International Journal of Scientific Engineering and Applied Science (IJSEAS) – Volume-2, Issue-4,April 2016
ISSN: 2395-3470
www.ijseas.com

Analysis of the Prony method resolution with spatial correlation of noise interference

Galina Glebova, Gennadii Zhbankov, Olga Maltseva, Elena Vinnik
Institute for Physics, Southern Federal University,
Rostov-on-Don, Russia

Abstract
Comparative analysis of the Prony method and the spatial filtering methods with different resolution is carried out using a computer modeling. The study was performed for conditions when the signal is received from a local source on the background of uncorrelated and spatially correlated noise. Evaluation of the local source parameters obtained by the Prony method is presented in the form of the radiation pattern, which makes it possible not only to more clearly present the results, but also to compare the Prony method with spatial filtering methods based on common characteristics, such as beam width, level of background noise. The study showed that the accuracy for evaluating source parameters by the Prony method was comparable to the methods of high and very high resolution.

Keywords: *Computer Modeling, Spatial Filtering, Prony Method, Resolution Ability.*

1. Introduction

Increased interest in the Prony method is largely due to the development of computer technology, as well as the need to develop algorithms with high resolution to detect signals in the background of strong interference [1]. For example, the Prony method is widely used for solving problems in such areas of science and technology as seismic, medicine, radiolocation [2-4]. In presented study, we carried out a comparative analysis of the Prony method and spatial filtering methods having high [5] and ultra-high resolution [6].

Spectral and energy characteristics do not fully characterize the field of noise and in a number of practical problems, such as the analysis of the directional properties of antennas, it is necessary to take into account the spatial correlation functions of the field. Results of many papers on optics [7], electronics [8], acoustics [9] show that a set of random uncorrelated sources forms a correlated field for monochromatic radiation. One of purposes of this paper is determination of the spatial spectra when receiving signals on the background of uncorrelated, so and spatially correlated noise.

Investigation of the Prony method effectiveness consisting of three successive stages of solutions was usually performed based on statistical characteristics calculated using computer modeling [10]. Authors of the paper [11] have proposed transformation of signal parameter estimates found using the Prony method. This transformation allows us to present the results of solving the system of equations in a form comparable to the spatial filtering methods and calculate common signal characteristics of the receiving arrays, such as beam width, background noise and etc.

The purpose of this paper is a comparative analysis of different methods on the basis of an integrated approach that includes both noise and signal modeling field like [12], and modeling of various signal processing algorithms.

2. Algorithms of processing

It is assumed that the receiving array includes M receiver elements, arranged equidistantly along the abscissa axis. The phase center of the receiving array is located at the origin of coordinates. After filtering, the narrowband signal from the signal source at the *m*-receiver element can be represented as

$$U_m = \sqrt{S}\exp[-jkd(m-1)\cos\theta], \qquad (1)$$

where S is the power of the signal, $k = 2\pi/\lambda$ - the wave number, d - inter-element distance, θ - azimuth of a source. The signal emitted by the source is received in admixture with a noise. It is believed that the signals of the sources and noises are statistically independent and obey the normal distribution law. Characteristics of Gaussian variables are completely determined by their covariance matrix, whose elements are equal

$$K_{ml} = \langle U_l \cdot U_m^* \rangle, \quad m,l=1,\dots,M. \qquad (2)$$

Here, the symbol "*" denotes the Hermitian conjugate. The measured signal is the sum of signal matrix **K**(s) and additive noise matrix **K**(n):

$$\mathbf{K}=\mathbf{K}(s)+\mathbf{K}(n). \quad (3)$$

To form the spatial spectrum, correlation is calculated between the received signal vector U and its model, which corresponds to the law of propagation in the medium,

$$\Phi(\theta)=\mathbf{W}^*(\theta)\mathbf{U}\mathbf{U}^*\,\mathbf{W}(\theta)=\mathbf{W}^*(\theta)\mathbf{K}\,\mathbf{W}(\theta), \quad (4)$$

where $\mathbf{W}(\theta)$ is the scan vector, agreed with the propagation law in the medium, θ is the direction of the signal source, the K is a measure of covariance matrix of the received signals. Equation (4) corresponds to the method of Bartlett, which has a standard resolution. Evaluation of the spatial spectrum by high and ultra-high resolution methods: Capon's method [5] and the MUSIC [6], respectively, is also performed using the covariance matrix of the received signals (2)

$$\Phi(\theta)=\left(\mathbf{W}^*(\theta)\mathbf{K}^{-1}\mathbf{W}(\theta)\right)^{-1}, \quad (5)$$

$$\Phi(\theta)=\left(\mathbf{W}(\theta)^*\mathbf{B}\mathbf{B}^*\,\mathbf{W}(\theta)\right)^{-1}. \quad (6)$$

And here **B** is a linear combination of younger eigenvectors of covariance matrix **K** experimentally measured. When using a linear equidistant array, the covariance matrix K is a Toeplitz matrix. The averaged values of the elements of the matrix **K**, for which *n* = *i-j* = *const*, are denoted by F_n. The theoretical values of the components of *F* when receiving signals from *L* local sources can be written as

$$F_n(P)=\sum_{l=1}^{L} S_l x_l^n . \quad (7)$$

Here S_l corresponds to the power value of *l* local source, $x_l=\exp(-j\varphi_l)$, φ_l is the phase difference of signal from the *l* local source in two adjacent receiving elements. The parameter φ_l is associated with a spatial angle of signal arrival from *l* local source by relationship $\varphi_l=(2\pi/\lambda)d\cdot\cos\theta_l$. Note, for approximating signal model as a sum of *L* exponentials (7) the number of spatial samples should not be less than $2\cdot L$, that is, $M\geq 2\cdot L$.

The unknown quantities x_l (7) are the roots of a polynomial

$$x^L+\gamma_1 x^{L-1}+\ldots+\gamma_L=0, \quad (8)$$

the coefficients of which satisfy the system of linear equations

$$F_{n+L}+F_{n+L-1}\gamma_1+\ldots+F_n\gamma_L=0,\ n=0,1,\ldots,2M-1 \quad (9)$$

After finding the values of x_l, these values are substituted into (7) and a linear system of equations is decided to get S_l. In the general case, the parameter *L*, which determines the model order (the number of spatial frequency or the number of local signal sources) is not known also and needs be included in the number of unknown quantities. A number of authors indicate that the choice of the model order is rather complicated, the process of selection cannot be automated and is an informal [13]. But at the same time an optimal choice of this parameter largely determines the efficiency and quality of interpretation of the data processing results.

Usually, accuracy of parameter estimation by the Prony method is determined by statistical modeling, which allows calculating the mean and standard errors of parameter estimation. In this paper, statistics of measurements are used for determination of function $\Phi(\theta)$: signal powers belonging to a given interval of angles $\Delta\theta$ are adding up. As a result of this conversion, an analogue of spatial spectrum is obtained. This analogue enables to perform a comparison of fundamentally different methods using a common characteristic of receiving arrays, namely directional pattern.

3. Results of modeling

In the calculations, it is assumed that the number of elements in the linear receiving array is M = 21. Inter-element distance is equal to d = 0.5λ, where λ is a wave length corresponding to the frequency of the received signal. Transmission band and the time of observation satisfy to a requirement $\Delta f \cdot \Delta T = 1$. A local source is in the far zone in the direction of 30º. Calculation of response output of signal processing by different algorithms is performed using the same sets of input signals of simulated realizations and noise. The spatial spectrum for methods of Bartlett, Capon, MUSIC was averaged over 100 realizations. Parameters of the source S and θ calculated by the Prony method for the same 100 realizations are used to create dependence Φ(θ): signal powers, trapped in a spatial angle interval Δθ = 0.1º, are added up. As a result of this conversion, analog of directional characteristics is obtained.

It is easy to show that when receiving a signal from a single local source on the background of spatially uncorrelated noise, for which $\mathbf{K}(n) = \mathbf{NE}$, the parametric method Prony accurately determines the direction of the source by setting model order *L* = 1. Indeed, the noise component is contained in the measured data only in an element F_0. Excluding it from the system of equations (7), we obtain a system of equations of smaller dimension, in which the experimental data do not contain noise. Therefore, the phase shift between adjacent elements φ, and hence the direction of the source can be determined accurately.

The simulation results shown in Figure 1 support the assumption that in a rather idealized situation where the receiving system receives a signal from a single local source and uncorrelated noise, the Prony method is able to determine the direction of the signal source precisely. Studies have shown that there are no false roots for the selected conditions of the numerical experiment and order of the model. This explains the absence of the lateral background in output response of the Prony method for any ratio s/n_{in}. It is necessary to emphasize that the noise and the signal components on the input of the receiving array elements were modeled as random variables distributed according to the normal law. The results show that the parametric Prony method, in which the selected model fits the observed data, provides more accurate estimates than the spatial filtering methods such as Bartlett, Capon and MUSIC.

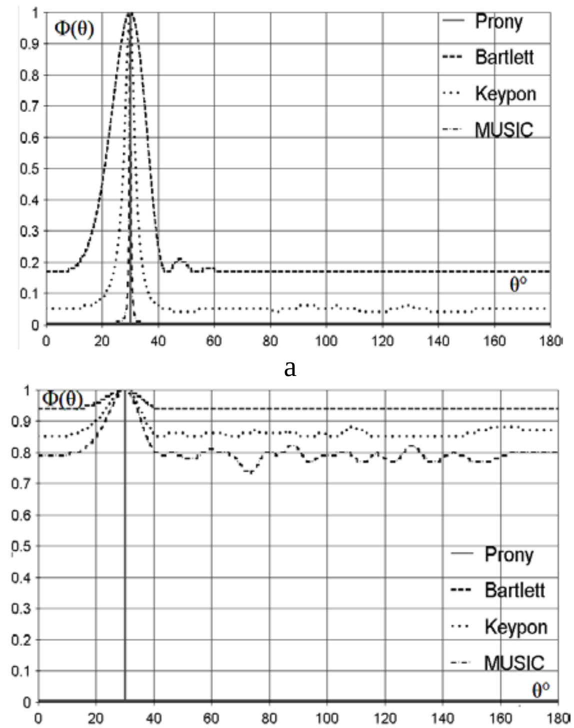
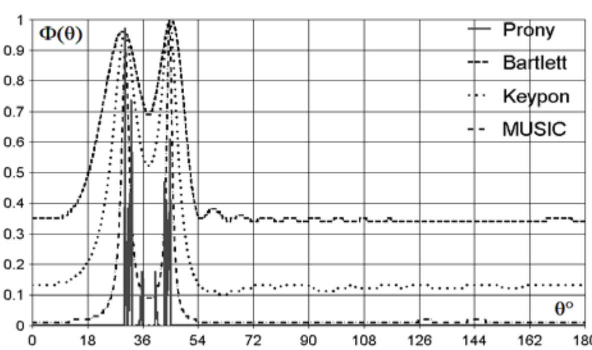
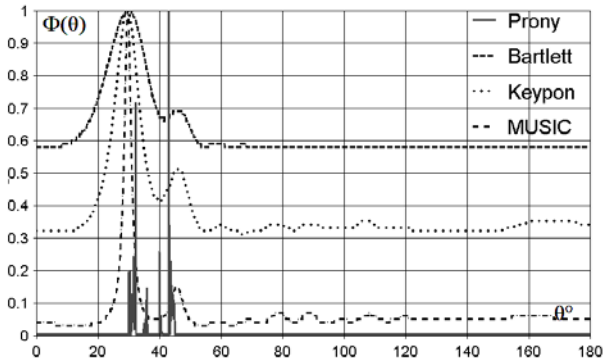


Fig. 1 Spatial spectra calculated when working on the background of spatially uncorrelated noise. Signal to noise ratio at the input of the receiver element equals s/n_{in} = 0.1 (a) and 0.01 (b).

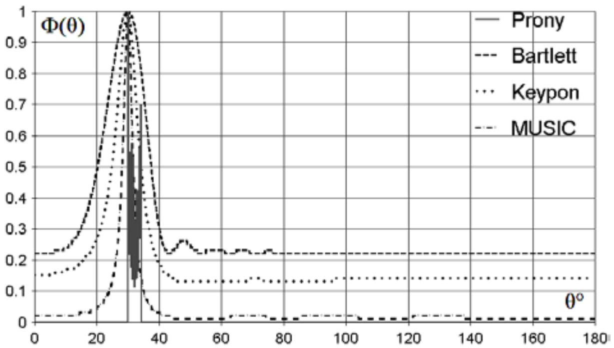
Results of similar calculations, but for two independent local sources of equal power are presented in Figure 2.



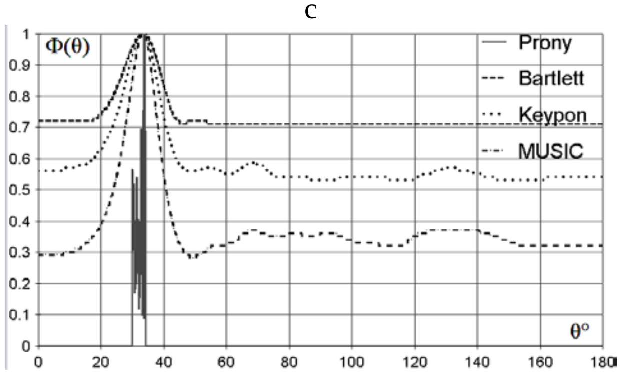
a



b



c



d

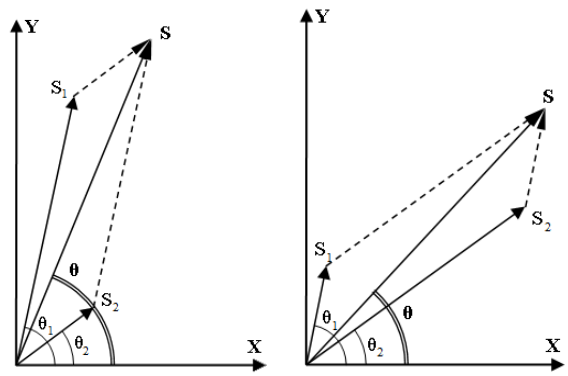
Fig. 2 Spatial spectra calculated when working on the background of spatially uncorrelated noise. Signal to noise ratio at the input of the receiving element of the antenna is equal to s/n_{in} =0.1 (a, b) s/n_{in} = 0.01 and (c, d). The true direction of the source signal equals a, b: $\theta_1 = 30$ °, $\theta_2 = 45$ °; c, d: $\theta_1 = 30$ °, $\theta_2 = 34.3$ °.

When the angular distance between the sources is of $\Delta\theta \approx 15$ ° all considered methods resolve sources for $s/n_{in} = 0.1$ and for $s/n_{in} = 0.01$. However, spatial filtering methods have a high level of background for $s/n_{in} = 0.01$. When the angular distance between the sources is $\Delta\theta \approx 4$ ° problem of resolution of closely spaced sources was doable only for the Prony method. Various spatial filtering methods have different resolution. For example, the Bartlett method has a limited spatial resolution, which is approximately equal to the reciprocal of the antenna aperture $\Delta\theta \approx \lambda/L$, measured in wavelengths of the received signal waves. For this situation the resolution of the two sources is possible with $\Delta\theta \geq 8.6°$. As noted in many papers, examining methods of ultra-high resolution, the MUSIC method is able to resolve closely spaced sources only with $s/n_{in} \geq 1$ [14]. For calculations presented in Figure 2 s/n_{in} << 1, so no one of the spatial filtering techniques fulfill the resolution of closely spaced sources.

Comparison of the results of Figures 1 and 2 shows that the shape of the main lobe of the Prony method differs significantly. In the first case, when there is a signal from a single local source it is practically δ function. The width of the main lobe is defined by width $\Delta\theta$ at the bottom of the interval, for which the spatial spectrum is built (in this case $\Delta\theta$ = 0.1). In the second case (when receiving signals from two local sources), main lobe has the form of two parallel vertical lines. The position of these lines is almost identical with the true coordinates of the sources and the power of output response between these lines has a random oscillating form.

High resolution of the Prony method for receiving signals from two closely spaced sources can be explained as follows. The signal from the source can be represented as a vector whose direction coincides with the angle of arrival of the signal, and a module corresponds to the amplitude of signal. When receiving signals from two closely spaced sources, the resulting vector is the sum of two vectors, and coincides with the diagonal of the parallelogram (Figure 3), at any ratio of source power.

The Prony method (defined by the model order L = 1) determines the angular coordinate equal angular coordinate of diagonal, which coincides with the values of either θ_1 or θ_2, or lies between them. The case when both sources simultaneously emit signals of the same power, and even the maximum power, is less likely than the case of the radiation of signals of different power. Therefore, a more likely is situation in which the diagonal of a parallelogram "is pressed" to one of its sides, and as a consequence, the evaluation of the angular coordinate of diagonal direction is approached to θ_1 or θ_2 (Figure 3(a) and Figure 3(b), respectively).

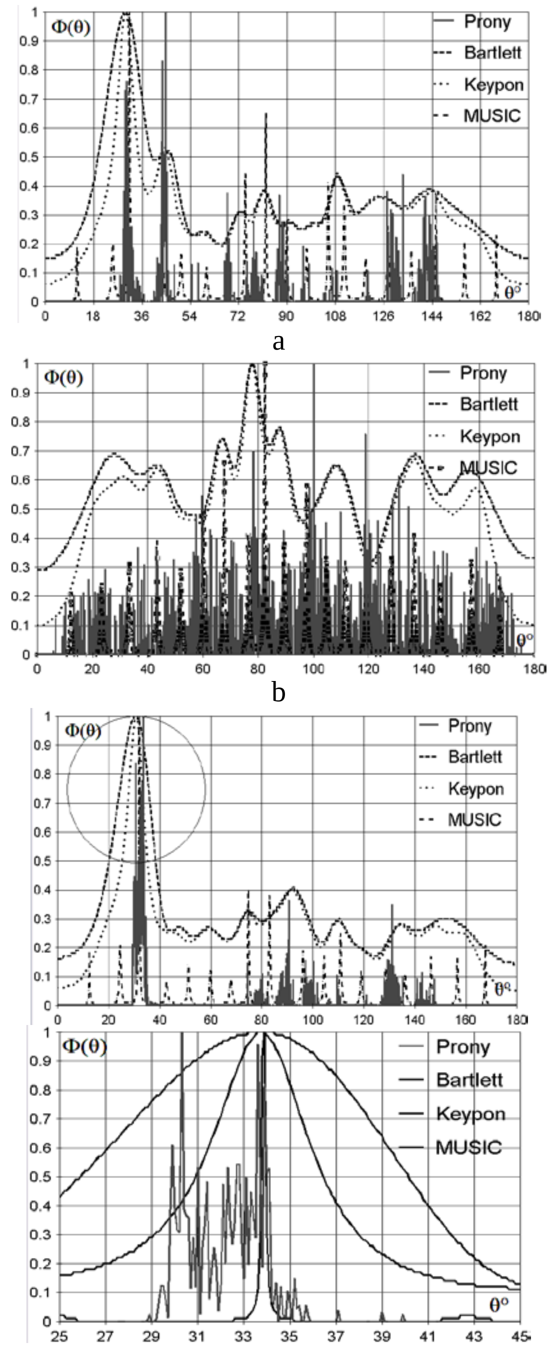


a b

Fig. 3 The signal from the two closely-spaced sources of random signals with different ratio of their power: a) S1> S2 and b) S1 <S2.

As the experiment, theory and simulation shows, signal power variations can be observed not only for broadband random signals in the time and spectral domain, but also for harmonic signals. For example, in the propagation of the signals in the waveguide due to the correlated interference of components (normal rays or waves) signal powers change substantially if the source is moving.

Further the modeling results are presented for case of receiving signals on a background of a spatially correlated noise (Figure 4). To calculate the noise correlation matrix the physical model is used according to which the noises are an independent set of point sources, uniformly distributed on the surface [7]. The amplitude of the radiation for all local sources is distributed over Rayleigh law. Random initial phases are uniformly distributed in the range [0,2π]. Analysis of the Prony method performance when working at a background of spatially correlated noise showed that the optimal value of the order of the model, in which the signal from the source is determined by the best way, is 10, i.e., it is equal to the maximum possible number for a given number of spatial M = 21 samples.



c

Fig. 4 Spatial spectra calculated when working at sea background noise. Signal to noise ratio at the input of the receiving element of the array is equal to s/n_{in} =0.1 (a, c), and s/n_{in} = 0.01 (b). The true direction of the signal source equals a, b: $\theta_1 = 30$ °, $\theta_2 = 45$ °; c: $\theta_1 = 30$ °, $\theta_2 = 34.3$ °.

As can be seen from the presented results, all the used methods have significantly pronounced side lobes that can be mistaken for a false signal sources. None of these methods define the direction of the signal sources in case of $s/n_{in} = 0.01$, even when the angular distance between them is equal to 15º (Figure 4(b)). The main lobe at the Prony method is different from δ-function (as is the case when working on the background of uncorrelated noise) and its width is comparable with the width of the main lobe of the MUSIC method (Figure 4(a)). However, closely spaced sources in case of $s/n_{in} = 0.1$ are resolved only using the Prony method (Figure 4(c)).

4. Conclusions

The transformation of parameters of the source signals obtained by the Prony method, namely histogram, allowed obtaining a visual comparison of results of this method with spatial filtering methods. The optimum value of the order is different for different models of the noise. In particularly, when receiving signals at background of noise spatially uncorrelated the best spatial resolution was obtained with closely spaced source L = 1. When receiving signals at background of the correlated noise such resolution was obtained with L = 10, i.e., with the maximum possible value.

In all presented methods, the form of the output signal is different for different noise models. For example, when working on the spatially uncorrelated noise, level of background noise is smooth. When receiving a signal on the background of spatially correlated noise, lateral background of all these methods has the pronounced lobes, which can be mistaken for a false signal sources.

The Prony method allows resolving closely spaced sources when receiving signals on spatially uncorrelated background noise in case of $s/n_{in} = 0.01$ and when receiving signals on the background of correlated noise in case of $s/n_{in} = 0.1$. While none of the spatial filtering methods resolve these sources at the given conditions. Thus, it can be argued that for the considered models of signal and noise, the Prony method does not yield in sense of its resolution to the MUSIC method having ultra-high resolution.

Acknowledgments

This work was supported by the Ministry of Education and Science of the Russian Federation under Grant №3.961.2014/K.

References

[1] S. P. Marple, Digital Spectral Analysis and Its Applications. Englewood Cliffs, NJ: Prentice Hall, 1987.
[2] A. B. Baev, and Yu. V. Kuznetsov, “The use of higher-order statistics of the digital signal processing UWB radar”, in 2nd International conference «Digital processing of signals and its application», MZNTI, RNTOREC of A.S.Popov, IEEE, 1999.
[3] A. A. Mashanov, and A. I. Mashanov, “Analysis of the dynamics of metabolic parameters in blood lymphocytes by the Prony method”, in The international correspondence scientific conference «Problem of a modern agrarian science» - Krasnoyarsk, 2010, pp. 106-107.
[4] E. V. Verstakov, “The method of parametric representations of two-dimensional data on the basis of the Prony method”, in The international scientifically-practical conference "Info-2010", MIEM, 2010, pp. 207-210.
[5] J. Capon, “High resolution frequency wave number spectrum analysis”, Proc. IEEE, Vol. 57, 1969, pp. 1408-1418.
[6] V. F. Pisarenko, “The retrieval of harmonics from a covariance function”, Geophys. J. Roy. Astron. Soc., Vol. 33, 1973, pp. 347-366.
[7] G. B. Parrent, “Studies in the theory of partial coherence”, Opt. Acta, Vol. 6, 1959, pp. 286-296.
[8] S. M. Rytov, “Theory of electrical fluctuations and heat radiation”, Moscow, 1953.
[9] B. F. Kuryanov, “Spatial correlation of fields emitted by random sources on a plane”. Acoustic Journal, Vol. 9, 1963, pp. 442-448.
[10] A. L. Shestakov, A. S. Semenov, and O. L. Ibryaeva, “Evaluation of the carrier frequency of the random sequence of pulses by the Prony method”, Vestnik Yuzhno-Uralsk State University. The series "Mathematical modeling and programming", Vol. 37, 2009, pp. 106-115.
[11] E. V. Vinnik, G. M. Glebova, and T, N. Larina, “Comparison of the resolution of the methods Prony and Bartlett when working on the background noise of the sea” [electronic resource], Engineering bulletin of Don, Vol. 4, 2013, available from:

http://www.ivdon.ru/magazine/archive/n4y2013/1976.

[12] G. M. Glebova, G. N. Kuznetsov, and O. E. Shimko, "Vector-scalar noise field formed by the rough sea surface", Acoustic Journal, Vol. 59, 2013, pp. 453-463.

[13] G. Mitrofanov, and B. Priimenko, "Basics and application of the method of Prony filtration", Seismic prospecting process engineerings, Vol. 3, 2011, pp. 93-108.

[14] S. A. Zotov, E. S. Makarov, and Nechayev Yu., "Super-resolution methods in problems of direction finding", Informational processes and process engineering in a society and economy, Vol. 3, 2006, pp12-26.