A Quasi-differencing Transformation of Dynamic Panel Data Models with Unobservable Individual Effects

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The quasi-differencing approach assumes time-varying individual effects and thus defines the orthogonality conditions separately for each time period. Thereby, the quasi-differencing approach can satisfy the orthogonality conditions more closely and produce smaller standard errors than the first-differencing approach, which assumes time-invariant individual effects and defines its orthogonality conditions for the entire period. Empirical results support the superiority of the quasi-differencing over the first-differencing approach for realistically finite samples.

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JEL classication: C13, C33, C36

I. Introduction

In this study we consider a dynamic panel data model which includes lagged dependent variables as regressors and compare two approaches which are designed to remove the individual effects: the quasi-differencing (QD) and the first-differencing (FD) approach. Assuming that individual effects vary over time, the QD approach includes a product term of the individual effect multiplied by a time-varying coefficient, and eliminates the product term by a quasi-differencing transformation [3, 4]. In contrast, the FD approach, currently the most widely employed, eliminates the time-invariant individual effect by subtracting the equation for time period *t*-1 from the one for *t*.

This study focuses on estimation efficiency and suggests the use of the QD approach, particularly for realistically finite samples. After removing the individual effects, the two approaches estimate their transformed models using the generalized method of moment (GMM) with appropriate instrumental variables (IVs) [1, 2]. The standard errors and thus the efficiency of the estimators depend on how closely the IVs satisfy the orthogonality conditions. The empirical findings reveal that the QD approach dominates over the FD one for estimation of dynamic panel data models. It is because the QD can define the orthogonality conditions separately for each time period but the FD has to define its orthogonality conditions for the entire period.

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II. The Quasi-differencing Approach

We consider the following dynamic panel data model which allows for time-specific and individual effects. For cross-sectional unit *i* (=1,..., *M*) and time period *t* (=1, ..., *T*),

$$y_{it} = \alpha\ y_{i,t-1} + \beta\ x_{i,t-1} + \delta_t + \psi_t f_i + u_{it} \tag{1}$$

where the error term u_{it} is uncorrelated between units and time periods, and also satisfies the orthogonality conditions $E[y_{is}u_{it}] = E[x_{is}u_{it}] = 0 \quad (s < t)$. The time-specific effect (δ_t) is common to all cross-sectional units. This model allows individual effects to vary over time as the time-invariant individual effect f_i is multiplied by a time-varying coefficient ψ_t [4].

Applying the quasi-differencing transformation used in [3] and [4], we can eliminate the time-varying individual effects.

$$y_{it} = \theta_{1t} y_{i,t-1} + \theta_{2t} x_{i,t-1} + \theta_{3t} y_{i,t-2} + \theta_{4t} x_{i,t-2} + d_t + v_{it} \tag{2}$$

where $\theta_{1t} = \alpha + r_t$, $\theta_{2t} = \beta$, $\theta_{3t} = -\alpha\ r_t$, $\theta_{4t} = -\beta\ r_t$, $d_t = \delta_t - r_t\delta_{t-1}$ and $v_{it} = u_{it} - r_t u_{i,t-1}$

with $r_t = \psi_t / \psi_{t-1}$. Lagged values, $z_{it} = [y_{i,t-2}, \cdots, y_{i1}, x_{i,t-2}, \cdots, x_{i1}, 1]'$, can be used as instrumental variables for Eq.(2). Because of the time-varying coefficients, the orthogonality conditions are defined separately for each *t* [4]. For each *t*,

$$Z_t^{QD}{}'V_t \xrightarrow{M\to\infty} 0 \tag{3}$$

where Z_t^{QD} is a matrix of instrumental variables for the QD and $V_t = [v_{1t}, \cdots, v_{Nt}]'$ is a vector of disturbances in time *t*. By applying the nonlinear GMM, the lag coefficients (α, β) are estimated, along with the ratios of the time-varying coefficients for the individual effects $(r_t = \psi_t / \psi_{t-1})$.

If ψ_t is constant over time, then $r_t = 1$ and Eq.(2) becomes the first-differenced specification.

$$\Delta y_{it} = \alpha\ \Delta y_{i,t-1} + \beta\ \Delta x_{i,t-1} + \Delta\delta_t + \Delta u_{it} \tag{4}$$

where Δ denotes the difference between time period t and t-1. Because of the time-invariant coefficients, the instrumental variables for the FD specification satisfy the orthogonality conditions defined for the entire period.

$$Z^{FD}{}'\Delta U \xrightarrow{M\to\infty} 0 \tag{5}$$

where Z^{FD} is a matrix of instrumental variables for the FD and ΔU is a $M(T-1)\times 1$ vector of the differenced-disturbances for the entire period.

By choosing estimates of the time-varying parameters separately for each period, the QD can satisfy the orthogonality conditions more closely than the FD. Therefore, the QD approach is expected to produce smaller standard errors than the FD approach because the deviations from orthoganality conditions are used for the calculation of the standard errors.

III. Simulated Data and Estimation Results

To evaluate the estimation performance of the QD and the FD approach, data are generated using the following specification.

$$\begin{aligned} y_{it} &= \alpha\ y_{i,t-1} + \beta\ x_{i,t-1} + \delta_t + \psi_t f_i + u_{it} \\ x_{it} &= \gamma_1 y_{i,t-1} + \gamma_2 x_{i,t-1} + w_{it} \end{aligned} \tag{6}$$

After data are generated for t = 1 to 32, the first 20 observations are discarded to minimize any effects of the starting values. The values assigned for the parameters are $\alpha = 0.7$, $\beta = 0.3$, $\gamma_1 = 0.1$, $\gamma_2 = 0.7$, $\sigma_y = 0.2$, and $\sigma_x = 1$ Values for the time-specific effects (δ_t's) and for the individual effects (f_i) are independently drawn from uniform distributions $\delta_t \sim Uniform(-0.5, 0.5)$ and $f_i \sim Uniform(-2, 2)$, respectively. The lagged variables on the right-hand side are correlated with these effects through the dynamic relations in the dynamic model. Table 1 shows that the lagged regressors are highly correlated with the individual effects; the correlation coefficients are 0.981 and 0.792. This emphasizes the importance of controlling for the individual effects.

Table 1

Summary statistics and correlation coefficients of the explanatory variables

Variable	mean	s.d.	min	max	correlation coefficient with individual effects (f_i)
$y_{i,t-1}$	-0.392	5.829	-12.106	12.528	0.981
$x_{i,t-1}$	-0.137	2.347	-6.604	5.791	0.792

Note: The number of cross-sectional units is *M*=100 and the data period is *t* =21 ~ 32. The total number of observations is 1,200.

Table 2 reports the estimation results for two cases. One is for the time-varying individual effects and the other is for the time-invariant ones.

(1) Case 1: time-varying ψ_t, thus time-varying individual effects ($\psi_t f_i$)

Since ψ_t and ψ_{t-1} are different, the FD specification cannot eliminate the individual effects. The resulting component $(\psi_t - \psi_{t-1}) f_i$ causes a bias even though instrumental variables are employed for estimation. Case 1 in table 2 shows that the FD estimates of (α, β) are (0.383, 0.073) when their true values are (0.7, 0.3). The true values are not included in the 99% confidence intervals, $0.240 < \alpha < 0.383$ and $-0.064 < \beta < 0.073$. In contrast, the QD approach correctly and accurately estimates the regression coefficients with small standard errors. Therefore, if the individual effects vary over time, the QD approach outperforms the FD.

Table 2

Estimation results when the coefficient for the individual effect is time-varying or time-invariant.

	Quasi-differencing		First-differencing		se (FD) –
	estimate	se(QD)	estimate	se(FD)	se(QD)
	Case 1: when individual effects are time-varying				
α	0.665	0.026	0.383	0.073	0.047
β	0.306	0.024	0.073	0.070	0.046

	Case 2: when individual effects are time-invariant				
α	0.683	0.010	0.172	0.045	0.009
β	0.303	0.008	-0.088	0.050	0.012

Note: The number of cross-sectional units is M=100. Time dummies were included in all of the regressions in this study, but estimates of their coefficients are not reported in the table.

(2) Case 2: constant $\psi_t = 1$

We examine whether the QD outperforms the FD even when the individual effects are constant over time. Case 2 in table 2 shows that when the number of units is relatively small with *M*=100, the standard errors of QD are smaller than the ones of FD; the differences are 0.009 for α and 0.012 for β . Even when the parameters are constant over time, their estimates in one period could be different from those in another period due to sampling errors for small-sized samples. If so, QD is expected to produce smaller standard errors than FD because the QD instrumental variables has more flexibility in satisfying the orthogonality conditions which are defined separately for each time period.

IV. Conclusions

We examined herein whether the consistency and the efficiency in estimating dynamic panel data models can be improved when the sample size is realistically finite. It was shown theoretically and empirically that the FD approach is not valid when the individual effects are time-varying, and thus, its estimator is biased and inconsistent. In contrast, the QD approach, which is designed to account for time-varying individual effects, accurately estimates the parameters with small standard errors.

Even when the individual effects are constant, the empirical results demonstrate that the QD approach produces smaller standard errors than the FD approach for finite-sized samples. This is because the orthogonality conditions of QD are defined separately for each time period, and thus, its instrumental variables can satisfy the conditions closely. In contrast, since the orthogonality conditions of FD are defined for the entire time period, the FD instrumental variables do not have so much flexibility as the QD ones in satisfying the conditions.

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