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Analytical Investigation of the Effects of Oxidation Formation on Heat Transfer under the Effect of Spray Cooling

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Abstract
Water spray cooling is extensively applied in heat treatment and metallurgical processes where precise temperature control is essential. During steel plate cooling, the formation of oxide layers can significantly influence heat transfer and cooling intensity. This study develops an analytical model to evaluate the effects of varying oxide layer thicknesses on cooling performance, supported by experimental measurements and numerical simulations. The time-dependent heat diffusion problem in a multilayer medium under convective boundary conditions was solved using the method of separation of variables, following the homogenization of non-homogeneous boundary conditions via superposition. Eigenvalue functions for each layer were derived, with their roots determined by the Newton method, and corresponding coefficients established to validate these solutions. The full temperature profile of the medium was formulated and computationally implemented in Mathematica. Comparative analysis confirms the model's effectiveness, providing valuable insights into the influence of oxide layers on steel cooling processes and advancing the understanding of spray cooling mechanisms.
Keywords: Heat diffusion equation, multilayered media, spray cooling, oxidation thickness, analytical solution, separation of variables.

1. Introduction

Heat diffusion in multilayered media plays a crucial role in several critical applications, including energy systems (fuel cells, electrochemical reactors), advanced materials (microelectronics, composites), and industrial processes (steel coil production, geological thermal analysis, and thermal management of austenitic stainless steel under water spray cooling) [1] [2]. Biological applications include drug carrier efficacy in tissues, infrared tissue diagnostics, and muscle thermogenesis studies [3]. In microelectronics and MEMS cooling, multilayered designs are essential for reducing overheating risks [4].

The heat conduction solution method for multilayered medium can also be used to analyze the effects of the oxide layer formed during the spray cooling process on heat transfer. Water spray cooling is widely accepted as an effective method for cooling hot steel surfaces and is a key component in the thermal processing of steel. It is an integral part of continuous casting and hot rolling processes and plays a critical role in steel production and final processing stages. Designing an effective cooling process requires careful consideration of various factors, particularly the oxide layers formed on the steel surface during the process. Oxide layers can significantly alter cooling properties, affecting heat transfer rates and cooling uniformity [5].

This study aims to examine the effect of oxide layers on cooling intensity in static cooling regimes. In this study, water at a temperature of 17°C is sprayed onto austenitic stainless-steel material at 1000°C, which is taken out of the furnace after thermal processing, for cooling purposes. It is assumed that an oxide layer with a thickness of 50 µm forms on the steel surface during the spraying cooling process [6].

2. Mathematical Model and Solution

A general mathematical formulation of the heat conduction problem in a multilayered (*n* layer) medium within a one-dimensional cartesian coordinate system has been established, and the boundary conditions required for the solution have been defined. For the analytical solution of the mathematical model, the superposition and separation of variables methods were employed, and the solution was obtained in closed form.

2.1 One-Dimensional Heat Conduction Equation

A composite material consisting of *n* layers, schematically illustrated in Fig. 1, has been examined. As shown in Fig. 1, the scenario where different fluids flow along the inner and outer surfaces of the structure in cartesian coordinates has been considered. The temperature of each layer is denoted as $Ti(x,t)$, where *i* represents the layer number. The layers are assumed to have isotropic thermal properties, and thermal contact resistance at the interfaces has not been considered. Consequently, the temperatures at the interfaces are set to be equal. On the surface of the first layer ($x=X_0$), heat transfer occurs via convection due to fluid flow. Similarly, convective heat transfer takes place between the outermost layer's external surface ($x=X_n$) and the ambient environment. At the initial time (t=0), each layer possesses a specific and known temperature.

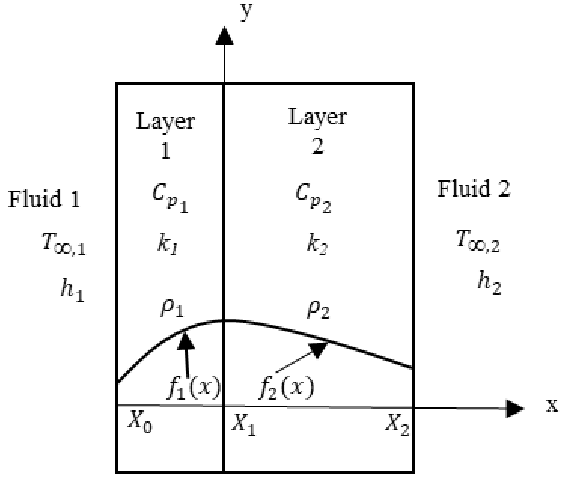

Fig. 1 Schematic view of an 2 layer cartesian structure with inner and outer surfaces subjected to convection.

Eq (1) describes heat conduction in a material over time and space. Here, $T_i(x,t)$ represents the temperature at a specific location x and time t in a material with thermal diffusivity α_i , which is a property that determines how quickly heat spreads through the material. The left-hand side of the equation accounts for the time rate of change of temperature at a given position, while the right-hand side represents how temperature varies with position in the material, indicating the temperature gradient. The second derivative of temperature with respect to position x captures how the heat diffuses across the material. Essentially, this equation shows that the rate at which temperature changes over time at any given location is proportional to the curvature of the temperature profile along the material. This relationship is fundamental in modeling heat flow and is commonly used in scenarios involving steady-state and transient heat conduction in solid materials.

$$\frac{1}{\alpha_i}\frac{\partial T_i(x,t)}{\partial t}=\frac{\partial^2 T_i(x,t)}{\partial x^2} \qquad (1)$$

2.2 Boundary and Initial Conditions

From the conservation of energy at the inner surface of the first layer:

$$k_1 \frac{\partial T_1(X_0,t)}{\partial x} = -h_{in}\left(T_{\infty_1} - T_1(X_0,t)\right) \quad (2)$$

From the conservation of energy at the interfaces $x = X_i \quad (i = 1,2,3,\dots,n-1)$:

$$k_i \frac{\partial T_i(X_i,t)}{\partial x} = k_{i+1} \frac{\partial T_{i+1}(X_{i+1},t)}{\partial x} \quad (3)$$

and temperature continuity at the interfaces under the assumption of zero thermal contact resistance,

$$T_i(X_i,t) = T_{i+1}(X_{i+1},t) \quad (4)$$

the outer surface of the outermost layer $x = X_n$

$$k_n \frac{\partial T_n(X_n,t)}{\partial x} = h_{out}\left(T_{\infty_2} - T_n(X_n,t)\right) \quad (5)$$

Initial conditions at $(t = 0)$;

$$T_i(x,t=0) = T_{0_i} \quad (6)$$

When the boundary condition expressions are reformulated, it becomes evident that the interface boundary conditions are homogeneous, while those at the inner surface of the first layer and the outer surface of the outermost layer are non-homogeneous in structure.

2.3 Separation of Variables Method

The Separation of Variables Method is applicable only to homogeneous equations. Due to the non-homogeneous nature of the boundary conditions in our problem, this method cannot be used. Therefore, the general equation is split into two parts as,

$$T_i(x,t) = \bar{T}_i\,(x,t) + T_{ss_i}(x) \quad (7)$$

where $\bar{T}_i\,(x,t)$ represents the transient component and $T_{ss_i}(x)$ denotes the steady-state component. This superposition approach allows the application of boundary conditions to the modified system. Equation (8) is used to find $T_{ss_i}(x)$.

$$T_{ss_i}(x) = p_i\,x + q_i \quad (8)$$

p_i and q_i are determined by applying the boundary conditions. Equation (9) is used to find $\bar{T}_i(x,t)$.

$$\bar{T}_i(x,t) = \sum_{m=1}^{\infty} \left(DD_m\, e^{-\lambda_m{}^2 t}\right)\left(a_{i,m} \sin\left(\frac{\lambda_m}{\sqrt{\alpha_i}} x\right) + b_{i,m} \cos\left(\frac{\lambda_m}{\sqrt{\alpha_i}} x\right)\right) \quad (9)$$

The coefficients DD_m, $a_{i,m}$ and $b_{i,m}$ are determined from initial conditions (initial temperature distribution) and boundary conditions (such as temperatures at the edges of the material). The exponential decay $e^{-\lambda_m{}^2 t}$describes how the temperature modes decay over time, with faster-decaying modes contributing less to the overall temperature as time progresses. Sine and cosine functions represent the spatial variations of temperature. There is a detailed explanation of the solution to the equations [6]. The general equation for the time-dependent temperature distribution is given in Equation (10).

$$T_i(x,t) = \sum_{m=1}^{\infty} \left(DD_m\, e^{-\lambda_m{}^2 t}\right)\left(a_{i,m} \sin\left(\frac{\lambda_m}{\sqrt{\alpha_i}}x\right) + b_{i,m} \cos\left(\frac{\lambda_m}{\sqrt{\alpha_i}}x\right)\right) + T_{ss_i}(x) \qquad (10)$$

3. Statement of the Problem

This study focuses on determining the temperature of the oxide layer formed on high-temperature austenitic stainless steel during water-spray cooling after processing, as well as the temperature distribution within the material. The study references the numerical and experimental work of M. Chabicovsky and colleagues [5]. This problem was solved using an analytical model developed in the thesis and implemented via Mathematica software [6]. The results were compared with Chabicovsky's numerical and experimental findings [5].

3.1. Oxide Layer Formation

A schematic of the problem and boundary conditions are shown in Fig. 2. Water at 17°C is sprayed onto austenitic stainless steel (initially at 1000°C after heat treatment) for cooling. During the spray cooling process, a 50 µm-thick oxide layer is assumed to form on the steel surface.

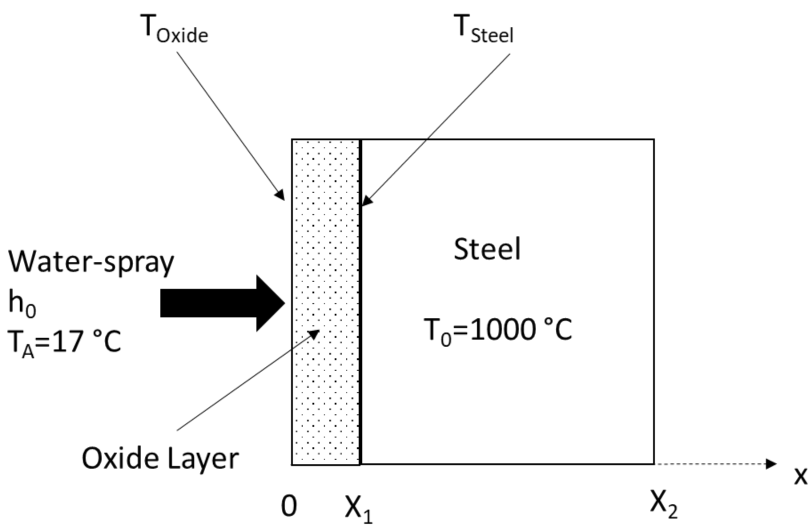

Fig. 2 Spray Cooling Process of Austenitic Stainless Steel

In Fig. 2, T_{oxide} represents the temperature on the oxide layer surface, while T_{Steel} denotes the temperature at the interface between the steel substrate and the oxide layer (below the oxide layer). In the figure, the oxide layer thickness is labeled as X_1 (m) and X_2 (m).

The spray cooling parameters for the material and oxide layer used in this study are provided in Table 1. Among the parameters listed in the table, the oxide layer thickness varies with time and temperature. The effective heat transfer coefficient (h_0), included in the parameters, ranges between 500–2300 W/m²·K depending on temperature. The thermophysical properties of the material (density, thermal conductivity, specific heat, etc.) are assumed as constant. The thermal diffusivity of the materials is defined as α_i.

Table 1. Spray Cooling Process Parameters for Austenitic Stainless Steel [5]

Parameter	Value
X_0	0 mm
X_2	25 mm
k_1	23,4 W/m K
k_2	0,2 W/m K
α_1	5,21774x10^{-6} m^2/s
α_2	4,35578x10^{-8} m^2/s
h_0	500-2300 W/m^2 K
T_A	17 °C
T_0	1000 °C

In this study, while investigating the effects of oxidation formation under spray cooling on heat transfer, the material and the oxide layer formed on its surface were modeled as two separate layers. The impact of oxidation on heat transfer was analyzed using a layered heat diffusion model developed specifically for this work.

3.2 Solution and Results for the Steel Surface Beneath the Oxide Layer

To determine the temperature profile of the steel surface beneath the oxide layer, experimental and numerical results from the referenced paper for a 0.025 mm oxide thickness were extracted at 50-second intervals from their graphical representations. In this study, time-dependent temperature variation was solved using the program developed for the analytical model. Comparative graphs between the experimental/numerical results from the paper and the current analytical results are presented in Fig. 3 and Fig. 4.

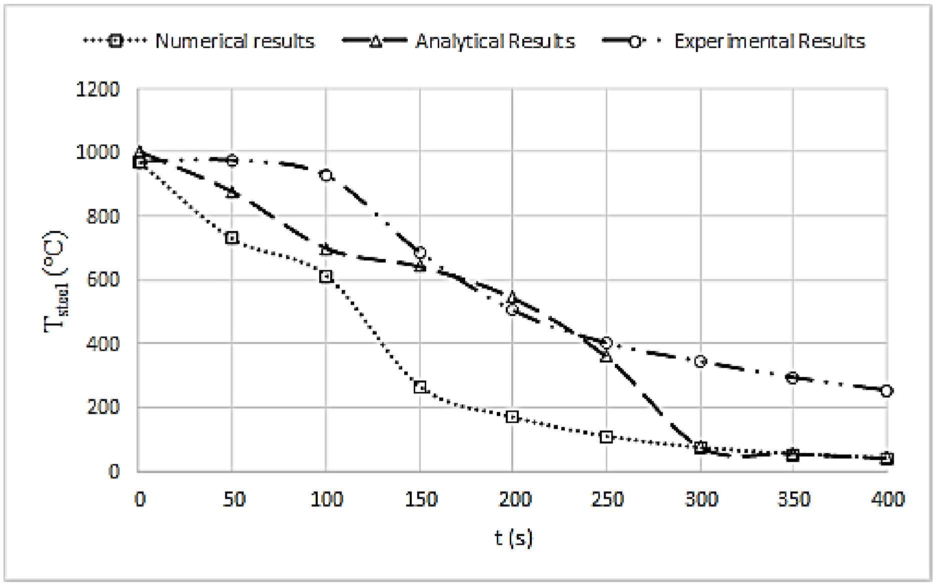

Fig. 3 Numerical, analytical, and experimental results for T_{Steel} (steel-oxide interface temperature)

In Fig. 3, it was determined that the experimental and analytical results are consistent with each other regarding the temperature change over time of the steel surface with a 0.025 mm oxide layer. However, during the cooling period before t = 150 s, additional insulation effects are observed in the experimental data. The theoretical reason for this is the water striking the surface at a temperature higher than its own evaporation temperature during cooling, causing evaporation, and the vapor providing additional insulation.

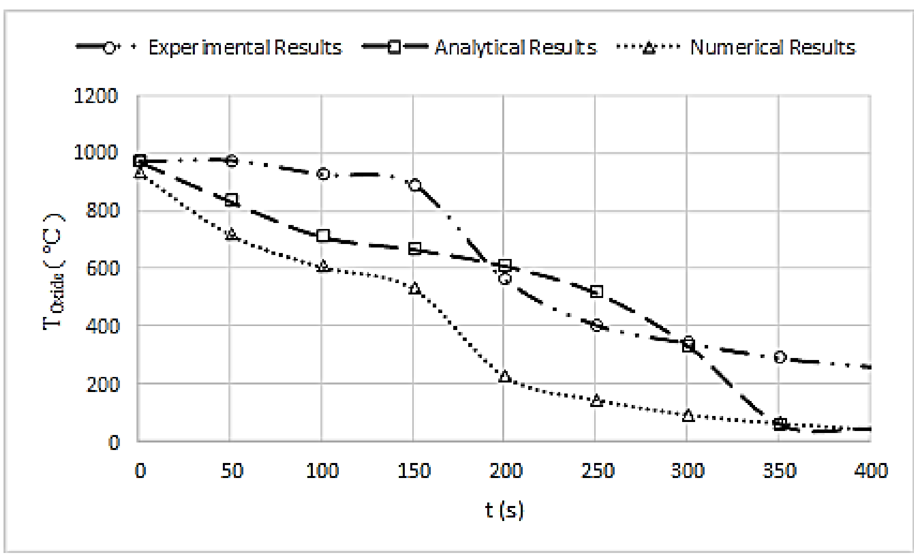

Fig. 4 Numerical, analytical, and experimental results for T_{Oxide} (oxide surface temperature)

Upon examining Fig. 4, it can be observed that the experimental and analytical results follow a consistent profile. Additionally, in the cooling period before t = 150 s, insulation effects are observed in the experimental data. This may be due to the water striking a surface at a temperature higher than its own evaporation temperature during cooling, causing evaporation, and the vapor providing additional insulation.

In this application, steel at 1000°C is cooled with 17°C water, and the effects of the oxide layer, which has a very low thermal conductivity, are investigated. Experimental results show that, particularly during the vaporization period, the measured temperatures indicate additional insulation beyond that of the oxide layer. Although deviations exist between the analytical and experimental results, a holistic evaluation of the process reveals consistent time-temperature profiles. The developed analytical solution is concluded to be useful for studying the effects of the oxide layer.

4. Conclusions

In this study, an analytical mathematical model was developed using the separation of variables method to analyze the effects of the oxide layer formed during the spray cooling process on heat transfer. The model solves heat conduction problems in multilayer systems and is implemented through a code written in Mathematica. This study analyzes the cooling of steel at 1000°C with 17°C water, focusing on the influence of the oxide layer, characterized by its low thermal conductivity. Experimental results show that, especially during vaporization, temperatures exhibit insulation effects beyond those caused by the oxide layer alone. While some deviations exist between analytical and experimental results, a comprehensive evaluation of the process reveals agreement in time-temperature profiles. The developed analytical solution is validated as an effective tool for studying the effects of oxide layers in such processes.

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