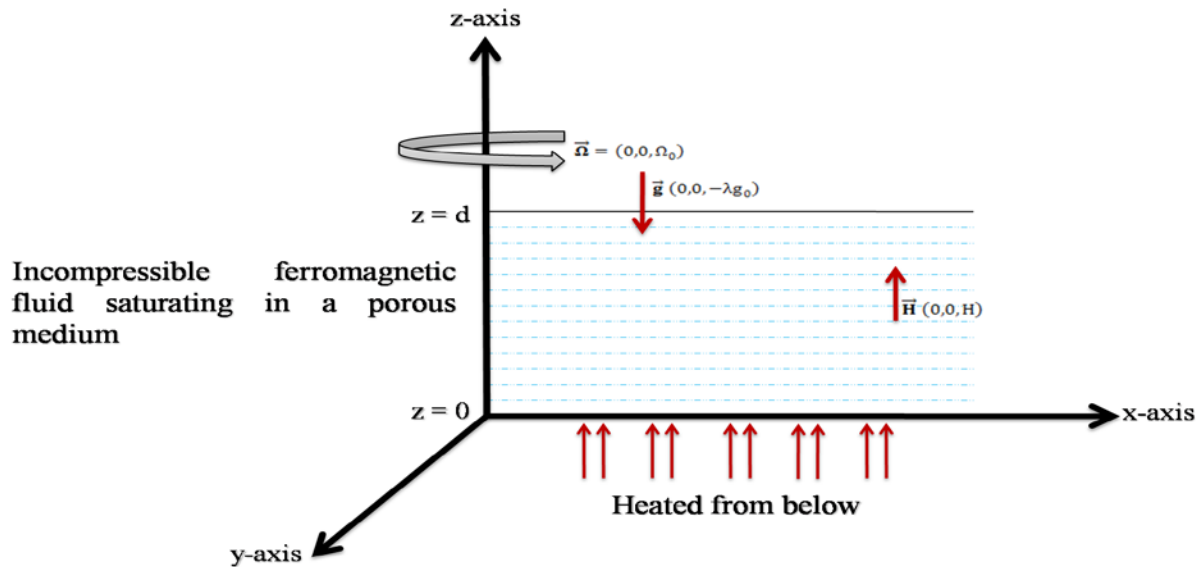


Fig.1: Geometrical Configuration

The governing equations of motion of a ferromagnetic fluid under the Boussinesq approximation, saturating a porous medium following darcy's law are as follows:

The equation of continuity, conservation of momentum, temperature and equation of state of ferromagnetic fluids through porous medium are

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\frac{1}{\varepsilon} \left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon} (\vec{q} \cdot \nabla) \vec{q} \right] = -\frac{1}{\rho_0} \nabla p - \frac{\rho}{\rho_0} \lambda g_0 \hat{e}_z - \frac{1}{k_1} \nu \vec{q} + \frac{M \nabla \vec{H}}{\rho_0} + \frac{\mu_e}{4\pi \rho_0} (\nabla \times \vec{H}) \times \vec{H} + \frac{2}{\varepsilon} (\vec{q} \times \vec{\Omega}) \tag{2}$$

$$E \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa \nabla^2 T \tag{3}$$

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \tag{4}$$

where, $\vec{q}(u, v, w)$ = fluid velocity, p = the fluid pressure, ρ = fluid density, ρ_0 = reference density, T = temperature, T_0 = reference temperature, g_0 = gravitational acceleration, α = thermal coefficient of expansion, μ_e = magnetic permeability, ν = kinematic viscosity, κ = thermal diffusivity, $E = \left[\varepsilon + (1 - \varepsilon) \frac{\rho_s c_s}{\rho_0 c_f} \right]$, ρ_s, c_s = density and specific heat of solid/porous material, ρ_0, c_f = density and specific heat of fluid.

Since ferromagnetic fluids respond so rapidly to a magnetic torque, so we assume the following conditions to be hold

$$\vec{M} \times \vec{H} = 0 \quad (5)$$

In ferrohydrodynamics, the free charge and the electric displacement are assumed to be absent, therefore Maxwell's equations becomes

$$\nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{H} = 0 \quad (6)$$

In Chu formulation of electrohydrodynamics, the relation between the magnetic field, magnetization and magnetic induction is given by

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad (7)$$

Here, \vec{M} stands for magnetization, \vec{H} stands for the magnetic field intensity and \vec{B} for magnetic induction.

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field and temperature, which can be expressed as

$$\vec{M} = \frac{\vec{H}}{H} M(\vec{H}, T) \quad (8)$$

Where, $\vec{H} = (0, 0, H)$, i.e. $\vec{H} = H\hat{e}_z$, \hat{e}_z is the unit vector along z-axis and H is the uniform magnetic field of the fluid layer and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad H = |\vec{H}|, \quad M = |\vec{M}| \quad \text{and} \quad B = |\vec{B}|$$

The Maxwell's equations in the presence of Hall currents is given by

$$\varepsilon \frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{q} \times \vec{H}) + \varepsilon \eta \nabla^2 \vec{H} - \frac{\varepsilon}{4\pi N e} \nabla \times [(\nabla \times \vec{H}) \times \vec{H}] \quad (9)$$

$$\text{and} \quad \nabla \cdot \vec{H} = 0 \quad (10)$$

Generally, for completing a system, it is necessary that the equation of state will specify M in two thermodynamics variables (say H and T), but in present study, we consider that the magnetization is independent of the magnetic field intensity i.e. $M = M(T)$. Thus, as a first approximation, we assume that

$$M = M_0[1 - \gamma(T - T_0)] \quad (11)$$

Where M_0 is the magnetization at $T = T_0$ and $\gamma = \frac{1}{M_0} \left(\frac{\partial M}{\partial T} \right)_H$

The basic state is assumed to be quiescent state which is given by

$$\vec{q} = \vec{q}_b = (0, 0, 0), \quad \rho = \rho_b(z), \quad p = p_b(z), \quad \vec{M} = \vec{M}_b = \vec{M}_b(z), \quad \vec{H} = \vec{H}_b = \vec{H}_b(z), \quad \vec{B} = \vec{B}_b$$

$$T = T_b(z) = -\beta z + T_0, \quad \rho = \rho_b = \rho_0(1 + \alpha\beta z), \quad M = M_0(1 + \gamma\beta z) \quad (12)$$

3. The Perturbations Equations

Let \vec{q}' , p' , ρ' , M' , θ and \vec{h} (h_x, h_y, h_z) denote respectively the small perturbations in \vec{q} , p , ρ , M , T and \vec{H} . Therefore the new variables becomes

$$\vec{q} = \vec{q}_b + \vec{q}', p = p_b + p', M = M_b + M', \vec{H} = \vec{H}_b + \vec{h}, T = T_b + \theta$$

Applying these perturbations and linearising equations (1) – (11), we get

$$\nabla \cdot \vec{q}' = 0 \quad (13)$$

$$\frac{1}{\varepsilon} \frac{\partial \vec{q}'}{\partial t} = -\frac{1}{\rho_0} \nabla p' + \lambda g_0 \alpha \theta \hat{e}_z - \frac{1}{k_1} v \vec{q}' - \frac{\gamma M_0 \nabla H}{\rho_0} \theta \hat{e}_z + \frac{\mu_e H}{4\pi \rho_0} (\nabla \times \vec{h}) \times \hat{e}_z + \frac{2\Omega_0}{\varepsilon} (\vec{q}' \times \hat{e}_z) \quad (14)$$

$$E \frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta \quad (15)$$

$$\varepsilon \frac{\partial \vec{h}}{\partial t} = H \nabla \times (\vec{q}' \times \hat{e}_z) + \varepsilon \eta \nabla^2 \vec{h} - \frac{\varepsilon H}{4\pi N_e} \nabla \times [(\nabla \times \vec{h}) \times \hat{e}_z] \quad (16)$$

$$\nabla \cdot \vec{h} = 0 \quad (17)$$

Writing the scalar components of equation (14) and eliminating $\nabla p'$, u , v , h_x , h_y between them by using equations (13) – (17), we get

$$\left(\frac{1}{\varepsilon} \frac{\partial}{\partial t} + \frac{1}{k_1} v\right) \nabla^2 w = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \left(\lambda g_0 \alpha - \frac{\gamma M_0 \nabla H}{\rho_0}\right) \theta + \frac{\mu_e H}{4\pi \rho_0} \nabla^2 \frac{\partial h_z}{\partial z} - \frac{2\Omega_0}{\varepsilon} \frac{\partial \zeta_z}{\partial z} \quad (18)$$

Again from equation (14), taking z-component, we get

$$\left(\frac{1}{\varepsilon} \frac{\partial}{\partial t} + \frac{1}{k_1} v\right) \zeta_z = \frac{\mu_e H}{4\pi \rho_0} \frac{\partial \xi}{\partial z} + \frac{2\Omega_0}{\varepsilon} \frac{\partial w}{\partial z} \quad (19)$$

From equation (15), taking z-component, we obtain

$$\left(E \frac{\partial}{\partial t} - \kappa \nabla^2\right) \theta = \beta w \quad (20)$$

From equation (16), taking z-component, we get

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2\right) h_z = \frac{H}{\varepsilon} \frac{\partial w}{\partial z} - \frac{H}{4\pi N_e} \frac{\partial \xi}{\partial z} \quad (21)$$

Again from equation (16), taking z-component, we obtain

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2\right) \xi = \frac{H}{\varepsilon} \frac{\partial \zeta_z}{\partial z} + \frac{H}{4\pi N_e} \nabla^2 \left(\frac{\partial h_z}{\partial z}\right) \quad (22)$$

4. Normal Mode Analysis

Now we analyze the perturbations into normal modes by assuming the following forms of perturbation quantities

$$[w, \theta, \xi, \zeta_z, h_z] = [W(z), \Theta(z), X(z), G(z), K(z)] \exp(ik_x x + ik_y y + \sigma t) \quad (23)$$

Where k_x, k_y are wave numbers along x and y directions respectively, $a = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number of the disturbance and σ is the growth rate. (Complex constant) For functions with this dependence on x, y and t, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -a^2$, $\nabla^2 = \frac{\partial^2}{\partial z^2} - a^2$

Using equation (23), equations (18) – (22) in non-dimensional form becomes

$$\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1}\right) (D^2 - a^2)W = -\frac{a^2 d^2 \alpha \lambda}{\nu} \left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda}\right) \Theta + \frac{\mu_e H d}{4\pi \rho_0 \nu} D(D^2 - a^2)K - \frac{2\Omega_0 d^3}{\varepsilon \nu} D \quad (24)$$

$$\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1}\right) G = \frac{\mu_e H d}{4\pi \rho_0 \nu} DX + \frac{2\Omega_0 d}{\nu \varepsilon} DW \quad (25)$$

$$(D^2 - a^2 - E\sigma p_1)\Theta = -\frac{\beta d^2}{\kappa} W \quad (26)$$

$$(D^2 - a^2 - \sigma p_2)K = -\frac{H d}{\varepsilon \eta} DW + \frac{H d}{4\pi N e \eta} DX \quad (27)$$

$$(D^2 - a^2 - \sigma p_2)X = -\frac{H d}{\varepsilon \eta} DG - \frac{H}{4\pi N e \eta d} D(D^2 - a^2)K \quad (28)$$

Where, we have expressed the coordinates in non-dimensional parametric form by using the following non-dimensional parameters, $(x, y, z) = (dx^*, dy^*, dz^*)$, $a = \frac{a^*}{d}$, $\sigma = \frac{\sigma^* \nu}{d^2}$, $D = \frac{d}{dz^*}$, $p_1 = \frac{\nu}{\kappa}$ is the prandtl number, $p_2 = \frac{\nu}{\eta}$ is the magnetic prandtl number and $P_1 = \frac{k_1}{d^2}$ (dropping * for convenience)

5. Exact solution for free boundaries

Here, we have considered that both the boundaries are free and perfect conductor of heat. The boundary conditions for the problem are (Chandrasekhar, 1981)

$$W = D^2 W = 0, \Theta = DG = 0, K = DX = 0 \text{ when } z = 0 \text{ and } 1 \quad (29)$$

Eliminating Θ, K, X and G from equations (24) – (28), we obtain

$$\begin{aligned} \lambda a^2 R_f W = & \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1} \right) (D^2 - a^2 - E\sigma p_1)(D^2 - a^2)W \\ & + \frac{(D^2 - a^2 - \sigma p_2)(D^2 - a^2 - E\sigma p_1)}{\varepsilon} \left[Q(D^2 - a^2) \left[\left\{ \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1} \right) \varepsilon (D^2 - a^2 - \sigma p_2) + QD^2 \right\} + 2\sqrt{MT_A}D^2 \right] \right. \\ & \left. + T_A \{ (D^2 - a^2 - \sigma p_2)^2 + MD^2(D^2 - a^2) \} \right] \\ & \cdot D^2 \left[\left\{ \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1} \right) \varepsilon (D^2 - a^2 - \sigma p_2) + QD^2 \right\} \{ (D^2 - a^2 - \sigma p_2)^2 + MD^2(D^2 - a^2) \} \right. \\ & \left. - MQD^4(D^2 - a^2) \right]^{-1} \cdot W \end{aligned} \tag{30}$$

Where $R_f = \left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda} \right) \frac{\alpha \beta d^4}{\nu \kappa}$ is the Rayleigh number for ferromagnetic fluids with varying gravity field. If $\lambda = 1$, then this reduces to general Rayleigh number (Aggarwal and Makhija, 2012). $Q = \frac{\mu_e H^2 d^2}{4\pi \rho_0 \nu \eta}$ is the Chandrasekhar number, $M = \left(\frac{H}{4\pi N e \eta} \right)^2$ is the Hall parameter and $T_A = \left(\frac{2\Omega_0 d^2}{\nu} \right)^2$

If $\lambda > 0$, $g_0 > \frac{M_0 \gamma \nabla H}{\lambda \rho_0 \alpha}$, then $R_f < R$, this implies that the convection starts in the ferrofluid at a higher thermal Rayleigh number and If $\lambda < 0$, then $R_f > R$, which implies that the convection starts in the ferrofluid at a lower thermal Rayleigh number.

Using the boundary conditions (29) we can show that all the even order derivatives of W must vanish at boundaries $z = 0$ and 1 . Hence the proper solution for W characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{31}$$

Where, W_0 is a constant. Substituting the proper solution (31) in equation (30), we get

$$\begin{aligned} R_1 = & \left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P} \right) (1 + x + i\sigma_1 E p_1) \frac{(1 + x)}{x\lambda} \\ & + \frac{(1 + x + i\sigma_1 p_2)(1 + x + i\sigma_1 E p_1)}{\lambda x \varepsilon^2} \left[Q_1 \varepsilon (1 + x) \left[\left\{ \left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P} \right) \varepsilon (1 + x + i\sigma_1 p_2) + Q_1 \varepsilon \right\} \right. \right. \\ & \left. \left. + 2\sqrt{MT_{A_1}} \right] + T_{A_1} \{ (1 + x + i\sigma_1 p_2)^2 + M(1 + x) \} \right] \\ & \cdot \left[\left\{ \left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P} \right) (1 + x + i\sigma_1 p_2) + Q_1 \right\} \{ (1 + x + i\sigma_1 p_2)^2 + M(1 + x) \} - MQ_1(1 + x) \right]^{-1} \end{aligned} \tag{32}$$

where $R_1 = \frac{R_f}{\pi^4}$, $Q_1 = \frac{Q}{\varepsilon \pi^2}$, $x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $P = \pi^2 P_1$, $T_{A_1} = \frac{T_A}{\pi^4}$

Equation (32) is the required dispersion relation including the effect of Hall current, rotation and magnetic field on a layer of ferromagnetic fluid saturating in a porous medium under the influence of varying gravity field. In the absence of rotation and constant gravity field, this

relation agrees with the dispersion relation derived by Aggarwal and Makhija (2012) for Ferromagnetic fluid, if solute concentration is removed from his study.

6. The Case of Stationary Convection

For the case of stationary convection, the marginal state will be characterized by $\sigma_1 = 0$, therefore the dispersion relation (32) reduces to

$$R_1 = \frac{1}{P} \frac{(1+x)^2}{\lambda x} + \frac{\frac{(1+x)}{\lambda x \epsilon^2} \left[\frac{Q_1 \epsilon^2 (1+x)}{P} + Q_1^2 \epsilon^2 + 2\sqrt{MT_{A_1}} \epsilon Q_1 + T_{A_1} (1+x+M) \right]}{\left(\frac{1+x+M}{P} + Q_1 \right)} \quad (33)$$

The dispersion relation (33) expresses the modified Rayleigh number R_1 as a function of the rotation parameter (T_{A_1}), medium permeability (P), Hall current parameter (M), magnetic field parameter (Q_1) and dimensional wave number x . In the absence of rotation ($T_{A_1} = 0$), the above Rayleigh number reduces to

$$R_1 = \frac{1}{P} \frac{(1+x)^2}{\lambda x} + \frac{Q_1 (1+x)}{\lambda x} \frac{\left[\frac{(1+x)}{P} + Q_1 \right]}{\left(\frac{1+x+M}{P} + Q_1 \right)} \quad (34)$$

which agree with the expression for R_1 derived by Aggrawal and Makhija (2012) if the solute gradient S_1 is vanishing.

In the above expression (34), if we remove Hall current parameter (M), the expression for Rayleigh number R_1 will become identical with the expression for R_1 derived by Sharma et al (71) in the absence of solute gradient.

In order to investigate the effects of rotation (T_{A_1}), Hall current (M) and magnetic field (Q_1), we examine the behavior of $\frac{dR_1}{dT_{A_1}}$, $\frac{dR_1}{dM}$ and $\frac{dR_1}{dQ_1}$ analytically.

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x)}{\lambda x \epsilon^2} \frac{\left[\frac{\sqrt{M} \epsilon Q_1}{\sqrt{T_{A_1}}} + 1 + x + M \right]}{\left(\frac{1+x+M}{P} + Q_1 \right)} \quad (35)$$

This equation confirms that for stationary convection, rotation has a stabilizing effect if $\lambda > 0$ and destabilizing effect for $\lambda < 0$.

$$\frac{dR_1}{dM} = \frac{(1+x)Q_1}{\lambda x \epsilon^2} \frac{\left[\frac{\epsilon(1+x)}{P} + \sqrt{MT_{A_1}} + \epsilon Q_1 \right] \left[\frac{\sqrt{T_{A_1}}}{\sqrt{M}} - \frac{\epsilon}{P} \right]}{\left(\frac{1+x+M}{P} + Q_1 \right)^2}$$

(36)

This equation shows that the Hall current has stabilizing effect on the system if $T_{A_1} > \frac{\varepsilon^2}{P^2} M$ and $\lambda > 0$. In the absence of rotation, Hall current has destabilizing effect on the system for $\lambda > 0$ and stabilizing effect for $\lambda < 0$.

$$\frac{dR_1}{dQ_1} = \frac{\frac{(1+x)}{x\varepsilon^2} (1+x+M) \left[\frac{\varepsilon^2(1+x)}{P^2} + \frac{2Q_1\varepsilon^2}{P} + \frac{2\varepsilon\sqrt{MT_{A_1}}}{P} - T_{A_1} \right]}{\lambda \left(\frac{1+x+M}{P} + Q_1 \right)^2} \quad (37)$$

This shows that in the absence of rotation, magnetic field has stabilizing effect on the system for $\lambda > 0$ and destabilizing effect for $\lambda < 0$, but if rotation is present on the system the stabilizing effect of magnetic field depends on the condition that $T_{A_1} < \frac{4M\varepsilon^2}{P^2}$ and $\lambda > 0$.

The dispersion relation (33) is analyzed numerically also. In Fig.2, R_1 is plotted against modified rotation parameter T_{A_1} for $M = 10$, $P = 0.13$, $\varepsilon = 0.15$, $Q_1 = 10$, $\lambda > 0$ ($\lambda = 2$), $x = 1, 8, 15$. In Fig.3, R_1 is plotted against modified rotation parameter T_{A_1} for $M = 10$, $P = 0.13$, $\varepsilon = 0.15$, $Q_1 = 10$, $\lambda < 0$ ($\lambda = -2$), $x = 1, 8, 15$ and in Fig.4, R_1 is plotted against wave number x for $M = 10$, $P = 0.13$, $\varepsilon = 0.15$, $Q_1 = 10$, $\lambda = 2$, $T_{A_1} = 20, 80, 140$. Fig.2 and Fig.4 shows the stabilizing effect of rotation for $\lambda > 0$, as Rayleigh number increases with the increase in modified rotation parameter while Figure 3 shows the destabilizing effect of rotation for $\lambda < 0$. In fig.5, R_1 is plotted against Hall current parameter M for $Q_1 = 10$, $P = 0.13$, $\varepsilon = 0.15$, $\lambda = 2$, $x = 1, 8, 15$. In fig.6, R_1 is plotted against Hall current parameter for $Q_1 = 10$, $P = 0.13$, $\varepsilon = 0.15$, $\lambda = -2$, $x = 1, 8, 15$ and in fig.7, R_1 is plotted against wave number x for $P = 0.13$, $\varepsilon = 0.15$, $Q_1 = 10$, $\lambda = 2$, $T_{A_1} = 100$, $M = 0.1, 0.2, 0.3$. Fig.5 and 7 shows the stabilizing effect of Hall current as the Rayleigh number increases with the increase in Hall current parameter M for the case $\lambda > 0$ while Fig.6 shows the destabilizing effect of Hall current for $\lambda < 0$. In fig.8 shows the variation of R_1 with modified magnetic field parameter Q_1 for $T_{A_1} = 10$, $P = 0.13$, $\varepsilon = 0.15$, $\lambda = 2$, $x = 1, 8, 15$. In Fig.9 shows the variation of R_1 with modified magnetic field parameter Q_1 for $T_{A_1} = 10$, $P = 0.13$, $\varepsilon = 0.15$, $\lambda = -2$, $x = 1, 8, 15$ and in fig.10, R_1 is plotted against wave number for $T_{A_1} = 10$, $\varepsilon = 0.15$, $\lambda = 2$, $Q_1 = 100, 200, 300$. Fig.8 and Fig.10 shows the Rayleigh number increases with the increase in modified magnetic field parameter Q_1 which confirms the stabilizing effect of magnetic field on the system for the case $\lambda > 0$ while Fig.9 shows the destabilizing effect of Hall current for $\lambda < 0$.

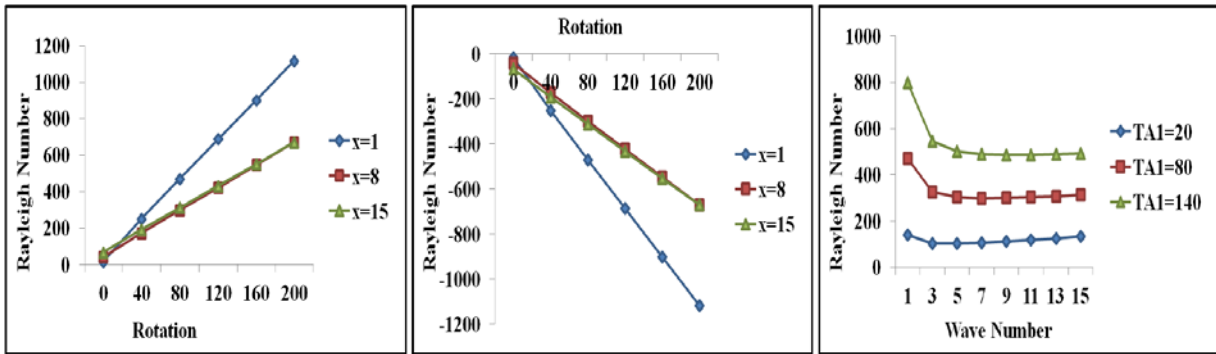


Fig.2: Variation of R_1 with T_{A1} for $\lambda > 0$, Fig.3: Variation of R_1 with T_{A1} for $\lambda < 0$, Fig.4: Variation of R_1 with x

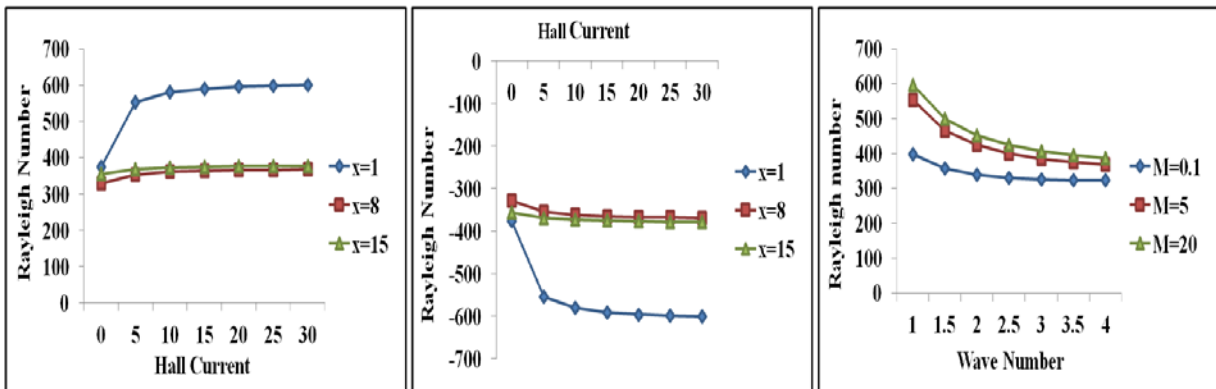


Fig.5: Variation of R_1 with M for $\lambda > 0$, Fig.6: Variation of R_1 with M for $\lambda < 0$, Fig.7: Variation of R_1 with x

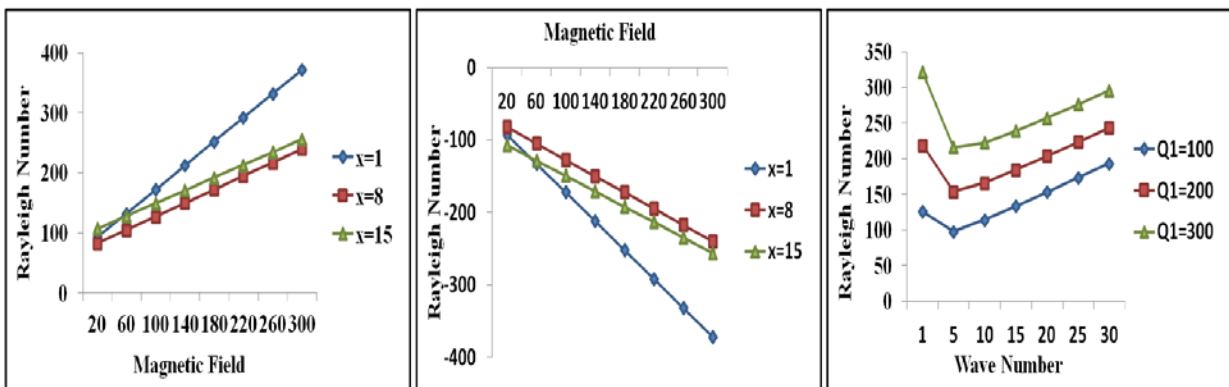


Fig.8: Variation of R_1 with Q_1 for $\lambda > 0$, Fig.9: Variation of R_1 with Q_1 for $\lambda < 0$, Fig.10: Variation of R_1 with x

7. The Case of Oscillatory Mode

Multiplying equation (24) by W^* (complex conjugate of W) and integrating over the range of values of z and making use of (25) – (28) together with the boundary conditions (29), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_1} \right] I_1 - \frac{\alpha a^2 \kappa}{v\beta} \left(\lambda g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha} \right) (I_2 + E p_1 \sigma^* I_3) + \frac{\mu_e \varepsilon \eta}{4\pi \rho_0 v} (I_4 + \sigma^* p_2 I_5) + d^2 \left[\left(\frac{\sigma^*}{\varepsilon} + \frac{1}{P_1} \right) I_6 + \frac{\mu_e \varepsilon \eta}{4\pi \rho_0 v} (I_7 - \sigma p_2 I_8) \right] = 0 \quad (38)$$

Where $I_1 = \int_0^1 (|DW|^2 + a^2|W|^2) dz$, $I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz$, $I_3 = \int_0^1 |\Theta|^2 dz$,
 $I_4 = \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2) dz$, $I_5 = \int_0^1 (|DK|^2 + a^2|K|^2) dz$, $I_6 = \int_0^1 |G|^2 dz$,
 $I_7 = \int_0^1 (|DX|^2 + a^2|X|^2) dz$, $I_8 = \int_0^1 |X|^2 dz$ (39)

Now putting $\sigma = i\sigma_i$ ($\sigma^* = -i\sigma_i$) in equation (38) and equating imaginary parts, we get

$$\sigma_i \left[\frac{1}{\varepsilon} I_1 + \frac{\alpha \kappa a^2}{v\beta} \left(\lambda g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha} \right) E p_1 I_3 - \frac{\mu_e \varepsilon \eta}{4\pi \rho_0 v} p_2 I_5 - \frac{d^2}{\varepsilon} I_6 - \frac{\mu_e \varepsilon \eta d^2}{4\pi \rho_0 v} p_2 I_8 \right] = 0 \quad (40)$$

This shows that σ_i may be either zero or non-zero which means exchange of stabilities is not satisfied for the problem.

In the absence of rotation and Hall current equation (40) reduces to

$$\sigma_i \left[\frac{1}{\varepsilon} I_1 + \frac{\alpha \kappa a^2}{v\beta} \left(\lambda g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha} \right) E p_1 I_3 \right] = 0 \quad (41)$$

If $\lambda g_0 > \frac{\gamma M_0 \nabla H}{\rho_0 \alpha}$, then $\sigma_i = 0$ (because all terms in the bracket are positive definite) which implies that the oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for $\lambda g_0 > \frac{\gamma M_0 \nabla H}{\rho_0 \alpha}$

8. Conclusions

We have concluded following results from the present study:

- 1- For stationary convection, when $T_{A1} > \frac{\varepsilon^2}{p^2} M$ and gravity increases upwards (i.e. $\lambda > 0$), the Hall Current has stabilizing effect on the system. In the absence of rotation, Hall current has to stabilize the system for the case $\lambda > 0$ and destabilizing effect for $\lambda < 0$.
- 2- When gravity increases upward (i.e. $\lambda > 0$), the rotation has stabilizing effect on the system whereas it has destabilizing effect for $\lambda < 0$.
- 3- For stationary convection, in the absence of rotation, magnetic field has stabilizing effect if $\lambda > 0$, while it has destabilizing effect when $\lambda < 0$, but in the presence of rotation, the stabilizing effect of magnetic field depends on the condition $T_{A1} < \frac{4M\varepsilon^2}{p^2}$ and $\lambda > 0$.
- 4- Principle of exchange of stabilities is not satisfied for the problem. In the absence of rotation and hall current, it is valid under the condition $\lambda g_0 > \frac{\gamma M_0 \nabla H}{\rho_0 \alpha}$.

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