*International Journal of Scientific Engineering and Applied Science (IJSEAS) - Volume-1, Issue-9, December 2015*
*ISSN: 2395-3470*
*www.ijseas.com*

# On the Class of Similarity Solution for electrically conducting Non-Newtonian fluids over a Vertical porous- elastic Surface

Pankaj Sonawane[1], M. G. Timol[2] and J.N.Salunke[3]

1. Department of Mathematics, Dhanaji Nana Mahavidyalaya, Faizpur, Dist Jalgaon, India
2. Department of mathematics, Veer Narmad South Gujarat University, Surat. India
3. Department of Mathematics, Swami Ramanand Tirth Marathawada University, Nanded. India

**Abstract**

Similarity solution is investigated for a free convective boundary layer flow of electrically conducting Non-Newtonian fluids over a vertical porous-elastic surface. The similarity equation is derived using one parameter linear group of transformation. Finally, this similarity equation which is highly non-linear ordinary differential equation is solved numerically for particular Non-Newtonian fluid so-called Powell-Eyring fluid.

**Keywords:** Similarity solution, Powell- Eyring fluid, Non-Newtonian Fluid, boundary layer flow.

## 1. Introduction

Most of the researchers in the field of fluid mechanics try to obtain the similarity solutions by introducing a general similarity transformation with unknown parameters into the differential equation obtaining in this way an algebraic system. The symmetries of a differential equation are those continuous group of transformations under which the differential equation remains invariant, that is, a symmetry group maps any solution to another solution. The interesting point is that, having obtained the symmetries of a specific problem, one can proceed further to find out the group invariant solutions, which are nothing but the well-known similarity solutions. The similarity solutions are quite popular because the result in the reduction of the independent variables of the problem. In our case, the problem under investigation is two-dimensional. Hence, any similarity solution will transform the system of partial differential equations into a system of ordinary differential equations.

To obtain symmetry of a differential equation is equivalent to the determination of the transformation group associated with this symmetry. In Olver [1]; Bluman and Kumei [2]; Ibragimov [3, 4], one can find the general theory if Lie groups as well as the implied methods for determining transformation group via the infinitesimal generator components. An alternative way being based on exterior calculus for determining the transformation group so-called deductive group can be found in Moran and Gaggioli [5]. It is worth nothing that there is an extensive literature where the methods arising from exterior calculus are used to attack symmetry problems of continuum mechanics (Suhubi [6, 7], Pakdemirli and Suhubi [8], Kalpakides [9, 10], Koureas [11, 12]).

This procedure is applied to a boundary layer problem which arises from the motion of an elastic surface into an electrically conducting, incompressible, viscous non-Newtonian fluid. We mention here the some work of Sakiadis [13]; Erickson [14]; Tsou [15]; Gupta and Gupta [16]. It is remarkable that all of them have used the above described heuristic method to obtain the similarity transformations and the associated similarity solutions of the problem. That is, assuming particular boundary conditions and considering a particular form of the magnetic field, they try to fit a similarity solution in these data.

We consider the most general form for the boundary conditions and the magnetic field function involved in the system. Both the specific form of the functions on the boundaries and the form of magnetic field arise as a consequence of the requirement to respect the obtained symmetries. The similarity equations obtained are more general and systematic along with auxiliary conditions. Recently this method has been successfully applied to various non-linear problems (see Malek [17]; Darji and Timol [18, 19]; Adnan [20]).

The boundary layer flow of Newtonian fluids past stretching sheet was first discussed by Crane[21].Later on same problem was extended by several authors, few of these Soundalgekar and Ramana Murthy[22]; Grubka[23]; Dutta [24]; Jeng[25]; Dutta[26];Chen and Char[27] for different physical situations, due to its important applications to polymer industry. These studies restrict their analyses to Newtonian fluids. Flow due to a stretching sheet also occurs in thermal and moisture treatment of materials, particularly in processes involving continuous pulling of a sheet through a reaction zone, as in metallurgy, textile and paper industries, in the manufacture of polymeric sheets, sheet glass and crystalline materials. It is well known that a number of industrial fluids such as molten plastics, polymeric liquids, food stuffs or slurries exhibit non-Newtonian character. Therefore a study of flow and heat transfer in non-Newtonian fluids is of practical importance.

In recent years several industries deal with the non-Newtonian fluids under the influence of magnetic field. In view of this, some researchers Sarpakaya[28]; Saponkov[29];Martinson and Pavlov[30]; Samokhen[31];Andersson[32];Cortell[33]; Liao[34] have presented works on MHD flow and heat transfer in an electrically conducting power law fluid over a stretching sheet. However, in the literature rare work has been found regarding other non-Newtonian fluids. This may due to mathematical complication of its strain-stress relationship.

In the present paper, Similarity solution is investigated for a free convective boundary layer flow of electrically conducting Non-Newtonian fluids over a vertical porous and elastic surface. The class of all non-Newtonian fluids is characterized by the property that its stress tensor component $\tau_{ij}$ can be related to the strain rate component $e_{ij}$ by the arbitrary continuous functional relation

$$\tau_{ij,} = G(e_{ij}) \qquad (1)$$

**Problem Formulation**

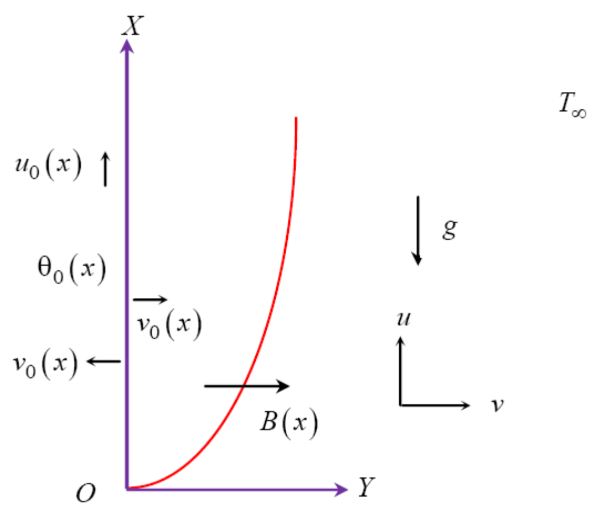


**Figure 1: Boundary layer around the stretching surface**

We consider a free convective, laminar boundary layer flow of an electrically conducting incompressible viscous power law fluid over a vertical porous and elastic surface. The surface is stretched vertically upward along the positive $x$-axis, with a prescribed velocity

$$y = 0, \qquad u(x,0) = u_0(x) \qquad (2)$$

While the origin $(x, y) = (0,0)$ is kept fixed. The $y$-axis is vertical to the surface, as it is depicted in Figure 1. Also, due to the fact that the elastic surface is porous, there is a component of the velocity of the fluid which has vertical direction to the surface given by

$$y = 0, \qquad v(x, 0) = v_0(x) \tag{3}$$

The motion of the surface within the fluid creates a boundary layer, which is extended along the $x$-axis. The whole system is under the influence of a magnetic field $B(x)$ which applies to the $y$-direction. We consider that the temperature of the surface changes along the $x$-axis and its distribution is described by a given function $T_0(x)$. The stress-strain relation, under the boundary layer assumption can be found in the form of arbitrary function with only non-vanishing component. Then the relation (20) can be given by $\tau_{yx}$. Then equation (1) can be given by

$$\tau_{yx} = G\left(\frac{\partial u}{\partial y}\right) \tag{4}$$

Under the assumption that the viscous dissipation term in the energy equation and the induced magnetic field can be neglected, the basic boundary layer equations of the mass, momentum and energy for the steady flow of Boussinesq type are respectively as follows, with the stress-strain relationship given by (4)

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

Momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}(\tau_{yx}) - \frac{\sigma B^2}{\rho}u + g\beta(T - T_\infty) \tag{6}$$

Energy

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{7}$$

Where $\sigma$ -electric conductivity, $\beta$ -volumetric coefficient of thermal expansion, $\rho$-mass density and $\alpha$ -thermal diffusivity, which are assumed to be constants. Also, $g$-gravity field assumed to be parallel to the $x$-axis, $T = T(y, x)$ - temperature field and $T_\infty$ - temperature at infinity. Therefore the boundary conditions of the problem of the form

$$y = 0, u(x, 0) = u_0 \ , v(x, 0) = v_0, \qquad \theta = \theta_0 \tag{8}$$

$$y = \infty, u(x, y) = 0 \ , \theta = 0 \tag{9}$$

Where $\theta = T - T_\infty$ , $\theta_0 = T_0 - T_\infty$ is a prescribed function along the boundary surface $y = 0$
With the stress-strain relationship

$$\tau_{yx} = G\left(\frac{\partial u}{\partial y}\right) \tag{10}$$

The above equation can be made dimensionless using following quantities,

$$x^* = \frac{G_r}{L}x \ , y^* = \frac{y}{L}(R_{e_x}.G_{r_x})^{\frac{1}{2}} \ , u^* = \frac{u}{U_0} \ , R_{x_e} = \frac{U_0 L}{v}$$

$$v^* = \frac{v}{U_0}\left(\frac{R_{e_x}}{G_{r_x}}\right)^{\frac{1}{2}}, \tau^*_{yx} = \frac{\tau_{yx}}{\rho U_0^2}\left(\frac{R_{e_x}}{G_{r_x}}\right)^{-\frac{1}{2}},$$

$$\theta^* = \frac{\theta}{(T_0 - T_\infty)}$$

$$\theta_0^* = \frac{\theta}{(T_0 - T_\infty)} \ , P_r = \frac{v}{\alpha} \ , G_r = \frac{L^3}{v^2}g\beta(T_0 - T_\infty) \tag{11}$$

Introducing above non-dimensional quantities in equations (5) to (7),

We get
Continuity

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{12}$$

Momentum

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\partial}{\partial y^*}\left(\tau^*{}_{y^*x^*}\right)-M^*u^*+\lambda\theta^* \quad (13)$$

Energy

$$u^*\frac{\partial \theta^*}{\partial x^*}+v^*\frac{\partial \theta^*}{\partial y^*}=\frac{1}{P_r}\frac{\partial^2\theta^*}{\partial {y^*}^2} \quad (14)$$

With the stress-strain relationship

$$\tau^*{}_{yx}=G\left(\frac{\partial u^*}{\partial y^*}\right) \quad (15)$$

With the boundary conditions

$$y=0, u^*(x,0)=u_0^*, v^*(x,0)=v_0^*, \theta^*=\theta_0^* \quad (16)$$

$$y=\infty, u^*(x,y)=0\ , \quad \theta^*=0 \quad (17)$$

Introducing stream function $\psi$ such that,

$$u^*=\frac{\partial\psi^*}{\partial y^*} \quad ,v^*=-\frac{\partial\psi^*}{\partial x^*} \quad (18)$$

Equation of continuity (12) gets satisfied identically, equation (13)-(17) become

$$\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial {y^*}^2}=\frac{\partial}{\partial y^*}\left(\tau_{yx}^*\right)-M^*\frac{\partial\psi^*}{\partial y^*}+\lambda\theta^* \quad (19)$$

$$\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}=\frac{1}{P_r}\frac{\partial^2\theta^*}{\partial {y^*}^2} \quad (20)$$

$$\tau^*{}_{yx}=G\left(\frac{\partial^2\psi^*}{\partial {y^*}^2}\right) \quad (21)$$

With boundary conditions

$$y=0, \frac{\partial\psi^*}{\partial y^*}=u_0\ , \frac{\partial\psi^*}{\partial x^*}=v_0, \quad \theta^*=\theta_0^* \quad (22)$$

$$y=\infty, \quad \frac{\partial\psi^*}{\partial y^*}=0\ , \quad \theta^*=0 \quad (23)$$

By using linear group transformation

$$\bar{x}^*=D^{\alpha_1}x^*\ , \quad \bar{y}^*=D^{\alpha_2}y^*\ ,$$
$$\bar{\psi}^*=D^{\alpha_3}\psi^*, \quad \bar{\theta}^*=D^{\alpha_4}\theta^*\ ,$$
$$\bar{\tau}_{yx}^*=D^{\alpha_5}\tau_{yx}^*\ , \quad \bar{M}^*=D^{\alpha_6}M^* \quad (24)$$

Where $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6$ and P are constants For the dependent and independent variables. From Eq. (24) one obtains

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{\alpha_1}}=\left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{\alpha_2}}=\left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{\alpha_3}}=\left(\frac{\bar{\theta}^*}{\theta^*}\right)^{\frac{1}{\alpha_4}}=\left(\frac{\bar{\tau}_{yx}^*}{\tau_{yx}^*}\right)^{\frac{1}{\alpha_5}}=\left(\frac{\bar{M}^*}{M^*}\right)^{\frac{1}{\alpha_6}}=D \quad (25)$$

Introducing the linear transformation, given by equation (25), into the equations (19)-(21) result in

$$D^{\alpha_1+2\alpha_2-2\alpha_3}\frac{\partial\bar{\psi}^*}{\partial\bar{y}^*}\frac{\partial^2\bar{\psi}^*}{\partial\bar{x}^*\partial\bar{y}^*}-D^{\alpha_1+2\alpha_2-2\alpha_3}\frac{\partial\bar{\psi}^*}{\partial\bar{x}^*}\frac{\partial^2\bar{\psi}^*}{\partial{\bar{y}^*}^2}=D^{\alpha_2-\alpha_5}\frac{\partial}{\partial\bar{y}^*}\left(\bar{\tau}_{yx}^*\right)-D^{\alpha_2-\alpha_3-\alpha_6}\bar{M}^*\frac{\partial\bar{\psi}^*}{\partial\bar{y}^*}+\lambda D^{-\alpha_4}\bar{\theta}^* \quad (26)$$

$$D^{\alpha_1+\alpha_2-\alpha_3-\alpha_4}\frac{\partial\bar{\psi}^*}{\partial\bar{y}^*}\frac{\partial\bar{\theta}^*}{\partial\bar{x}^*}-D^{\alpha_1+\alpha_2-\alpha_3-\alpha_4}\frac{\partial\bar{\psi}^*}{\partial\bar{x}^*}\frac{\partial\bar{\theta}^*}{\partial\bar{y}^*}=\frac{1}{P_r}D^{2\alpha_2-\alpha_4}\frac{\partial^2\bar{\theta}^*}{\partial{\bar{y}^*}^2} \quad (27)$$

And

$$D^{\propto_5}\bar{\tau}_{yx} = G\left(p^{-\alpha_3+2\alpha_2}\ \frac{\partial^2\bar{\psi}}{\partial\bar{y}^2}\right) \quad (28)$$

The differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations are obtained

$$\alpha_1+2\alpha_2-2\alpha_3=-\alpha_4 \quad (29)$$
$$\alpha_2-\alpha_5=-\alpha_4 \quad (30)$$
$$\alpha_2-\alpha_3-\alpha_6=-\alpha_4 \quad (31)$$
$$\alpha_1+\alpha_2-\alpha_3-\alpha_4=2\alpha_2-\alpha_4 \quad (32)$$
$$-\alpha_3+2\alpha_2 = 0 \quad (33)$$

and

$$\alpha_5 = 0 \tag{34}$$

By Solving above equations, we get

$$\alpha_2 = \frac{1}{3}\alpha_1 = \frac{1}{2}\alpha_3 = -\frac{1}{2}\alpha_6 \text{ and } \alpha_5 = 0 \tag{35}$$

Introducing equation (35) into equation(25) results in

$$\eta = \frac{y^*}{x^{*\frac{1}{3}}}, \qquad \psi^* = f(\eta)x^{*\frac{2}{3}},$$

$$\theta^* = G(\eta)x^{*-\frac{1}{3}} \qquad \tau^*_{yx} = H(\eta),$$

$$M^* = x^{*-\frac{2}{3}} m \tag{36}$$

With boundary conditions, equations (22)-(23) becomes

$$\eta = 0, f'(0) = u_0, f(0) = v_0, G(0) = \theta_0$$

$$\eta \to \infty, f'(\infty) = 0, \quad G(\infty) = 0 \tag{37}$$

Introducing equation (36) in equations (19)-(23), we get following similarity equation

$$f'^2(\eta) - 2f(\eta)f''(\eta) - 3H'(\eta) + 3m.f'(\eta) - 3\lambda G(\eta) = 0 \tag{38}$$

$$2f(\eta)G'(\eta) + f'(\eta)G(\eta) + \frac{1}{P_r}G''(\eta) = 0 \tag{39}$$

$$H(\eta) = G(f''(\eta)) \tag{40}$$

With boundary conditions

$$\eta = 0, \quad f(0) = c_1, f'(0) = c_2, G(0) = c_3$$

$$\eta \to \infty, \quad f'(\infty) = 0, \quad G(\infty) = 0 \tag{41}$$

**Result and Discussions**

Many Non-Newtonian fluid models based on functional relationship between shear-stress and rate of the strain, are available in real world applications Bird [35]. Among these models most research work is so far carried out on power-law fluid model, this is because of its mathematical simplicity. On the other hand rest of fluid models is mathematically more complex and the natures of partial differential equations governing these flows are too non-linear boundary value type and hence their analytical or numerical solution is bit difficult. For the present study the partial differential equation model, although mathematically more complex, is chosen mainly due to two reasons. Firstly, it can be deduced from kinetic theory of liquids rather than the empirical relation as in power-law model. Secondly, it correctly reduces to Newtonian behavior for both low and high shear rate. This reason is somewhat opposite to Pseudo plastic system whereas the power-law model has infinite effective viscosity for low shear rate and thus limiting its range of applicability.

Mathematically, the Powell-Eyring model can be written as (Bird [35], Skelland [36])

$$\tau_{yx} = \mu\frac{\partial u}{\partial y} + \frac{1}{B}\sin h^{-1}\left(\frac{1}{C}\frac{\partial u}{\partial y}\right) \tag{42}$$

Where B and C are rheological parameters

Introducing the dimensionless quantities into equation (42) and using similarity variables, we get

$$H'(\eta) = f''' + \frac{\epsilon_1 f'''}{\sqrt{1 + \epsilon_2 f''^2}} \tag{43}$$

Where $\epsilon_1 = \frac{1}{\mu BC}$, $\epsilon_2 = \frac{\rho u_0^3 G_r}{\mu L C^2}$ are referred as rheological flow parameters.

Substituting the value from (43), the system (38-40) reduce to,

$$F''' = \frac{\frac{1}{3}\left(f'^2 - 2ff'' + 3mf' - \lambda G\right)\sqrt{1 + \epsilon_2 f''^2}}{\epsilon_1 + \sqrt{1 + \epsilon_2 f''^2(\eta)}} \tag{44}$$

$$2f(\eta)G'(\eta) + f'(\eta)G(\eta) + \frac{1}{P_r}G''(\eta) = 0 \tag{45}$$

Also the dimensionless local skin-friction confident $C_{fx}$ expression is given by

$$\frac{1}{2}C_{fx}\sqrt{Re_x Gr_x} \equiv \tau_w \tag{46}$$

Where $\tau_w$ is local shear stress. That is $\tau_w = \tau_{yx}|_{y=0}$

In terms of defined rheological flow parameters (46) yields,

$$\frac{1}{2}C_{fx}\sqrt{Re_x Gr_x} = \epsilon_2 f''(0) + \frac{\epsilon_1}{\sqrt{\epsilon_2}}\sin h^{-1}\left(\sqrt{\epsilon_1} f''(0)\right) \tag{47}$$

In order to face numerically problem (38)-(41), we have used a numerical solver of MATLAB package which solves any two-point boundary value problem for ODEs by collocation. To enhance the effect of magnetic field, without loss of generality, each parameter assumed appropriately in boundary conditions (41). The numerical solutions are produced graphically in Figures (2)-(4).

Figure (2) shows that boundary layer decrease as the magnetic field increase.

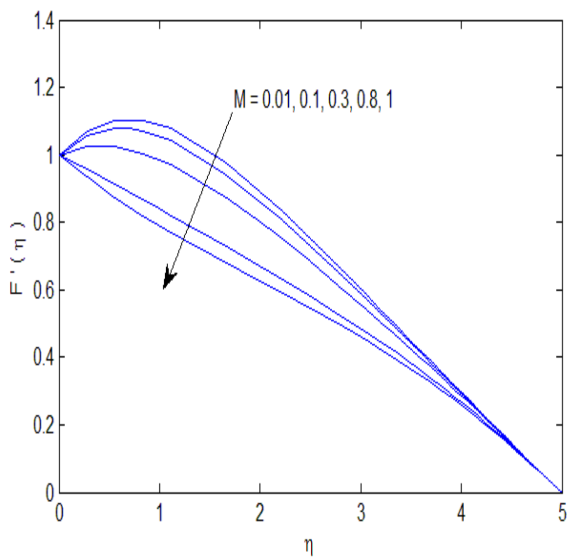


**Figure 2: Influence of magnetic field on horizontal velocity**

Figure (3) depicts behavior of $f''(\eta)$ throughout the domain. In particular it is interesting to observe that, as M increases $f''(0)$ decrease and hence the local shear-stress (see Table 1), which decreases local skin-friction $C_f$.

| M | 0.01 | 0.1 | 0.3 | 0.8 | 1 |
|---|---|---|---|---|---|
| $f''$ | 0.3374 | 0.2725 | 0.1385 | -0.1477 | -0.2473 |
| $\tau_w$ | 0.6687 | 0.5417 | 0.2766 | -0.2949 | -0.4921 |

**Table 1: Local shear stress**

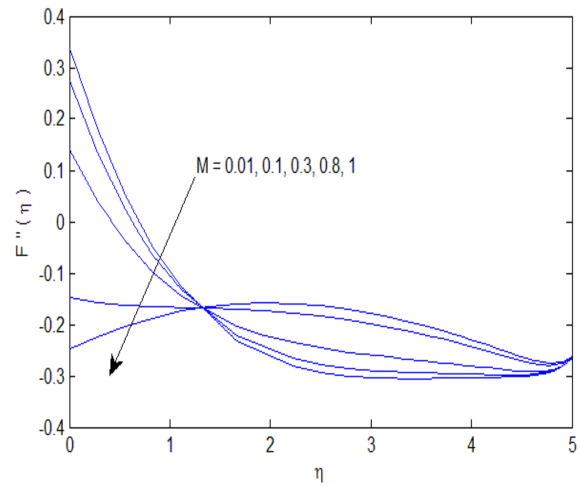


**Figure 3: Influence of magnetic field on shear stress within boundary layer domain**

Influence of magnetic field on thermal boundary layers displayed by Figure 4. It shows that increase in magnetic field will precisely increase thermal boundary layer within the boundary layer domain.

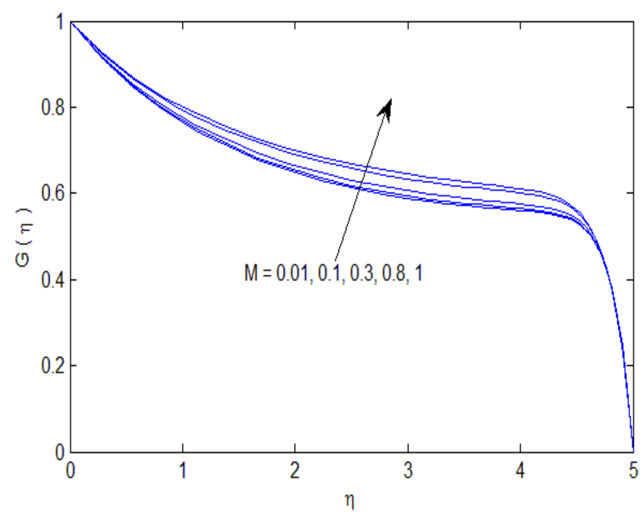


**Figure 4: Thermal boundary layer domain under the effect of magnetic field**

## Conclusion

Similarity solution is produced for a free convective boundary layer flow of electrically conducting Non-Newtonian fluids over a vertical porous and elastic surface. The governing system of Partial differential equations transformed into the system of Ordinary differential equations subject to the similarity requirement, by employing the derived transformations. Numerical solutions for special Non-Newtonian fluid so-called Powell-Eyring fluid are produced by MATLAB computational algorithm. An interesting effect of magnetic field is observed. All the numerical solutions are generated for dimensionless quantity and hence it is executed for all types of under considered fluids. An interesting effect of magnetic field is observed.

**References:**


1. Olver P.J., Application of Lie Group to Differential Equations, Spring-Verlag, 2nd Edition, New York, 1993.
2. Bluman J.W. and Kumai S. Symmetries and Differential Equations , Springer-Verlag, New York, 1989.
3. Ibragimov N.H., Transformation group Applied to Mathematical Physics , D. Reidel , Dordrecht, 1985.
4. Ibragimov N.H., Elementary Lie Group Analysis and Ordinary Differential Equations , Wiley , Chichester , 1999.
5. Moran M.J. and Gaggioli R. A. , Reduction of the number of variables in system of partial differential equations with auxiliary conditions ,SIAM J. Appl. Math. 16 , pp. 202-215,1968.
6. Suhubi E.S., Isovector fields and similarity solutions for general balance equations, Int. J Eng. Sci 29(1), pp.133-150, 1991.
7. Suhubi E. S., Symmetry groups and similarity solutions for radial motions of compressible heterogeneous hyper elastic spheres and cylinders , Int. J. Eng. Sci. 32 (5), pp.817-837, 1994.
8. Pakdemirli M and Suhubi E.S. , Similarity solutions of boundary-layer equations for second order fluids Int. J. Eng. Sci. 30 (5) ,pp. 611-629, 1992.
9. Kalpakides V.K., Isovector fields and similarity solution of nonlinear thermo-elasticity, Int. J. Eng. Sci. 36 (10), pp. 1103-1126, 1998.
10. Kalpakides V.K., on the symmetries and similarity solutions one-dimensional non-linear thermoelasticity, Int. J. Eng. Sci. 39 (16), pp 1863-1879, 2001.
11. Koureas Th., Charalmbopoulos A. and Kalpakides V.K., On Isovector fields and similarity solutions of generalized dynamic thermo elasticity, Int. Eng. Sci. 39 (18), pp.2071-2087, 2001.
12. Koureas Th., Charalmbopoulos A. and Kalpakides V.K., Symmetry groups and Group-invariant solutions of 3D non-linear thermo elasticity, Int. J.Eng. Sci. 41 (6), pp.547-568, 2003.
13. Sakiadis B.C., Boundary-layer behaviour on continuous solid surfaces, AIChE. 7(1), pp. 26-28, 1961.
14. Erickson L.E., Fan L.T. and Fox V.G. , Heat and mass transfer on a moving continuous flat plate with suction or injection , Ind. Eng. Chem. Fundam. 5, pp. 19-25, 1966.
15. Tsou F.K., Sparrow E.M. and Goldstein R.J., Flow and heat transfer in the boundary layer on a continuous moving surface, Int. J. Heat Mass Transfer 10 (2), pp.219-235, 1967.
16. Gupta P.S. and Gupta A.S., Heat and mass transfer on a stretching sheet with suction or blowing, Can. J. Chem. Eng. 55 (6) , pp. 744-746, 1977 .
17. Malek M.B., Badran N.A. and Hassan H.S., Solution of the Rayleigh Problem for a power law non-Newtonian conducting fluid via group method, Int. J. Eng. Sci 40, pp. 1599-1609, 2002.
18. Darji R.M and Timol M. G., Deductive group theoretic Analysis for MHD flow of a Sisko fluid in a porous medium , Int. J. of

Appl. Math and Mech. 7 (19), pp.49-58, 2011.
19. Darji R.M and Timol M. G., Deductive group invariance analysis of boundary layer equations of a special non-Newtonian fluid over a stretching sheet, Int. J. of Maths. and Sci. Comp.2 (2), pp.54-58, 2012.
20. Adnan K.A., Hasmani A.H. and Timol M.G., A new family of similarity solutions of three dimensional MHD boundary layer flows of non-Newtonian fluids using new systematic group-theoretic approach, Applied Mathematical Sciences 5(27), pp.1325-1336. 2011.
21. Crane L.J., Flow past a stretching plate, ZAMP 21, pp.645-647, 1970.
22. Soundalgekar V. M. and Ramana Murthy T. V., Heat transfer in the flow past a continuous moving plate with variable temperature, Warme-und Stoffubertragung 14, pp. 91-93, 1980.
23. Grubka L.G. and Bobba K. M., Heat transfer characteristics of a continuous stretching surface with variable temperature, ASME J. Heat Transfer 107, pp. 248-250, 1985.
24. Datta B.K., Roy P. and Gupta S. (1985), Temperature field in the flow over a stretching sheet with uniform heat flux, Int. Common. Heat Mass Transfer 12, pp. 89-94, 1985.
25. Jeng D.R., Chang T.C. And Dewitt K., Momentum and Heat transfer on a continuous moving surface, ASME J. Heat Transfer 108 , pp. 532-539, 1986.
26. Dutta B.K. Heat transfer from a stretching sheet with uniform suction or blowing, Acta Mech. 78, pp. 255-262, 1989.
27. Chen C.K. and Char M.I., Heat transfer of a continuous stretching surface with suction or blowing, J. Math. Anal. Appl.135, pp.568-580, 1988.
28. Sarpakaya T., Flow of non-Newtonian fluids in a magnetic field, AIChE. 7, pp. 324-328, 1961.
29. Saponkov Y., Similar solutions of steady boundary layer equations in Magnetohydrodynamic Power law conducting fluids, Mech. Fluid Gas .6 , pp. 77-82.
30. Martinson L. K. and Pavlov K.B., Unsteady shear flows of a conducting fluid with a Rheological power law, MHD 7, pp.182-189, 1971.
31. Samokhen V. N., on the boundary-layer equation of MHD of dilatants fluid in a transverse magnetic field, MHD 3, pp. 71-77, 1987.
32. Andersson H. I., Bech K.H. and Dandapat B.S., Magnetohydrodynamic flow of a power-law fluid over a stretching sheet, Int. J. Non-Linear Mech.27,pp.929-936,1992.
33. Cortell R., A note on magneto hydrodynamic flow of a power law fluid over a stretching sheet, Appl. Math. Comput. 168, pp. 557-566, 2005.
34. Liao, Shi-jun. , On the analytic solution of Magnetohydrodynamic flows of non-Newtonian fluids over a stretching sheet , J. Fluid Mech. 488, pp. 189-212, 2005.
35. Bird R.B., Stewart W.E. and Lightfoot E.M., Transport Phenomena, John Wiley, New York .1960.
36. Skelland A.H.P., Non-Newtonian flow and heat transfer, John Wiley, New York, 1967.