

Thermo Mechanical Analysis of Steel Reinforced Composite Disc

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Abstract

This paper deals with the elastic stress analysis of a steel wire reinforced composite disc with aluminum matrix subjected to temperature specified thermal gradient across the thickness, for the case of radially inward heat flow with linearly-increasing temperature distribution. The values of tangential and radial stress components that have occurred under the effect of the temperature gradient across the outer surface of the disc towards the inner surface have been obtained by both analytical and numerical method using FEA.

Keywords: Composite disc, Tangential stress, Radial stress, MatLAB, ANSYS.

1 Introduction

Composite materials have successfully substituted the traditional materials in several light weight and high strength applications. The reasons why composites are selected for such applications are mainly their high strength-to-weight ratio, high tensile strength at elevated temperatures, high creep resistance and high toughness.

2 Analysis of Hollow Composite Disc

2.1 Thermal Elastic Stress Solution

The equilibrium equation for the plane stress case can be written as follows:

$$r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0 \quad (1)$$

If the stress distribution is symmetrical to an axis that passes from 'O' and that is perpendicular to x-y plane, stress components cannot be dependent on 'θ' and become the functions of 'r' only. Because of the symmetry, the value of $\tau_{r\theta}$ shear stress is equal to zero.

As steel fiber-reinforced orthotropic aluminium composite disc is thin, the case of plane stress is valid. In this case, strain-stress correlation that takes place on an orthotropic composite disc under the influence of temperature can be written as follows:

$$\epsilon_r = \frac{du}{dr} = a_{rr}\sigma_r + a_{r\theta}\sigma_\theta + \alpha_r T \quad (2)$$

$$\epsilon_\theta = \frac{u}{r} = a_{r\theta}\sigma_r + a_{\theta\theta}\sigma_\theta + \alpha_\theta T \quad (3)$$

Relationship between linearly-increasing temperature and equation are as follows:

$$\frac{T}{T_0} = \frac{r-a}{b-a}, \quad T = T_0 \frac{r-a}{b-a} \quad (4)$$

Here α_r and α_θ are thermal expansion coefficients in radial and tangential directions. T represents the magnitude of the temperature distribution. Constants of the a_{rr} , $a_{r\theta}$ and $a_{\theta\theta}$ elasticity matrix are as follows:

$$a_{\theta\theta} = \frac{1}{E_\theta}, \quad a_{rr} = \frac{1}{E_r}, \quad \text{and} \\ a_{r\theta} = a_{\theta r} = -\frac{\nu_{r\theta}}{E_r} = -\frac{\nu_{\theta r}}{E_\theta},$$

where E_r and E_θ are the modulus of elasticity in the radial and tangential directions, respectively. Stress components as in Eq. (5), obtained from an F stress function, secure the equilibrium equation in Eq. (1).

$$\sigma_r = \frac{F}{r} \quad \text{and} \quad \sigma_\theta = \frac{dF}{dr} \quad (5)$$

$$\varepsilon_r = \frac{du}{dr} \quad \text{and} \quad \varepsilon_\theta = \frac{u}{r} \quad (6)$$

Accordingly, Eqs. (2) and (3) can be written as follows:

$$\varepsilon_r = \frac{du}{dr} = a_{rr} \frac{F}{r} + a_{r\theta} \frac{dF}{dr} + \alpha_r T \quad (7)$$

$$\varepsilon_\theta = \frac{u}{r} = a_{r\theta} \frac{F}{r} + a_{\theta\theta} \frac{dF}{dr} + \alpha_\theta T \quad (8)$$

From the above equation,

$$\varepsilon_r = \frac{d}{dr} (r \cdot \varepsilon_\theta) \quad (9)$$

If these equations are put in their place in the compatibility equation (Eq. (9)), differential equation of the F stress function is as follows:

$$\text{Considering } k^2 = \frac{a_{rr}}{a_{\theta\theta}} = \frac{E_\theta}{E_r}$$

$$\varepsilon_r = \frac{d}{dr} (r \cdot \varepsilon_\theta)$$

$$a_{rr} \frac{F}{r} + a_{r\theta} \frac{dF}{dr} + \alpha_r T = \frac{d}{dr} \left\{ r \cdot \left[a_{r\theta} \frac{F}{r} + a_{\theta\theta} \frac{dF}{dr} + \alpha_\theta T \right] \right\}$$

$$a_{rr} \frac{F}{r} + a_{r\theta} \frac{dF}{dr} + \alpha_r T = \frac{d}{dr} \left[a_{r\theta} \cdot r + a_{\theta\theta} \cdot r \cdot \frac{dF}{dr} + \alpha_\theta \cdot r \cdot T \right]$$

$$= \left[a_{r\theta} + a_{\theta\theta} \cdot r \cdot \frac{d}{dr} \left(\frac{dF}{dr} \right) + a_{\theta\theta} \cdot 1 \cdot \left(\frac{dF}{dr} \right) + \alpha_\theta \cdot \frac{d}{dr} (r \cdot T) \right]$$

$$r^2 F'' + rF' - k^2 F = \frac{(\alpha_r - \alpha_\theta)T}{a_{\theta\theta}} r - \frac{\alpha_\theta T}{a_{\theta\theta}} r^2 \quad (10)$$

Fig. 1 represents to find the distribution of the stresses when $T = 0^\circ\text{C}$ in the internal surface and under temperature distribution linearly increasing towards the outer surface $T = T_0$.

Taking stress function F as follows:

$$F = C_1 r^k + C_2 r^{-k} + Er + Dr^2 \quad (11)$$

σ_r and σ_θ stress components can be found from stress function F as follows:

$$\sigma_r = C_1 r^{k-1} + C_2 r^{-k-1} + E + Dr \quad (12)$$

$$\sigma_\theta = C_1 k r^{k-1} + C_2 k r^{-k-1} + E + 2Dr \quad (13)$$

Here C1 and C2 are arbitrary integral constants and E and D constants are as follows:

$$D = \frac{(\alpha_r - 2\alpha_\theta)T_0}{a_{\theta\theta}(4-k^2)(b-a)} \quad E = \frac{(\alpha_\theta - \alpha_r)T_0}{a_{\theta\theta}(1-k^2)(b-a)} \quad (14)$$

C1 and C2 arbitrary integral constants can be found with the use of limit conditions mentioned below: $\sigma_r=0$ at $r=a$ and $\sigma_r=0$ at $r=b$

$$C_1 = E \frac{a^{k+1} - b^{k+1}}{b^{2k} - a^{2k}} + D \frac{a^{k+2} - b^{k+2}}{b^{2k} - a^{2k}} \quad (15)$$

$$C_2 = E \frac{a^{2k} b^{k+1} - b^{2k} a^{k+1}}{b^{2k} - a^{2k}} + D \frac{a^{2k} b^{k+2} - b^{2k} a^{k+2}}{b^{2k} - a^{2k}} \quad (16)$$

Tangential and radial stress components under the load of linearly-increasing temperature can be found from the above equations.

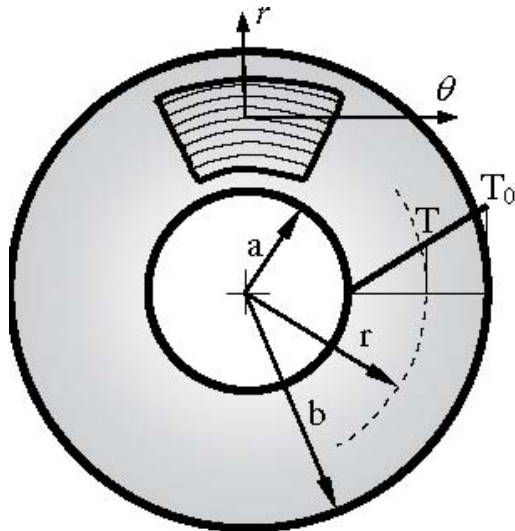


Figure. Circular disc with linearly increasing temperature distribution.

r - radial direction

θ - tangential direction

a = 15 mm, inner radius of the disc

b = 75 mm, outer radius of the disc

T - temperature at any point

T₀ - temperature at outer surface of the disc

3 Methodology

Analytical analysis by using Mat lab program and Numerical analysis by using ANSYS and validating the results.

Forming the equations for composite circular disc with hole and to find out the tangential stress and radial stress by using Mat lab program.

Varying the volume fraction of steel wires 20%,25%,30%,35%,40% and then find out the stresses.

Taking the different matrix materials of Al & CI check which material is suitable based on stresses.

3.1 Determination of E_r , E_{θ} , α_r & α_{θ} for Composite Disc based on volume fraction

$$\left. \begin{aligned} i. E_{r,c} &= E_{f,r}V_f + E_{m,r}V_m \\ ii. E_{\theta,c} &= \frac{E_{f,\theta}E_m}{E_mV_f + E_fV_m} \\ iii. \alpha_{r,c} &= \alpha_{f,r}V_f + \alpha_{m,r}V_m \\ iv. \alpha_{\theta,c} &= \frac{\alpha_{f,\theta}\alpha_m}{\alpha_mV_f + \alpha_fV_m} \end{aligned} \right\} \quad (17)$$

where,

r - radial, θ - tangential directions.

$E_f = E_{\text{steel}}$ & $E_m = E_{\text{aluminium}}$

$\alpha_f = \alpha_{\text{steel}}$, & $\alpha_m = \alpha_{\text{aluminium}}$

$V_f = V_{\text{steel}}$ & $V_m = V_{\text{aluminium}}$

3.2 Determination of Factor Of Safety for case of 20% steel wires 80% matrix material

$$\left. \begin{aligned} i. \sigma_{c,y} &= \sigma_{y,s} \cdot 0.2 + \sigma_{y,al} \cdot 0.8 \\ ii. \sigma_{c,y} &= \sigma_{y,s} \cdot 0.2 + \sigma_{y,CI} \cdot 0.8 \\ iii. \sigma_{von} &= (\sigma_r^2 + \sigma_{\theta}^2 - \sigma_r \cdot \sigma_{\theta})^{\left(\frac{1}{2}\right)} \\ iv. FOS &= \frac{\sigma_{c,y}}{\sigma_{von}} \end{aligned} \right\} \quad (18)$$

where,

$\sigma_{c,y}$ - yield stress of composite

σ_{von} = Von Mises stress of composite

$\sigma_{y,s}$ = yield stress of steel

$\sigma_{y,al}$ = yield stress of Aluminium

$\sigma_{y,CI}$ = yield stress of Cast Iron

3.3 Material Properties

Mechanical properties of Steel, Aluminium and Cast Iron

Metals	E (MPa)	ν	α (1/°C)
Steel	2.1×10^5	0.3	11.1×10^{-6}
Aluminium	0.675×10^5	0.34	23.8×10^{-6}
Cast Iron	1.00×10^5	0.23	9×10^{-6}

Mechanical properties of composite disc for 80% matrix and 20% steel reinforcement.

Composite	Al matrix with Steel reinforcement	CI matrix with Steel reinforcement
E_{θ} (MPa)	89500	122000
E_r (MPa)	71500	110100
α_{θ} ($1/^{\circ}\text{C}$)	18.6×10^{-6}	8.4×10^{-6}
α_r ($1/^{\circ}\text{C}$)	21.6×10^{-6}	9.5×10^{-6}

From the thermal stress equations formulated for the composite disc, the tangential and radial stresses have been calculated across the radial thickness for various thermal gradients (at outer radius temperature fixed maximum and at inner radius temperature fixed at zero) for the case of 20% volume fraction of steel reinforcement with 80% of Al matrix. Fig. 1a shows radius of disc vs radial stress. Fig. 1b shows radius of disc vs tangential stress for various thermal gradients of 20% volume fraction of steel reinforcement with 80% of Al matrix.

For Aluminium - Steel

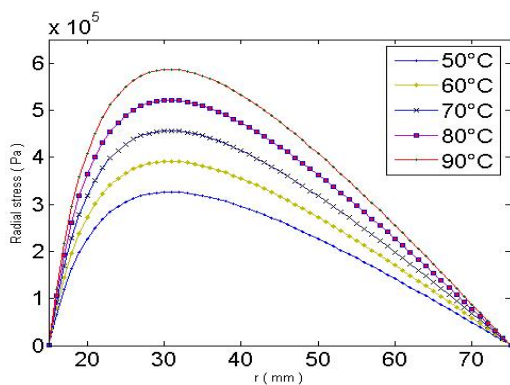


Fig. 1a Thermal radial stress obtained from analytical method for the composite disc

From these Figures it is found that tangential maximum stress is 4 times the radial maximum stress radially inward heat flow. The maximum tensile tangential stress occurs at the inner radius and maximum compressive tangential stress occurs at outer radius this is due to higher temperature towards outer side in compare with inner side.

In general both radial and tangential stress increases with increase of thermal gradient.

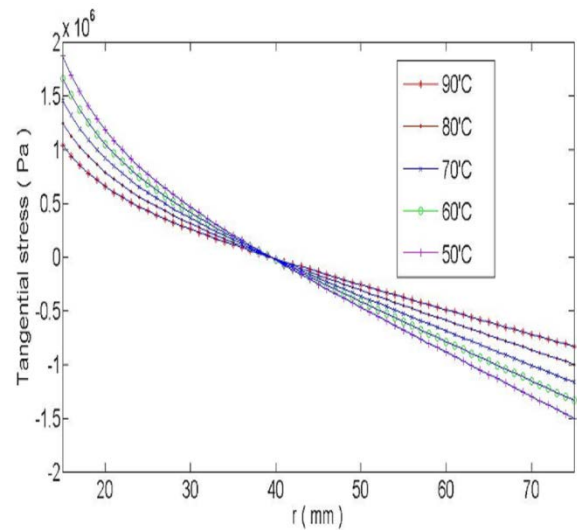


Fig. 1b Thermal tangential stress obtained from analytical method for the composite disc

Similarly Fig. 2a and Fig. 2b shows radius vs radial/tangential stress respectively for the case of 20% volume fraction of steel reinforcement with 80% of CI as matrix material for various thermal gradients. The trends were found to be similar to the Al matrix with steel reinforcement that was already discussed.

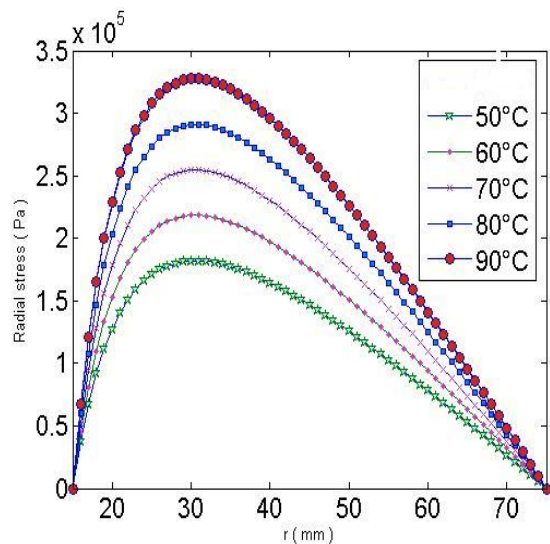


Fig. 2a Thermal radial stress obtained from analytical method for the composite disc

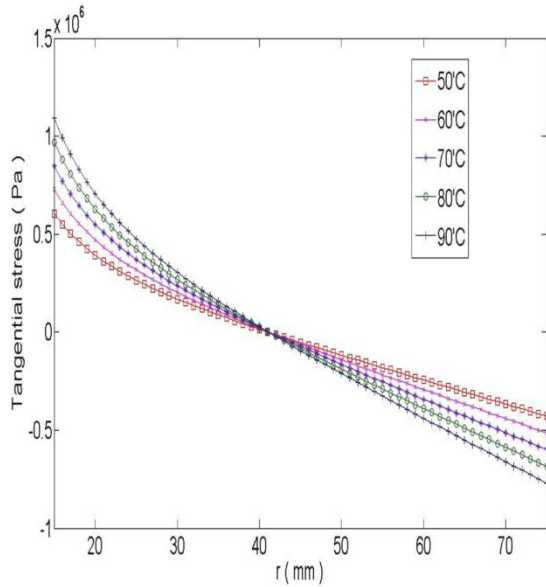


Fig. 2b Thermal tangential stress obtained from analytical method for the composite disc

The factor of safety which were calculated based on yield strength for the composite disc with reference to the maximum tangential stress for CI and Al as two independent matrix materials with 20% steel reinforcement using Eq. (18) and the values were listed in Table 1 and the same is shown Fig. 3. It is found that Al with steel reinforcement gives better FOS in compare with CI-Steel reinforcement for the thermal gradients. Hence further analysis is carried out only for Al with steel reinforcement.

TABLE 1 Temperature gradient vs Factor of safety

Temperature gradient in °C	FOS of CI matrix with Steel reinforcement	FOS of Al matrix with Steel reinforcement
50	2.4	2.6
60	2.4	2.5
70	2.35	2.4
80	2.3	2.35
90	2.2	2.3
95	2.1	2.1

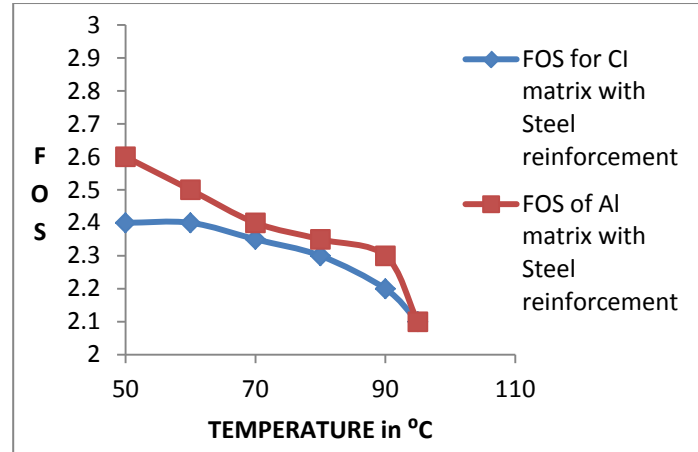


Fig. 3 Temperature gradient in °C vs factor of safety

Fig. 4 to Fig. 7 shows radius vs radial/tangential stress for various volume fractions of steel reinforcement of 25%, 30%, 35% and 40% with Aluminium as matrix material respectively. FOS values were calculated using Eq. (18) based on yield strength of composite disc with reference to maximum tangential stress for each of volume fraction are listed in Table 2 also shown in Fig. 8. It is found that by increasing volume fraction of steel reinforcement does not improve the mechanical strength of the composite because of FOS gets decreased. The reason is due to increase in steel reinforcement causes more mismatch in coefficient of thermal expansion that increases stress levels. Hence further the analysis carried out 20% volume fraction of steel reinforcement with Al as matrix material.

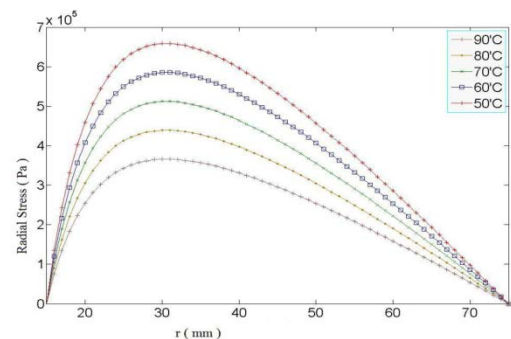


Fig. 4a Radius vs radial stress for $V_f = 25\%$

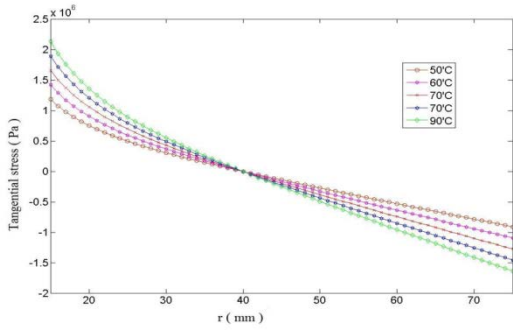


Fig. 4b Radius vs tangential stress for $V_f = 25\%$

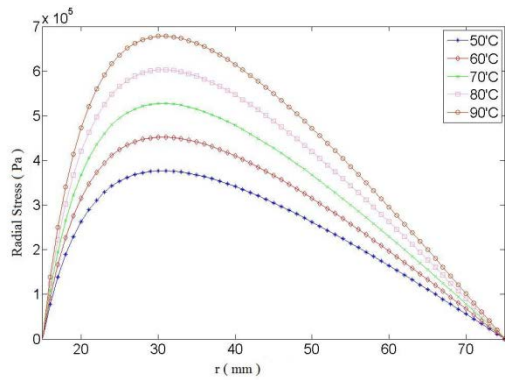


Fig. 5a Radius vs radial stress for $V_f = 30\%$

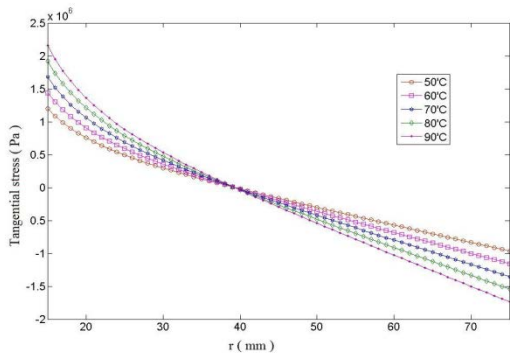


Fig. 5b Radius vs tangential stress for $V_f = 30\%$

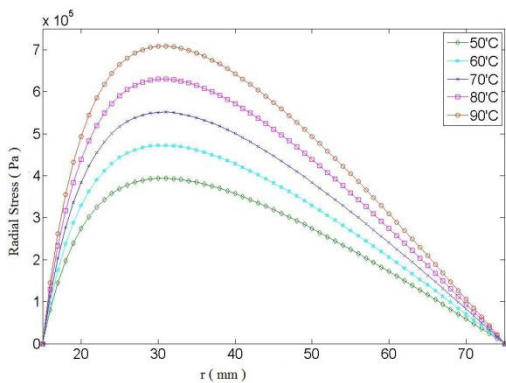


Fig. 6a Radius vs radial stress for $V_f = 35\%$

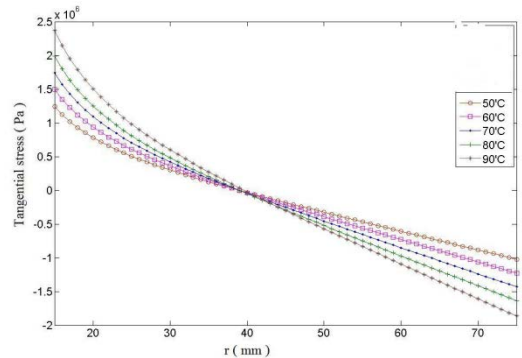


Fig. 6b Radius vs tangential stress for $V_f = 35\%$

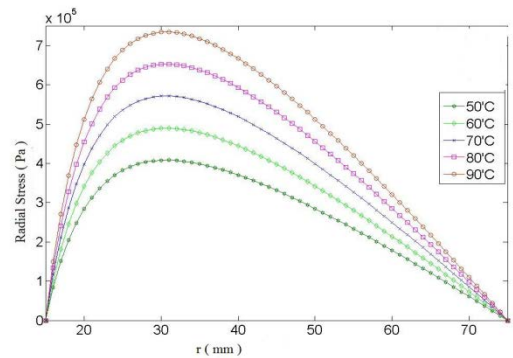


Fig. 7a Radius vs radial stress for $V_f = 40\%$

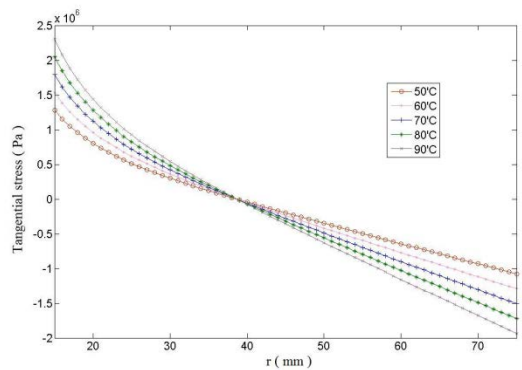


Fig. 7b Radius vs tangential stress for $V_f = 40\%$

TABLE 2 Various Volume fraction of steel vs Factor of safety for 50°C thermal gradient

Volume fraction of steel in %	Volume matrix of Al in %	Factor of safety
20	80	2.5
25	75	2.4
30	70	2.3
35	65	2.2
40	60	2.2

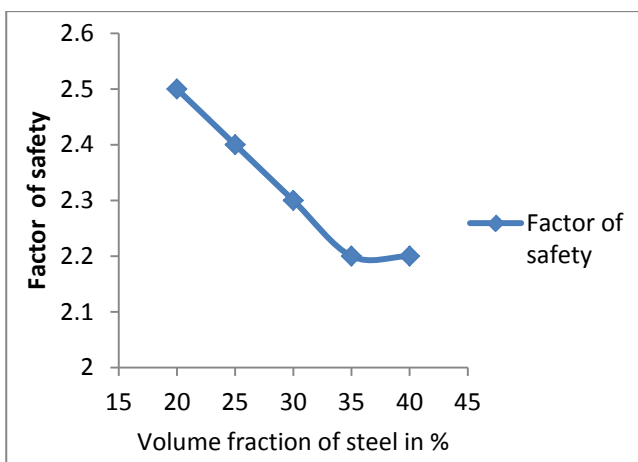


Fig. 8 Various Volume fraction of steel vs factor of safety for 50°C thermal gradient

3.4 Finite Element Analysis

The composite disc has been modelled, the boundary conditions are applied and the analysis was carried out using commercial FEA software of ANSYS for one case of 20% steel reinforcement with Al as matrix to validate the analytical results.

Fig. 9 & Fig. 10 shows the radial and tangential stress distribution respectively across the radial thickness by FEA. The FEA results of radial and tangential stress compared with analytical results for the case of 20% Steel reinforcement with Al as matrix material and listed in Table 3 (Fig. 11) and Table 4 (Fig. 12) respectively. The values are closely matching by maximum of 30 % for validation.

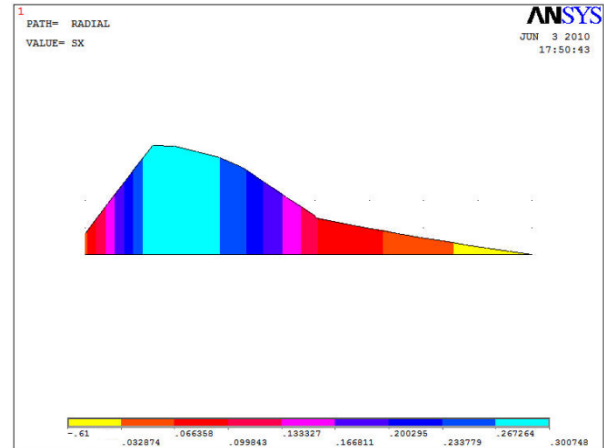


Fig. 9a Radius vs Radial Stress distribution by FEA

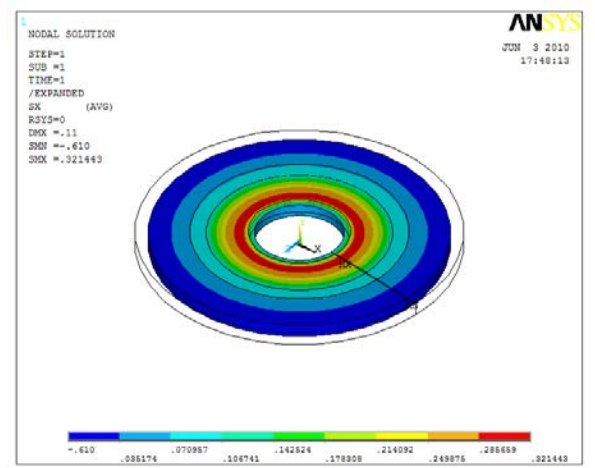


Fig. 9b Radius vs Radial Stress distribution in circular disc - FEA.

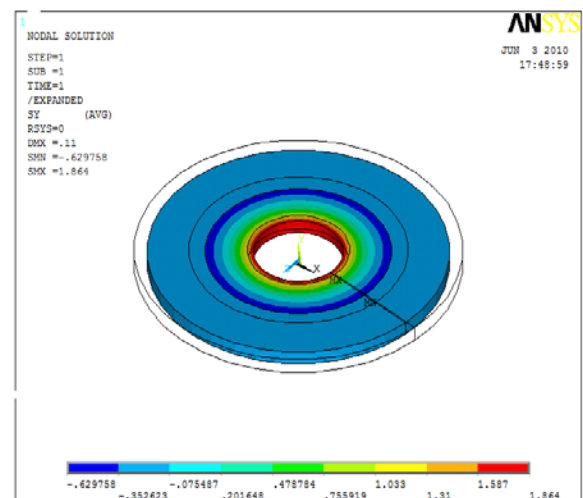


Fig. 10 Radius vs Tangential Stress distribution in circular disc - FEA.

TABLE 3 Comparison of radial stress in

S.No	Analytical approach		FEM (MPa)	% Variation
	Radius (mm)	Radial stress (MPa)		
1	15	0	0	0.00
2	20	0.21	0.178	15.24
3	25	0.25	0.234	6.40
4	30	0.31	0.295	4.84
5	35	0.32	0.321	-0.31
6	40	0.26	0.276	-6.15
7	45	0.21	0.229	-9.05
8	50	0.12	0.135	-12.50
9	55	0.093	0.102	-9.68
10	60	0.08	0.0809	-1.13
11	65	0.07	0.072	-2.86
12	70	0.051	0.044	13.73
13	75	0	0	0.00

analytical and finite element method for 50°C thermal gradient.

TABLE 4 Comparison of tangential stress in analytical and finite element method for 50°C thermal gradient.

S.No	Analytical approach		FEM (MPa)	% Variation
	Radius (mm)	Tangential stress (Mpa)		
1	15	1.73	1.86	-7.51
2	20	1.24	1.31	-5.65
3	25	0.78	0.91	-16.67
4	30	0.64	0.7	-9.37
5	35	0.5	0.47	6.00
6	40	0.29	0.24	16.24
7	45	0	0	0.00
8	50	-0.3	-0.26	13.33
9	55	-0.4	-0.35	12.50
10	60	-0.45	-0.42	6.67
11	65	-0.5	-0.56	-12.00
12	70	-0.65	-0.61	6.15
13	75	-1.01	-0.92	8.91

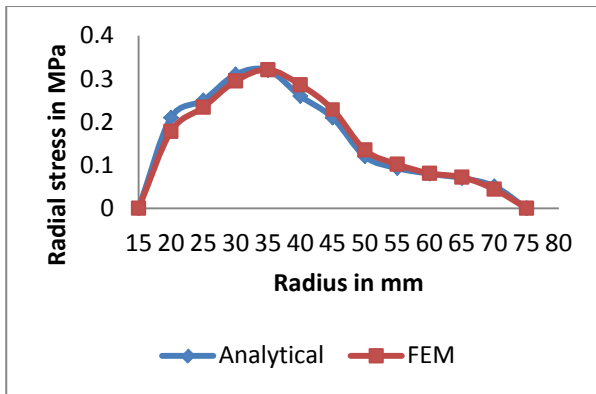


Fig. 11 Comparison of Radial Stress in Analytical Vs FEM at $\Delta T = 50^\circ\text{C}$

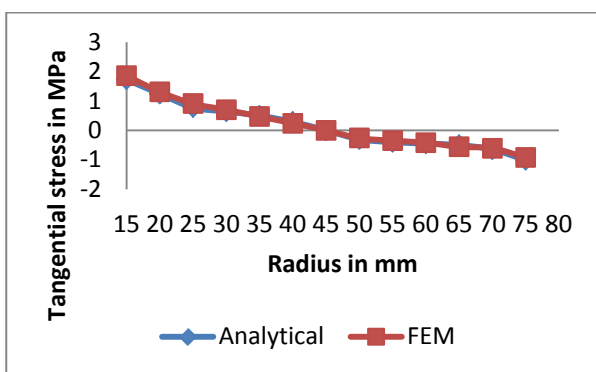


Fig. 12 Comparison of tangential Stress in Analytical Vs FEM at $\Delta T = 50^\circ\text{C}$

4 Conclusion

The following are the conclusions drawn from the analytical and numerical thermo-elastic stress analysis of a composite disc of aluminium matrix with steel reinforcement under thermal gradient across the radial thickness with radially inward heat flow.

1. Methodology has been formulated to analyse composite hollow disc made up of Al as matrix material with steel as reinforcement under thermal gradient across the radial thickness for radially inward heat flow which can be used for brake discs in automotive and aerospace applications.
2. Two different matrix materials of Al & CI with steel as reinforcement have been tried and it is found that Al with steel reinforcement gives better FOS of 2.6 based on yield strength for case of Al with 20%

Steel reinforcement under 50°C as thermal gradient across the radial thickness for radially inward heat flow.

3. Various volume fractions of 20%, 25%, 30%, 35% & 40% with Al as matrix materials have been tried and found that the lowest Al with 20% steel reinforcement, composite disc gives maximum FOS of 2.5 based on yield strength under 50°C thermal gradient across the radial thickness for the radially inward heat flow. This is due to the increasing steel reinforcement causes more mismatch in Co-efficient of thermal expansion of the two materials. Hence it is suggested that 20% of steel reinforcement with Aluminium as matrix materials can be chosen.

4. For the case of 20% steel reinforcement with Aluminium as matrix material, the composite disc has been analysed using FEA under 50°C thermal gradient across the radial thickness with radially inward heat flow and compared with analytical results. The result were closely matching with maximum variation of 16%.

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