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Electron–Phonon Interactions under the External Applied Electric Fields in the Normal Metallic and Superconducting States in Various Sized Materials

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Abstract

We elucidate the mechanism of the Ampère's law (experimental rule discovered in 1820) in normal metallic and superconducting states, on the basis of the theory suggested in our previous researches. The induced magnetic field in the Ampère's law is realized because the bosonic electronic state tries not to change the electronic structure by inducing the magnetic field. If an electron were not in the bosonic state, any induced magnetic field would not be observed. Furthermore, we discuss how the strength of the electric field are closely related to the electron–phonon interactions in the normal metallic states and the superconducting states. We formulate the electron–phonon coupling constants, which are very important physical parameters in the various research fields such as the normal metallic states and superconducting states, under the external applied electric field as well as under no external applied field.

***Keywords*:** Normal Metal, Superconductor, Electron–Phonon Interactions, Bose–Einstein Condensation, Ampère's Law, A Bosonic Electron.

1. Introduction

The effect of vibronic interactions and electron–phonon interactions [1–7] in molecules and crystals is an important topic of discussion in modern chemistry and physics. The vibronic and electron–phonon interactions play an essential role in various research fields such as the decision of molecular structures, Jahn–Teller effects, Peierls distortions, spectroscopy, electrical conductivity, and superconductivity. We have investigated the electron–phonon interactions in various charged molecular crystals for more than ten years [1–8]. In particular, in 2002, we predicted the occurrence of superconductivity as a consequence of vibronic interactions in the negatively charged picene, phenanthrene, and coronene [8]. Recently, it was reported that these trianionic molecular crystals exhibit superconductivity [9].

In general, in the vibronic interactions, we consider the first- and second-order processes. In the first-order processes, we must consider one electron systems in which a phonon is emitted or absorbed by an electron (Fig. 1). On the other hand, in the second-order processes, we must consider two electrons systems in which a phonon is exchanged between them (Fig. 2). Vibronic and Jahn–Teller stabilization energies for one electron have been calculated and discussed for a long time [1–8]. However, in these previous calculations and discussions, the only total electron–phonon coupling constants originating from both the first- and second-order processes have been well calculated, and the roles of the first- and second-order processes have not been clearly distinguished. Furthermore, in these discussions, the wave function for a randomly moving electron has been ambiguous, and the direction of motion of an electron has not been considered. This is the main reason why the roles of the first- and second-order processes have been ambiguous. Furthermore, in the conventional two-electrons theory [1–7], only the case where no external electric or magnetic field is applied has been considered. That is, the relationships between these energies and the strengths of the external applied electric field have not been discussed in detail.

In this article, we discuss how the opened-shell independent one electron is really stabilized in energy in view of the second-order processes in the vibronic and Jahn–Teller stabilization. In the previous researches, electrons and molecular vibrations have mainly been treated as particles and waves, respectively. On the other hand, electrons and molecular vibrations are more clearly treated as waves and particles, respectively, in this study, than those in the previous researches. By considering the wave characteristics of electrons and the direction of motion of an electron, we will clearly define the wave function for randomly moving one electron (Eq. (1)), which has not been clearly defined. By using this wave function, the roles of the first- and second-order processes in the vibronic interactions can be clearly distinguished in this study. We also discuss the relationships between the vibronic and Jahn–Teller stabilization energies, originating from the nondissipative second-order processes, and the electrical resistivity and Joule's heats, originating from the dissipative first-order processes in the vibronic interactions, as a function of the external applied electric field. Furthermore, in this article, we elucidate the mechanism of the Ampère's law (experimental rule

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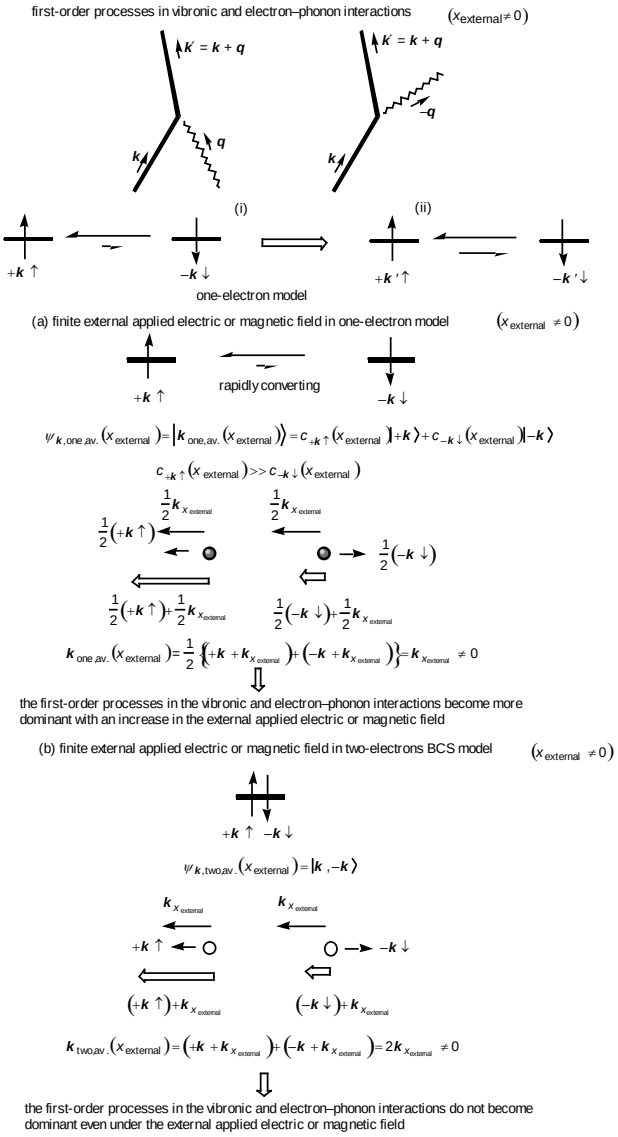

Fig. 1. First-order electron–phonon interactions processes under external electric field.

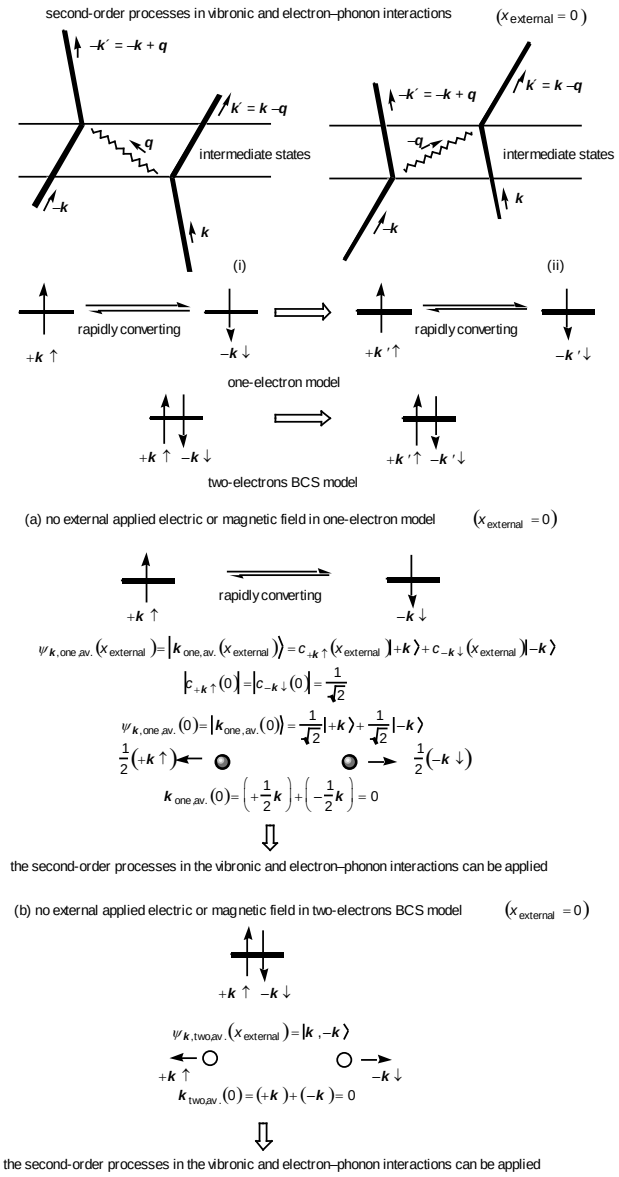

Fig. 2. Second-order electron–phonon interactions processes without any external electric field.

discovered in 1820) in normal metallic and superconducting states [10], on the basis of the theory suggested in our previous researches [1–7].

2. Vibronic Stabilization Energies under the Applied External Electric Field in Molecules

In this section, we will first show how the vibronic stabilization energies [1–6] are derived on the basis of our new one-electron model as well as the conventional two-electrons model.

2.1 One-Electron Model in Molecules

Let us consider an inert Fermi-sea, in which the electrons are treated as non-interacting [1–8]. To this Fermi-sea, one electron is added above the Fermi-surface.

This one added electron does not interact with the inert Fermi-sea, as shown in Fig. 3.

One electron occupies a plane-wave state $\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle$, in the absence of interactions, as shown in Fig. 2 (a),

$$\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle=\sqrt{P_{\boldsymbol{k}_{\text{ground}}}(T)}\left|\boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle+\sqrt{P_{\boldsymbol{k}_{\text{excited}}}(T)}\left|\boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle, \quad (1)$$

where

$$\left|\boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle = c_{+\boldsymbol{k}}\left|+\boldsymbol{k}\downarrow\right\rangle + c_{-\boldsymbol{k}}\left|-\boldsymbol{k}\uparrow\right\rangle, \tag{2}$$

$$\left|\boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle = c_{+\boldsymbol{k}}\left|+\boldsymbol{k}\uparrow\right\rangle + c_{-\boldsymbol{k}}\left|-\boldsymbol{k}\downarrow\right\rangle. \tag{3}$$

$$\begin{aligned}&\left\langle \boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\middle|\boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle \\ &= \left\langle \boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\middle|\boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle = c_{+\boldsymbol{k}}^2 + c_{-\boldsymbol{k}}^2 = 1,\end{aligned} \tag{4}$$

$$\begin{aligned}&\left\langle \boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\middle|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle \\ &= P_{\boldsymbol{k}_{\text{ground}}}(T)\left\langle \boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\middle|\boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle \\ &+ P_{\boldsymbol{k}_{\text{excited}}}(T)\left\langle \boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\middle|\boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle \\ &= 1.\end{aligned} \tag{5}$$

Here, the one extra electron in the absence of such a peculiar interaction is denoted by $\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle$, and in the presence of the interaction by $\left|\boldsymbol{K}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle$. Denoting the Hamiltonian of the system by

$$H = H_0 + V_{\text{eff}}, \tag{6}$$

then

$$H_0\left|\pm\boldsymbol{k}\right\rangle = \varepsilon_{\boldsymbol{k}}\left|\pm\boldsymbol{k}\right\rangle, \tag{7}$$

where $\varepsilon_{\boldsymbol{k}}$ denotes the single-particle energy of the non-interacting fermion system. Adding interactions, the exact Schrödinger equation for the one-particle problem defined above are given by

$$\begin{aligned}&H\left|\boldsymbol{K}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle \\ &= E_{\text{one}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\left|\boldsymbol{K}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle,\end{aligned} \tag{8}$$

where $E_{\text{one}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)$ denotes the exact one-particle energy above the Fermi-surface, in the presence of vibronic interactions. Assuming that the states $\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle$ form a complete set such that the exact one-particle eigenstate can be expanded in this basis, then

$$\begin{aligned}&\left|\boldsymbol{K}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle = \sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle \\ &= \sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left\{\sqrt{P_{\boldsymbol{k}_{\text{ground}}}(T)}\left|\boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle\right. \\ &\quad\left.+\sqrt{P_{\boldsymbol{k}_{\text{excited}}}(T)}\left|\boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle\right\} \\ &= \sum_{\boldsymbol{k}}\left\{a_{\boldsymbol{k}}\sqrt{P_{\boldsymbol{k}_{\text{ground}}}(T)}\left(c_{+\boldsymbol{k}}\left|+\boldsymbol{k}\downarrow\right\rangle + c_{-\boldsymbol{k}}\left|-\boldsymbol{k}\uparrow\right\rangle\right)\right. \\ &\quad\left.+a_{\boldsymbol{k}}\sqrt{P_{\boldsymbol{k}_{\text{excited}}}(T)}\left(c_{+\boldsymbol{k}}\left|+\boldsymbol{k}\uparrow\right\rangle + c_{-\boldsymbol{k}}\left|-\boldsymbol{k}\downarrow\right\rangle\right)\right\} \\ &= \sum_{\boldsymbol{k}} b_{\boldsymbol{k}}\left(c_{+\boldsymbol{k}}\left|+\boldsymbol{k}\right\rangle + c_{-\boldsymbol{k}}\left|-\boldsymbol{k}\right\rangle\right).\end{aligned} \tag{9}$$

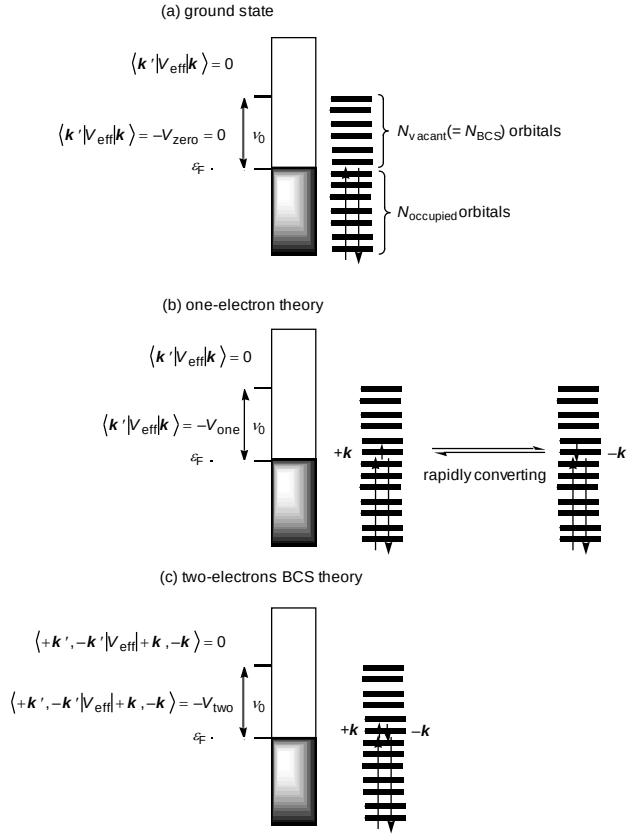

Fig. 3. Electronic states in the (a) ground state, (b) monoanionoic state, and (c) dianionic state.

Inserting Eq. (9) into Eq. (8), and then we obtain

$$\begin{aligned}&\left(H_0 + V_{\text{eff}}\right)\sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle \\ &= E_{\text{one}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}},c_{-\boldsymbol{k}}\right)\right\rangle.\end{aligned} \tag{10}$$

Considering the above orthogonality relation $\left\langle \boldsymbol{k}'\middle|\boldsymbol{k}\right\rangle = \delta_{\boldsymbol{k},\boldsymbol{k}'}$, we thus find

$$\begin{aligned}&\sum_{\boldsymbol{k}'}\left(c_{+\boldsymbol{k}'}\left\langle +\boldsymbol{k}'\right| + c_{-\boldsymbol{k}'}\left\langle -\boldsymbol{k}'\right|\right)H_0\left(c_{+\boldsymbol{k}}\left|+\boldsymbol{k}\right\rangle + c_{-\boldsymbol{k}}\left|-\boldsymbol{k}\right\rangle\right) \\ &= b_{\boldsymbol{k}}\varepsilon_{\boldsymbol{k}},\end{aligned} \tag{11}$$

$$\sum_{\boldsymbol{k}'}\left(c_{+\boldsymbol{k}'}\left\langle +\boldsymbol{k}'\right| + c_{-\boldsymbol{k}'}\left\langle -\boldsymbol{k}'\right|\right)E_{\text{one}}\left(c_{+\boldsymbol{k}}\left|+\boldsymbol{k}\right\rangle + c_{-\boldsymbol{k}}\left|-\boldsymbol{k}\right\rangle\right)$$

$$= b_{\boldsymbol{k}} E_{\text{one}}, \quad (12)$$

and thus

$$b_{\boldsymbol{k}} \left(\varepsilon_{\boldsymbol{k}} - E_{\text{one}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right) = -\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'} \left(c_{+\boldsymbol{k}'} \langle +\boldsymbol{k}'| + c_{-\boldsymbol{k}'} \langle -\boldsymbol{k}'|\right) \times |V_{\text{eff}}| \left(c_{+\boldsymbol{k}} |+\boldsymbol{k}\rangle + c_{-\boldsymbol{k}} |-\boldsymbol{k}\rangle\right). \quad (13)$$

The quantity $\langle \boldsymbol{k}'|V_{\text{eff}}|\boldsymbol{k}\rangle$ denotes a one-electron scattering matrix element from a one-particle state $|\boldsymbol{k}\rangle$ to a one-particle state $|\boldsymbol{k}'\rangle$.

We now consider that the electronic states $\boldsymbol{k}$ can be stabilized by V_{one} only when the scattering within the same electronic states $\boldsymbol{k}$ occurs. That is, we have

$$\langle +\boldsymbol{k}'|V_{\text{eff}}|+\boldsymbol{k}\rangle = \langle +\boldsymbol{k}'|V_{\text{eff}}|-\boldsymbol{k}\rangle = \langle -\boldsymbol{k}'|V_{\text{eff}}|+\boldsymbol{k}\rangle = \langle -\boldsymbol{k}'|V_{\text{eff}}|-\boldsymbol{k}\rangle = -V_{\text{one}}\delta_{\boldsymbol{k},\boldsymbol{k}'}, \quad (14)$$

$$\begin{aligned}
&\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'} \left(c_{+\boldsymbol{k}'} \langle +\boldsymbol{k}'| + c_{-\boldsymbol{k}'} \langle -\boldsymbol{k}'|\right) |V_{\text{eff}}| \left(c_{+\boldsymbol{k}} |+\boldsymbol{k}\rangle + c_{-\boldsymbol{k}} |-\boldsymbol{k}\rangle\right) \\
&= -V_{\text{one}} \sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'} \left(c_{+\boldsymbol{k}'} c_{+\boldsymbol{k}} + c_{-\boldsymbol{k}'} c_{-\boldsymbol{k}} + c_{+\boldsymbol{k}'} c_{-\boldsymbol{k}} + c_{-\boldsymbol{k}'} c_{+\boldsymbol{k}}\right) \times \delta_{\boldsymbol{k},\boldsymbol{k}'} \\
&= -V_{\text{one}} \sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'} \left(1 + 2c_{+\boldsymbol{k}'} c_{-\boldsymbol{k}}\right) \delta_{\boldsymbol{k},\boldsymbol{k}'} \\
&= -2V_{\text{one}} \sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'} \left(\frac{1}{2} + c_{+\boldsymbol{k}'} c_{-\boldsymbol{k}}\right) \delta_{\boldsymbol{k},\boldsymbol{k}'} \\
&= -2V_{\text{one}} f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) \sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'} \delta_{\boldsymbol{k},\boldsymbol{k}'} \\
&= -2b_{\boldsymbol{k}} V_{\text{one}} f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right), \quad (15)
\end{aligned}$$

where

$$f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) = \frac{1}{2} + c_{-\boldsymbol{k}} c_{+\boldsymbol{k}} = \frac{1}{2} + c_{-\boldsymbol{k}} \sqrt{1 - c_{-\boldsymbol{k}}^2}. \quad (16)$$

Thus, Eq. (13) takes the form

$$b_{\boldsymbol{k}} \left(\varepsilon_{\boldsymbol{k}} - E_{\text{one}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right) = 2 b_{\boldsymbol{k}} V_{\text{one}} f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right). \quad (17)$$

$$\Delta_{\text{vib,one}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) = 2V_{\text{one}} f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right). \quad (18)$$

The $\Delta_{\text{vib,one}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)$ denotes the stabilization energy of independent one electron as a consequence of the second-order process of the electron–phonon interactions.

2.2 Comparison of the One-Electron Theory with the Conventional Two-Electrons BCS Theory under the Applied Electric Field in Molecules

Let us next discuss the vibronic stabilization energy of independent two electrons in the one-electron theory ($\Delta_{\text{vib,pair,one}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)$).

The $\Delta_{\text{vib,pair,two}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)$ value can be defined as

$$\begin{aligned}
\Delta_{\text{vib,pair,two}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) &= 2\Delta_{\text{vib,one}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) \\
&= 4V_{\text{one}} - 4V_{\text{one}}\left(1 - f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right) \\
&= 4V_{\text{one}} f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right). \quad (19)
\end{aligned}$$

The electronic energy level itself for the electronic state is stabilized by $4V_{\text{one}}$ with the kinetic energy of supercurrent $4V_{\text{one}}\left(1 - f_{\text{Bose}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right)$.

3. Vibronic Stabilization Energies under the External Applied Electric Field in Solids

In this section, we will first show how the vibronic stabilization energies [1–6] are derived on the basis of our new one-electron model as well as the conventional two-electrons model under the external applied electric field in solids.

3.1 One-Electron Model in Solids

Let us consider an inert Fermi-sea, in which the electrons are treated as non-interacting. To this Fermi-sea, one electron is added above the Fermi-surface. This one added electron does not interact with the inert Fermi-sea, as shown in Fig. 3. One electron occupies a plane-wave state $\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle$, in the absence of interactions, as shown in Figs. 1 (a) and 2 (a),

$$\left|\boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle = \sqrt{P_{\boldsymbol{k}_{\text{ground}}}(T)} \left|\boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle + \sqrt{P_{\boldsymbol{k}_{\text{excited}}}(T)} \left|\boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle, \quad (20)$$

where

$$\left|\boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle = c_{+\boldsymbol{k}} \left|+\boldsymbol{k}\downarrow\right\rangle + c_{-\boldsymbol{k}} \left|-\boldsymbol{k}\uparrow\right\rangle, \quad (21)$$

$$\left|\boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle = c_{+\boldsymbol{k}} \left|+\boldsymbol{k}\uparrow\right\rangle + c_{-\boldsymbol{k}} \left|-\boldsymbol{k}\downarrow\right\rangle. \quad (22)$$

$$\begin{aligned}
&\left\langle \boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) \middle| \boldsymbol{k}_{\text{ground}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle \\
&= \left\langle \boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) \middle| \boldsymbol{k}_{\text{excited}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle \\
&= c_{+\boldsymbol{k}}^2 + c_{-\boldsymbol{k}}^2 = 1, \quad (23)
\end{aligned}$$

$$\left\langle \boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right) \middle| \boldsymbol{k}_{\text{one,av.}}\left(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}\right)\right\rangle$$

$$= P_{\boldsymbol{k}_{\text{ground}}}(T)\left\langle \boldsymbol{k}_{\text{ground}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \middle| \boldsymbol{k}_{\text{ground}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \right\rangle$$
$$+ P_{\boldsymbol{k}_{\text{excited}}}(T)\left\langle \boldsymbol{k}_{\text{excited}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \middle| \boldsymbol{k}_{\text{excited}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \right\rangle$$
$$= 1. \qquad (24)$$

Here, the one extra electron in the absence of such a peculiar interaction is denoted by $\left|\boldsymbol{k}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle$, and in the presence of the interaction by $\left|\boldsymbol{K}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle$. Denoting the Hamiltonian of the system by

$$H = H_0 + V_{\text{eff}}, \qquad (25)$$

then

$$H_0|\pm\boldsymbol{k}\rangle = \varepsilon_{\boldsymbol{k}}|\pm\boldsymbol{k}\rangle, \qquad (26)$$

where $\varepsilon_{\boldsymbol{k}}$ denotes the single-particle energy of the non-interacting fermion system. Adding interactions, the exact Schrödinger equation for the one-particle problem defined above are given by

$$H\left|\boldsymbol{K}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle$$
$$= E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\left|\boldsymbol{K}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle, \qquad (27)$$

where $E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})$ denotes the exact one-particle energy above the Fermi-surface, in the presence of vibronic interactions. Assuming that the states $\left|\boldsymbol{k}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle$ form a complete set such that the exact one-particle eigenstate can be expanded in this basis, then

$$\left|\boldsymbol{K}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle = \sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left|\boldsymbol{k}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle$$
$$= \sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left\{\sqrt{P_{\boldsymbol{k}_{\text{ground}}}(T)}\left|\boldsymbol{k}_{\text{ground}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle \right.$$
$$\left. + \sqrt{P_{\boldsymbol{k}_{\text{excited}}}(T)}\left|\boldsymbol{k}_{\text{excited}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle\right\}$$
$$= \sum_{\boldsymbol{k}}\left\{a_{\boldsymbol{k}}\sqrt{P_{\boldsymbol{k}_{\text{ground}}}(T)}\left(c_{+\boldsymbol{k}}|+\boldsymbol{k}\downarrow\rangle + c_{-\boldsymbol{k}}|-\boldsymbol{k}\uparrow\rangle\right)\right.$$
$$\left. + a_{\boldsymbol{k}}\sqrt{P_{\boldsymbol{k}_{\text{excited}}}(T)}\left(c_{+\boldsymbol{k}}|+\boldsymbol{k}\uparrow\rangle + c_{-\boldsymbol{k}}|-\boldsymbol{k}\downarrow\rangle\right)\right\}$$
$$= \sum_{\boldsymbol{k}} b_{\boldsymbol{k}}\left(c_{+\boldsymbol{k}}|+\boldsymbol{k}\rangle + c_{-\boldsymbol{k}}|-\boldsymbol{k}\rangle\right). \qquad (28)$$

Inserting Eq. (28) into Eq. (27), and then we obtain

$$\left(H_0 + V_{\text{eff}}\right)\sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left|\boldsymbol{k}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle$$
$$= E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\sum_{\boldsymbol{k}} a_{\boldsymbol{k}}\left|\boldsymbol{k}_{\text{one,av.}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right\rangle\!\!\right\rangle. \qquad (29)$$

Considering the above orthogonality relation $\langle\boldsymbol{k}'|\boldsymbol{k}\rangle = \delta_{\boldsymbol{k},\boldsymbol{k}'}$, we thus find

$$\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\left(c_{+\boldsymbol{k}'}\langle+\boldsymbol{k}'| + c_{-\boldsymbol{k}'}\langle-\boldsymbol{k}'|\right)H_0\left(c_{+\boldsymbol{k}}|+\boldsymbol{k}\rangle + c_{-\boldsymbol{k}}|-\boldsymbol{k}\rangle\right)$$
$$= b_{\boldsymbol{k}}\varepsilon_{\boldsymbol{k}}, \qquad (30)$$

$$\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\left(c_{+\boldsymbol{k}'}\langle+\boldsymbol{k}'| + c_{-\boldsymbol{k}'}\langle-\boldsymbol{k}'|\right)E_{\text{one}}\left(c_{+\boldsymbol{k}}|+\boldsymbol{k}\rangle + c_{-\boldsymbol{k}}|-\boldsymbol{k}\rangle\right)$$
$$= b_{\boldsymbol{k}}E_{\text{one}}, \qquad (31)$$

and thus

$$b_{\boldsymbol{k}}\left(\varepsilon_{\boldsymbol{k}} - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\right)$$
$$= -\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\left(c_{+\boldsymbol{k}'}\langle+\boldsymbol{k}'| + c_{-\boldsymbol{k}'}\langle-\boldsymbol{k}'|\right)$$
$$\times|V_{\text{eff}}|\left(c_{+\boldsymbol{k}}|+\boldsymbol{k}\rangle + c_{-\boldsymbol{k}}|-\boldsymbol{k}\rangle\right). \qquad (32)$$

The quantity $\langle\boldsymbol{k}'|V_{\text{eff}}|\boldsymbol{k}\rangle$ denotes a one-electron scattering matrix element from a one-particle state $|\boldsymbol{k}\rangle$ to a one-particle state $|\boldsymbol{k}'\rangle$.

Up to now, we consider quite general case, we now specialize to the case where this scattering matrix element is one of the somewhat peculiar nature described above, that is, stabilized by V_{one} in a thin shell within the energy of ν_0 from the Fermi-surface, and zero elsewhere, as shown in Fig. 3. That is, we have

$$\langle+\boldsymbol{k}'|V_{\text{eff}}|+\boldsymbol{k}\rangle = \langle+\boldsymbol{k}'|V_{\text{eff}}|-\boldsymbol{k}\rangle = \langle-\boldsymbol{k}'|V_{\text{eff}}|+\boldsymbol{k}\rangle$$
$$= \langle-\boldsymbol{k}'|V_{\text{eff}}|-\boldsymbol{k}\rangle = -V_{\text{one}}, \ \left(|\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}| < \nu_0\right)$$
$$= 0, \ \left(|\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}| > \nu_0\right) \qquad (33)$$

$$\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\left(c_{+\boldsymbol{k}'}\langle+\boldsymbol{k}'| + c_{-\boldsymbol{k}'}\langle-\boldsymbol{k}'|\right)|V_{\text{eff}}|\left(c_{+\boldsymbol{k}}|+\boldsymbol{k}\rangle + c_{-\boldsymbol{k}}|-\boldsymbol{k}\rangle\right)$$
$$= -V_{\text{one}}\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\left(c_{+\boldsymbol{k}'}c_{+\boldsymbol{k}} + c_{-\boldsymbol{k}'}c_{-\boldsymbol{k}} + c_{+\boldsymbol{k}'}c_{-\boldsymbol{k}} + c_{-\boldsymbol{k}'}c_{+\boldsymbol{k}}\right)$$
$$\times\Theta(\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}})\Theta\left(\nu_0 - |\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}|\right)$$
$$= -V_{\text{one}}\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\left(1 + 2c_{+\boldsymbol{k}'}c_{-\boldsymbol{k}}\right)$$
$$\times\Theta(\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}})\Theta\left(\nu_0 - |\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}|\right)$$
$$= -2V_{\text{one}}\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\left(\frac{1}{2} + c_{+\boldsymbol{k}'}c_{-\boldsymbol{k}}\right)$$
$$\times\Theta(\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}})\Theta\left(\nu_0 - |\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}|\right)$$
$$= -2V_{\text{one}}f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'}\Theta(\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}})$$
$$\times\Theta\left(\nu_0 - |\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}|\right), \qquad (34)$$

where

$$f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) = \frac{1}{2} + c_{-\boldsymbol{k}} c_{+\boldsymbol{k}} = \frac{1}{2} + c_{-\boldsymbol{k}} \sqrt{1 - c_{-\boldsymbol{k}}^2}. \quad (35)$$

Thus, Eq. (32) takes the form

$$b_{\boldsymbol{k}} \left(\varepsilon_{\boldsymbol{k}} - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \right) = 2V_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \times \sum_{\boldsymbol{k}'} b_{\boldsymbol{k}'} \Theta(\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}) \Theta(\nu_0 - |\varepsilon_{\boldsymbol{k}} - \varepsilon_{\text{F}}|). \quad (36)$$

It is ensured by the first Θ-function that the single-particle energies ε are above the Fermi-surface, and by the second one that the effective stabilization only is operative in a thin shell within the energy of ν_0 from the Fermi-surface. Here, it is more convenient to go over to energy integrals instead of $\boldsymbol{k}$-space integrals, by introducing the density of states $N(\varepsilon)$, and viewing the expansion coefficients as a function of energy ε. By using a Dirac delta function $\delta(\varepsilon - \varepsilon_{\boldsymbol{k}})$, the density of states $N(\varepsilon)$ can be defined as

$$\begin{aligned} N(\varepsilon) &\approx \sum_{\boldsymbol{k}} \delta(\varepsilon - \varepsilon_{\boldsymbol{k}}) = N_{\text{electron}}(\varepsilon) \, [\text{states / electron}] \\ &= N_{\text{spin}}(\varepsilon) \, [\text{states / spin}] = 2N_{\text{spin}}(\varepsilon) \, [\text{states / electron}] \\ &= N_{\text{BCS}}(\varepsilon) \, [\text{states / spin}] = 2N_{\text{BCS}}(\varepsilon) \, [\text{states / electron}]. \end{aligned} \quad (37)$$

Since $b_{\boldsymbol{k}}$ only depends on $\boldsymbol{k}$ via $\varepsilon_{\boldsymbol{k}}$, we may write Eq. (36) as follows

$$\begin{aligned} & b(\varepsilon_{\boldsymbol{k}}) \left(\varepsilon_{\boldsymbol{k}} - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \right) \\ &= 2V_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \\ &\quad \times \int_{-\infty}^{\infty} d\varepsilon \sum_{\boldsymbol{k}'} b(\varepsilon) \delta(\varepsilon - \varepsilon_{\boldsymbol{k}'}) \Theta(\varepsilon - \varepsilon_{\text{F}}) \Theta(\nu_0 - |\varepsilon - \varepsilon_{\text{F}}|) \\ &= 2V_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \\ &\quad \times \int_{-\infty}^{\infty} d\varepsilon b(\varepsilon) N(\varepsilon) \Theta(\varepsilon - \varepsilon_{\text{F}}) \Theta(\nu_0 - |\varepsilon - \varepsilon_{\text{F}}|) \end{aligned} \quad (38)$$

We then obtain, upon remaining variables

$$\begin{aligned} & b(\varepsilon) \left(\varepsilon_{\boldsymbol{k}} - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \right) \\ &= 2V_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \int_{\varepsilon_{\text{F}}}^{\varepsilon_{\text{F}} + \nu_0} d\varepsilon' N(\varepsilon') b(\varepsilon'). \end{aligned} \quad (39)$$

The right hand side of Eq. (39) is independent of ε, and hence $b(\varepsilon)$ must have the form

$$b(\varepsilon) = \frac{C}{\varepsilon - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})}, \quad (40)$$

where C is some normalization constant. Inserting $b(\varepsilon)$ in Eq. (40) into Eq. (39), we find

$$\begin{aligned} 1 &= 2V_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \int_{\varepsilon_{\text{F}}}^{\varepsilon_{\text{F}} + \nu_0} \frac{N(\varepsilon') d\varepsilon'}{\varepsilon' - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})} \\ &= 2V_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \int_{\varepsilon_{\text{F}}}^{\varepsilon_{\text{F}} + \nu_0} \frac{N_{\text{electron}}(\varepsilon') d\varepsilon'}{\varepsilon' - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})}. \end{aligned} \quad (41)$$

Considering the fact that we are looking into a thin shell around the Fermi-surface, and assuming that the density of states $N_{\text{electron}}(\varepsilon)$ varies slowly, we may simply replace it by its value on the Fermi surface, $N_{\text{electron}}(\varepsilon_{\text{F}})$. Introducing the dimensionless electron–phonon coupling constant $\lambda_{\text{one}} (\equiv 2V_{\text{one}} N_{\text{electron}}(\varepsilon_{\text{F}}) = 4V_{\text{one}} N_{\text{spin}}(\varepsilon_{\text{F}}) = 4V_{\text{one}} N_{\text{BCS}}(\varepsilon_{\text{F}}))$, we obtain

$$\begin{aligned} 1 &= 4V_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) N_{\text{BCS}}(\varepsilon_{\text{F}}) \\ &\quad \times \int_{\varepsilon_{\text{F}}}^{\varepsilon_{\text{F}} + \nu_0} \frac{d\varepsilon'}{\varepsilon' - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})} \\ &= \lambda_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) \ln \left\{ \frac{(\varepsilon_{\text{F}} + \nu_0) - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})}{\varepsilon_{\text{F}} - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})} \right\}. \end{aligned} \quad (42)$$

Then the energy difference between the states of one non-interacting particle on the Fermi-surface (ε_{F}), and the exact energy eigenvalue ($E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})$), is introduced, i.e., $\Delta_{\text{vib,one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) = \varepsilon_{\text{F}} - E_{\text{one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})$. In terms of this variable, Eq. (42) may be written

$$\frac{1}{\lambda_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})} = \ln \left\{ 1 + \frac{\nu_0}{\Delta_{\text{vib,one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})} \right\}, \quad (43)$$

and thus

$$\begin{aligned} \Delta_{\text{vib,one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) &= \frac{\nu_0}{e^{1/\{\lambda_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\}} - 1} \\ &\approx \nu_0 e^{-1/\{\lambda_{\text{one}} f_{\text{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\}}, \end{aligned} \quad (44)$$

where the last approximation follows if $\lambda_{\text{one}} << 1$ (if the effective one electron stabilization is small). Similar discussions can be made in other electrons at the Fermi-surface. The $\Delta_{\text{vib,one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})$ denotes the stabilization

energy of independent one electron as a consequence of the electron–phonon interactions.

3.2 Comparison of the One-Electron Theory with the Conventional Two-Electrons BCS Theory under the Applied Electric Field in Solids

Vibronic stabilization energy of independent two electrons in the one-electron theory ($\Delta_{\mathrm{vib,pair,one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})$) is denoted as

$$\begin{aligned}&\Delta_{\mathrm{vib,pair,one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}}) = 2\Delta_{\mathrm{vib,one}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\\ &= 2\nu_0 e^{-1/\lambda_{\mathrm{one}}} - 2\nu_0\left(e^{-1/\lambda_{\mathrm{one}}} - e^{-1/\{\lambda_{\mathrm{one}} f_{\mathrm{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\}}\right)\\ &= 2\nu_0 e^{-1/\{\lambda_{\mathrm{one}} f_{\mathrm{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\}}. \qquad (45)\end{aligned}$$

The electronic energy level itself for the electronic state is stabilized by $2\nu_0 \exp(-1/\lambda_{\mathrm{one}})$ with the kinetic energy of supercurrent $2\nu_0\left(\exp(-1/\lambda_{\mathrm{one}}) - \exp\left(-1/\{\lambda_{\mathrm{one}} f_{\mathrm{Bose}}(c_{+\boldsymbol{k}}, c_{-\boldsymbol{k}})\}\right)\right)$.

4. The Origin of the Ampère's Law

4.1 Theoretical Background

In this article, we consider the molecular systems for mathematical simplicity. On the other hand, we can easily apply this discussion to the case in the solids.

The wave function for an electron occupying the highest occupied crystal orbital (HOCO) in a material under the external applied field ($x_{\mathrm{in}} = B_{\mathrm{in}}$ or E_{in}) can be expressed as

$$\begin{aligned}&\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)\right\rangle\!\!\left\rangle((B_{\mathrm{out}}, B_{\mathrm{in}}); (E_{\mathrm{out}}, E_{\mathrm{in}}); B_{\boldsymbol{k}_{\mathrm{HOCO}}}; I_{\boldsymbol{k}_{\mathrm{HOCO}}})\right\rangle\!\!\left\rangle\\ &= \sqrt{P_{\mathrm{ground}}(T)}\left|\boldsymbol{k}_{\mathrm{HOCO,ground},0}(x_{\mathrm{in}})\right\rangle\!\!\left\rangle\right.\\ &+\sqrt{P_{\mathrm{excited}}(T)}\left|\boldsymbol{k}_{\mathrm{HOCO,excited},0}(x_{\mathrm{in}})\right\rangle\!\!\left\rangle\right., \qquad (46)\end{aligned}$$

where

$$\begin{aligned}&\left|\boldsymbol{k}_{\mathrm{HOCO,excited},0}(x_{\mathrm{in}})\right\rangle\!\!\left\rangle\right.\\ &= c_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}})\left|+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow\right\rangle\\ &+c_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}})\left|-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow\right\rangle, \qquad (47)\end{aligned}$$

$$\begin{aligned}&\left|\boldsymbol{k}_{\mathrm{HOCO,ground},0}(x_{\mathrm{in}})\right\rangle\!\!\left\rangle\right.\\ &= c_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}})\left|-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow\right\rangle\\ &+c_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}})\left|+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow\right\rangle, \qquad (48)\end{aligned}$$

$$P_{\mathrm{ground}}(T) + P_{\mathrm{excited}}(T) = 1, \qquad (49)$$

$$c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}}) + c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}}) = 1, \qquad (50)$$

$$c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}}) + c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}}) = 1. \qquad (51)$$

The magnetic field ($B_{\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}})(= B_{\mathrm{in}})$) at the condition of the external applied field x_{out} and the field felt by an electron x_{in} can be expressed as

$$\begin{aligned}&B_{\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}})\\ &= B_{\boldsymbol{k}_{\mathrm{HOCO}}\uparrow}(x_{\mathrm{out}}, x_{\mathrm{in}}) - B_{\boldsymbol{k}_{\mathrm{HOCO}}\downarrow}(x_{\mathrm{out}}, x_{\mathrm{in}}), \qquad (52)\end{aligned}$$

where

$$\begin{aligned}&B_{\boldsymbol{k}_{\mathrm{HOCO}}\uparrow}(x_{\mathrm{out}}, x_{\mathrm{in}})\\ &= P_{\mathrm{excited}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}})\\ &+P_{\mathrm{ground}}(T)c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}}), \qquad (53)\end{aligned}$$

$$\begin{aligned}&B_{\boldsymbol{k}_{\mathrm{HOCO}}\downarrow}(x_{\mathrm{out}}, x_{\mathrm{in}})\\ &= P_{\mathrm{excited}}(T)c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}})\\ &+P_{\mathrm{ground}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}}). \qquad (54)\end{aligned}$$

The electric field ($I_{\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}})(= E_{\mathrm{in}})$) at the condition of the external applied field x_{out} and the field felt by an electron x_{in} can be expressed as

$$\begin{aligned}&I_{\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}})\\ &= I_{+\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}}) - I_{-\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}}), \qquad (55)\end{aligned}$$

$$\begin{aligned}&I_{+\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}})\\ &= P_{\mathrm{excited}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}})\\ &+P_{\mathrm{ground}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}}), \qquad (56)\end{aligned}$$

$$\begin{aligned}&I_{-\boldsymbol{k}_{\mathrm{HOCO}}}(x_{\mathrm{out}}, x_{\mathrm{in}})\\ &= P_{\mathrm{excited}}(T)c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}})\\ &+P_{\mathrm{ground}}(T)c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,x_{\mathrm{in}}}(x_{\mathrm{out}} - x_{\mathrm{in}}). \qquad (57)\end{aligned}$$

Let us look into the energy levels for various electronic states when the applied field increases from 0 to x_{out} at 0 K in superconductor, in which the HOCO is partially occupied by an electron. The stabilization energy as a consequence of the electron–phonon interactions can be expressed as

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$$E_{\mathrm{SC,electronic}}(x_{\mathrm{out}}, x_{\mathrm{in}}) - E_{\mathrm{NM,electronic}}(0,0) = -2V_{\mathrm{one}} f_{\mathrm{Bose},0}(x_{\mathrm{in}}), \quad (58)$$

where the $-2V_{\mathrm{one}}$ denotes the stabilization energy for the electron–phonon interactions between an electron occupying the HOCO and the vibronically active modes (Fig. 4).

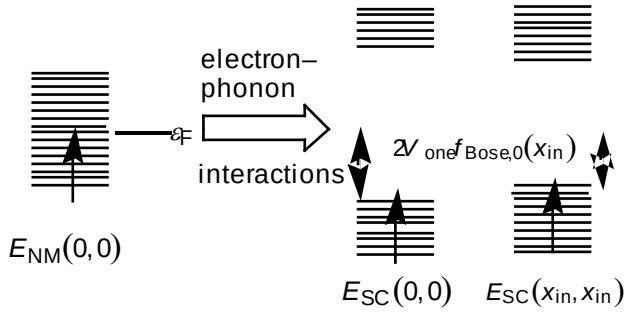

Fig. 4. Stabilization energy as a consequence of the electron–phonon interactions as a function of the external applied field.

The $f_{\mathrm{Bose},E_{\mathrm{in}}}(0) (= f_{\mathrm{Bose},0}(E_{\mathrm{in}}))$ denotes the ratio of the bosonic property under the internal field x_{in} ($c_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}}) = c_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}}) = c_{+\boldsymbol{k}_{\mathrm{HOCO}},0}(x_{\mathrm{in}})$ and $c_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}}) = c_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}}) = c_{-\boldsymbol{k}_{\mathrm{HOCO}},0}(x_{\mathrm{in}})$), and can be estimated as

$$f_{\mathrm{Bose},0}(x_{\mathrm{in}}) = f_{\mathrm{Bose},x_{\mathrm{in}}}(0) = \frac{1}{2} + c_{-\boldsymbol{k}_{\mathrm{HOCO}},0}(x_{\mathrm{in}})\sqrt{1 - c^2_{-\boldsymbol{k}_{\mathrm{HOCO}},0}(x_{\mathrm{in}})}. \quad (59)$$

The $f_{\mathrm{Bose},B_{\mathrm{in}}}(0) (= f_{\mathrm{Bose},0}(B_{\mathrm{in}}))$ denotes the ratio of the bosonic property under the internal field x_{in} ($c_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}}) = c_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}}) = c_{\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(x_{\mathrm{in}})$ and $c_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}}) = c_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}}) = c_{\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}})$), and can be estimated as

$$f_{\mathrm{Bose},0}(x_{\mathrm{in}}) = f_{\mathrm{Bose},x_{\mathrm{in}}}(0) = \frac{1}{2} + c_{\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}})\sqrt{1 - c^2_{\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(x_{\mathrm{in}})}. \quad (60)$$

4.2 New Interpretation of the Ampère's Law in the Normal Metallic States

Let us next apply the Higgs mechanism to the Ampère's law in the normal metallic states. Let us next consider the superconductor, the critical electric field of which is E_{c}. Below T_{C}, the bosonic Cooper pairs are in the superconducting states. We consider the case where the HOCO is partially occupied by an electron. We consider that the electric field is quantized by $\Delta E_{\mathrm{unit}} (= E_{\mathrm{c}} / n_{\mathrm{c}})$. The n_{c} value is very large and the quantization value of $E_{\mathrm{c}} / n_{\mathrm{c}}$ is very small ($E_{\mathrm{c}} / n_{\mathrm{c}} \approx 0$). That is, the jth quantized electric field E_j with respect to the zero electric field can be defined as

$$E_j = j\Delta E_{\mathrm{unit}}. \quad (61)$$

The ratio of the bosonic property under the internal electric field E_{in} with respect to the ground state for the zero electric field can be denoted as $f_{\mathrm{Bose},0}(E_{\mathrm{in}})$. We define the electronic state $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)\right\rangle\!\!\left\rangle((B_{\mathrm{out}}, B_{\mathrm{in}}); (E_{\mathrm{out}}, E_{\mathrm{in}}); B_{\boldsymbol{k}_{\mathrm{HOCO}}}; I_{\boldsymbol{k}_{\mathrm{HOCO}}})\right\rangle\!\!\left\rangle$, where the E_{out} denotes the induced electric field applied to the specimen, the E_{in} the induced electric field felt by the electron, the $B_{\boldsymbol{k}_{\mathrm{HOCO}}}$ the induced magnetic moment from the electron (the induced magnetic field $B_{\mathrm{induced},\boldsymbol{k}_{\mathrm{HOCO}}}$ or the change of the spin magnetic moment of an electron $\sigma_{\mathrm{spin},\boldsymbol{k}_{\mathrm{HOCO}}}$ *from the each ground state*), and the $I_{\boldsymbol{k}_{\mathrm{HOCO}}}$ the induced electric moment of an electron (canonical electric momentum $\boldsymbol{p}_{\mathrm{canonical},\boldsymbol{k}_{\mathrm{HOCO}}}$ or the electric momentum of an electron $\boldsymbol{v}_{\mathrm{em},\boldsymbol{k}_{\mathrm{HOCO}}}$).

Without any external applied electric field ($j = 0$; $E_{\mathrm{out}} = E_{\mathrm{in}} = 0$), the ratio of the bosonic property under the internal electric field 0 can be estimated to be $f_{\mathrm{Bose},0}(0) = 1$. Therefore, the electronic state pairing of an electron behaves as a boson,

$$f_{\mathrm{Bose},0}(0) = 1. \quad (62)$$

In such a case ($c_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(0) = c_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(0) = c_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(0) = c_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(0) = 1/\sqrt{2}$), there is no induced current and the magnetic fields, as expected,

$$B_{\boldsymbol{k}_{\mathrm{HOCO}}}(0,0) = B_{\boldsymbol{k}_{\mathrm{HOCO}}\uparrow}(0,0) - B_{\boldsymbol{k}_{\mathrm{HOCO}}\downarrow}(0,0) = \left\{P_{\mathrm{excited}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(0) + P_{\mathrm{ground}}(T)c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(0)\right\} - \left\{P_{\mathrm{excited}}(T)c^2_{-\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(0) + P_{\mathrm{ground}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(0)\right\} = 0, \quad (63)$$

$$I_{\boldsymbol{k}_{\mathrm{HOCO}}}(0,0) = I_{+\boldsymbol{k}_{\mathrm{HOCO}}}(0,0) - I_{-\boldsymbol{k}_{\mathrm{HOCO}}}(0,0) = \left\{P_{\mathrm{excited}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\uparrow,0}(0) + P_{\mathrm{ground}}(T)c^2_{+\boldsymbol{k}_{\mathrm{HOCO}}\downarrow,0}(0)\right\}$$

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$$-\left\{P_{\text{excited}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(0)+P_{\text{ground}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(0)\right\}$$
$$=0. \quad (64)$$

This can be in agreement with the fact that charges at rest feel no magnetic forces and create no magnetic fields. This is the bosonic ground normal metallic state for $j=0$ ($\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(0,0);0;0)\right\rangle\!\rangle$) (Fig. 5 (a)). It should be noted that the electronic states are in the ground normal metallic states when all the $\boldsymbol{p}_{\text{canonical}}$, $\boldsymbol{v}_{\text{em}}$, σ_{spin}, and B_{induced} values are 0 ($\boldsymbol{p}_{\text{canonical}}=0$, $\boldsymbol{v}_{\text{em}}=0$, $\sigma_{\text{spin}}=0$, and $B_{\text{induced}}=0$), and are in the excited normal metallic states when the $\boldsymbol{p}_{\text{canonical}}$, $\boldsymbol{v}_{\text{em}}$, σ_{spin}, or B_{induced} values are not 0 ($\boldsymbol{p}_{\text{canonical}}\neq 0$, $\boldsymbol{v}_{\text{em}}\neq 0$, $\sigma_{\text{spin}}\neq 0$, or $B_{\text{induced}}\neq 0$).

When the electric field ($I_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)=\Delta E_{\text{unit}}$) is applied, a Nambu–Goldstone boson formed by the fluctuation of the electronic state pairing of an electron $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(0,0);0;0)\right\rangle\!\rangle$ is absorbed by a photon (electric field) (Fig. 5 (b)). Therefore, a photon (electric field) has finite mass as a consequence of interaction with the Nambu–Goldstone boson formed by the fluctuation of the bosonic electronic state pairing of an electron. In such a case, the $I_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)$ and $B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)$ values for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0)\right\rangle\!\rangle$ state (Fig. 5 (b)) can be estimated as

$$\begin{aligned}I_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)=&\left\{P_{\text{excited}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right.\\&\left.+P_{\text{ground}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right\}\\&-\left\{P_{\text{excited}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right.\\&\left.+P_{\text{ground}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right\}\\&=0, \quad (65)\end{aligned}$$

and thus

$$\begin{aligned}B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)=&\left\{P_{\text{excited}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right.\\&\left.+P_{\text{ground}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right\}\\&-\left\{P_{\text{excited}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right.\\&\left.+P_{\text{ground}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right\}\\&=2P_{\text{excited}}(T)\left\{c^2_{+\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right.\\&\left.-c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right\}\\&=B_{\text{induced},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)=\Delta B_{\text{unit}}. \quad (66)\end{aligned}$$

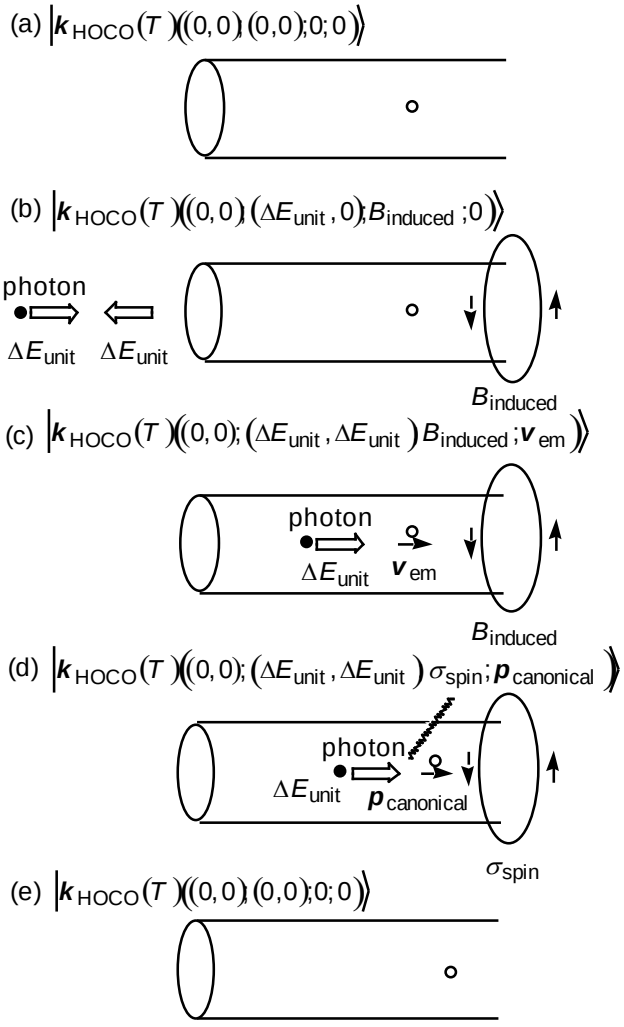

Fig. 5 The electronic states between $j=0$ and $j=1$ in normal metals.

Soon after the external electric field is applied, the momentum of the bosonic electronic state pairing of an electron cannot be changed but the magnetic field can be induced. It should be noted that the magnetic field $B_{\text{induced},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)$ is induced ($B_{\text{induced}}\neq 0$) but the spin magnetic moment of an electron with opened-shell electronic structure is not changed ($\sigma_{\text{spin}}=0$). This is very similar to the diamagnetic currents in the superconductivity in that the supercurrents are induced ($\boldsymbol{v}_{\text{em}}\neq 0$) but the total canonical momentum is zero ($\boldsymbol{p}_{\text{canonical}}=0$). The magnetic field is induced not because of the change of the each element of the spin magnetic moment of an electron ($\sigma_{\text{spin}}=0$) (similar to

the $\boldsymbol{p}_{\text{canonical}}$ in the superconducting states) but because of the change of the total magnetic momentum as a whole ($B_{\text{induced}} \neq 0$) (similar to the $\boldsymbol{v}_{\text{em}}$ in the superconducting states).

On the other hand, such excited bosonic electronic state pairing of an electron with the induced magnetic fields $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0\right)\right\rangle$ can be immediately destroyed because the initially applied electric field penetrates into the normal metallic specimen, and the electronic state becomes another bosonic excited normal metallic state for $j=0$ ($\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle\!\rangle$) (Fig. 5 (c)). In such a case, the $B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ and $I_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ values for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle\!\rangle$ state can be estimated as

$$\begin{aligned}
B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) &= B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0) \\
&= \left\{P_{\text{excited}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right. \\
&\quad \left. + P_{\text{ground}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right\} \\
&\quad - \left\{P_{\text{excited}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right. \\
&\quad \left. + P_{\text{ground}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right\} \\
&= 2P_{\text{excited}}(T)\left\{c^2_{+\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right. \\
&\quad \left. - c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right\} \\
&= B_{\text{induced},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0) = \Delta B_{\text{unit}}, \qquad (67)
\end{aligned}$$

$$\begin{aligned}
&I_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) \\
&\quad = \boldsymbol{v}_{\text{em},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) = \Delta E_{\text{unit}}. \qquad (68)
\end{aligned}$$

In the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle\!\rangle$ state, an electron receives the applied electric field ΔE_{unit}, and thus the superconducting current can be induced, and thus there is kinetic energy ($E_{\text{kinetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$) of the supercurrent.

That is, the expelling energy of the initially applied electric field ΔE_{unit} for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0\right)\right\rangle$ state is converted to the kinetic energy of the supercurrent for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle\!\rangle$ state. Both the supercurrent ($\boldsymbol{v}_{\text{em},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$) and the magnetic field ($B_{\text{induced},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0)$) can be induced under the condition of the opened-shell electronic structure with zero spin magnetic field and zero canonical momentum ($\sigma_{\text{spin}}=0$; $\boldsymbol{p}_{\text{canonical}}=0$). This is the origin of the Ampère's law.

On the other hand, such excited bosonic normal metallic states with supercurrents ($\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle\!\rangle$) can be immediately destroyed because of the unstable opened-shell electronic states, and the electronic state becomes another excited fermionic normal metallic state for $j=0$ ($\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\rangle$) (Fig. 5 (d)). It should be noted that the electronic $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\rangle$ state is now somewhat fermionic because the $\boldsymbol{p}_{\text{canonical}}$ value is not 0. In other words, the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\rangle$ state is closely related to the normal conducting states in that the normal metallic current with $\boldsymbol{p}_{\text{canonical}} \neq 0$ and $\boldsymbol{v}_{\text{em}}=0$ is induced by the applied electric field. In such a case, the $B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ and $I_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ values for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\rangle$ state can be estimated as

$$\begin{aligned}
B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) &= B_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0) \\
&= \left\{P_{\text{excited}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right. \\
&\quad \left. + P_{\text{ground}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right\} \\
&\quad - \left\{P_{\text{excited}}(T)c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right. \\
&\quad \left. + P_{\text{ground}}(T)c^2_{+\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right\} \\
&= 2P_{\text{excited}}(T)\left\{c^2_{+\boldsymbol{k}_{\text{HOCO}}\uparrow,0}(\Delta E_{\text{unit}})\right. \\
&\quad \left. - c^2_{-\boldsymbol{k}_{\text{HOCO}}\downarrow,0}(\Delta E_{\text{unit}})\right\} \\
&= \sigma_{\text{spin},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) = \Delta B_{\text{unit}}, \qquad (69)
\end{aligned}$$

$$\begin{aligned}
&I_{\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) \\
&\quad = \boldsymbol{p}_{\text{canonical},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) = \Delta E_{\text{unit}}. \qquad (70)
\end{aligned}$$

Such excited fermionic normal metallic states with currents and the induced magnetic field ($\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle\!\rangle$)

can be immediately destroyed because of the unstable opened-shell electronic states, and the induced current and the magnetic field can be immediately destroyed. Therefore, the electronic state tries to go back to the original ground bosonic metallic state for $j=0$ ($\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(0,0);0;0)\right\rangle$). In such a case, the $B_{\boldsymbol{k}_{\mathrm{HOCO}}}(0,0)$ and $I_{\boldsymbol{k}_{\mathrm{HOCO}}}(0,0)$ values for the $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(0,0);0;0)\right\rangle$ state can be estimated as Eqs. (63) and (64), respectively.

The ratio of the bosonic property $f_{\mathrm{Bose},\Delta E_{\mathrm{unit}}}(0)$ under the internal magnetic field ΔE_{unit} can be estimated as

$$f_{\mathrm{Bose},\Delta E_{\mathrm{unit}}}(0) = \frac{1}{2}+c_{-\boldsymbol{k}_{\mathrm{HOCO}},0}(\Delta E_{\mathrm{unit}})\sqrt{1-c^2_{-\boldsymbol{k}_{\mathrm{HOCO}},0}(\Delta E_{\mathrm{unit}})} < f_{\mathrm{Bose},0}(0)=1. \quad (71)$$

The $f_{\mathrm{Bose},\Delta E_{\mathrm{unit}}}(0)$ value is smaller than the $f_{\mathrm{Bose},0}(0)$ value. It should be noted that the $f_{\mathrm{Bose},E_{\mathrm{in}}}(0)$ value decreases with an increase in the $|E_{\mathrm{in}}|$ value. That is, the bosonic and fermionic properties decrease and increase with an increase in the $|E_{\mathrm{in}}|$ value, respectively.

In summary, because of the very large stabilization energy ($V_{\mathrm{kin,Fermi},\boldsymbol{k}_{\mathrm{HOCO}}\sigma}(0)\approx 35$ eV) for the Bose–Einstein condensation ($\boldsymbol{p}_{\mathrm{canonical}}=0$; $V_{\mathrm{kin,Bose},\boldsymbol{k}_{\mathrm{HOCO}}\sigma}(0)=0$ eV) [1–7], the electric and magnetic momentum of a bosonic electronic state pairing of an electron cannot be changed but the magnetic field can be induced soon after the electric field is induced. Therefore, the electronic state becomes $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},0);B_{\mathrm{induced}};0)\right\rangle$. On the other hand, such excited bosonic supercurrent states with the induced magnetic fields $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},0);B_{\mathrm{induced}};0)\right\rangle$ can be immediately destroyed because the applied electric field penetrates into the normal metallic specimen, and the electronic state becomes another bosonic excited supercurrent state for $j=0$ ($\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$). In the $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state, the supercurrent can be induced, and thus there is kinetic energy ($E_{\mathrm{kinetic}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})$). That is, the expelling energy of the initially applied electric field ΔE_{unit} for the $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},0);B_{\mathrm{induced}};0)\right\rangle$ state is converted to the kinetic energy of the supercurrent for the $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ state. Both the supercurrent ($\boldsymbol{v}_{\mathrm{em},\boldsymbol{k}_{\mathrm{HOCO}}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})\neq 0$) and the magnetic field ($B_{\mathrm{induced},\boldsymbol{k}_{\mathrm{HOCO}}}(\Delta E_{\mathrm{unit}},0)\neq 0$) can be induced under the condition of the opened-shell electronic structure with zero spin magnetic momentum and zero canonical momentum in a bosonic electronic state pairing of an electron ($\sigma_{\mathrm{spin}}=0$; $\boldsymbol{p}_{\mathrm{canonical}}=0$). This is the origin of the Ampère's law. On the other hand, such excited bosonic states with supercurrents $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}})\right\rangle$ can be immediately destroyed because of the unstable opened-shell electronic states, and the electronic state becomes another excited fermionic normal metallic state for $j=0$ ($\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});\sigma_{\mathrm{spin}};\boldsymbol{p}_{\mathrm{canonical}})\right\rangle$). The excited fermionic normal metallic $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});\sigma_{\mathrm{spin}};\boldsymbol{p}_{\mathrm{canonical}})\right\rangle$ state is very unstable and try to go back to the original ground bosonic metallic state for $j=0$ ($\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(0,0);0;0)\right\rangle$), and the induced electrical current and the induced magnetic field can be immediately dissipated.

4.3 Energy Levels for Various Electronic States

Let us look into the energy levels for various electronic states when the applied electric field (E_{out}) increases from 0 to ΔE_{unit} at 0 K in superconductor, in which the HOCO is partially occupied by an electron. The total energy $E_{\mathrm{total}}(x_{\mathrm{out}},x_{\mathrm{in}})$ for various electronic states with respect to the Fermi level before electron–phonon interactions at 0 K and $x_{\mathrm{out}}=x_{\mathrm{in}}=0$ (Fig. 4) can be expressed as

$$E_{\mathrm{total}}(x_{\mathrm{out}},x_{\mathrm{in}}) = E_{\mathrm{SC}}(x_{\mathrm{out}},x_{\mathrm{in}})-E_{\mathrm{NM}}(0,0) = E_{\mathrm{electronic}}(x_{\mathrm{out}},x_{\mathrm{in}})+E_{\mathrm{magnetic}}(x_{\mathrm{out}},x_{\mathrm{in}}). \quad (72)$$

At $E_{\mathrm{out}}=E_{\mathrm{in}}=0$, the electronic state is in the ground normal metallic $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(0,0);0;0)\right\rangle$ state for $j=0$. The electronic and magnetic energies for the $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)((0,0);(0,0);0;0)\right\rangle$ state can be expressed as

$$E_{\mathrm{electronic}}(0,0)=-2V_{\mathrm{one}}f_{\mathrm{Bose},0}(0)=-2V_{\mathrm{one}}, \quad (73)$$

$$E_{\mathrm{magnetic}}(0,0)=0. \quad (74)$$

The $E_{\text{electronic}}(\Delta E_{\text{unit}},0)$ value for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0)\right\rangle$ state can be estimated as

$$\begin{aligned}&E_{\text{electronic}}(\Delta E_{\text{unit}},0)\\&=-2V_{\text{one}}f_{\text{Bose},0}(0)+E_{I_{\boldsymbol{k}_{\text{HOCO}}}}(\Delta E_{\text{unit}},0)\\&=-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}}),\end{aligned}\quad(75)$$

where the $E_{I_{\boldsymbol{k}_{\text{HOCO}}}}(\Delta E_{\text{unit}},0)$ value denotes the expelling energy of the applied electric field, and is estimated as

$$\begin{aligned}&E_{I_{\boldsymbol{k}_{\text{HOCO}}}}(\Delta E_{\text{unit}},0)\\&=2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)\\&=2V_{\text{one}}\left(1-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)\end{aligned}\quad(76)$$

Furthermore, we must consider the magnetic energy ($E_{\text{magnetic}}(\Delta E_{\text{unit}},0)$) for the induced magnetic field ΔB_{unit} as a consequence of the applied electric field ΔE_{unit},

$$E_{\text{magnetic}}(\Delta E_{\text{unit}},0)=E_{B_{\text{induced}}}(\Delta E_{\text{unit}},0),\quad(77)$$

The total energy level for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0)\right\rangle$ state can be estimated as

$$\begin{aligned}&E_{\text{total}}(\Delta E_{\text{unit}},0)\\&=E_{\text{electronic}}(\Delta E_{\text{unit}},0)+E_{\text{magnetic}}(\Delta E_{\text{unit}},0)\\&=-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}})+E_{B_{\text{induced}}}(\Delta E_{\text{unit}},0).\end{aligned}\quad(78)$$

We can consider from Eqs. (75)–(78) that the energy for the excited normal metallic $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0)\right\rangle$ state is $-2V_{\text{one}}$ with the expelling energy of the applied electric field $2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)$ and the energy of the induced magnetic field $E_{\text{magnetic}}(\Delta E_{\text{unit}},0)$, and thus the total energy for the bosonic excited normal metallic $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0)\right\rangle$ state is $-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}})+E_{B_{\text{induced}}}(\Delta E_{\text{unit}},0)$. In other words, the energy for the initially applied electric field ΔE_{unit} is converted to the expelling energy of the applied electric field $2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)$ and the energy of the induced magnetic field $E_{B_{\text{induced}}}(\Delta E_{\text{unit}},0)$.

The $E_{\text{electronic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ value for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}})\right\rangle$ state can be estimated as

$$\begin{aligned}&E_{\text{electronic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=-2V_{\text{one}}f_{\text{Bose},0}(0)+E_{\boldsymbol{v}_{\text{em}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}}),\end{aligned}\quad(79)$$

where the $E_{\boldsymbol{v}_{\text{em}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ value denotes the kinetic energy of the supercurrent, and is estimated as

$$\begin{aligned}&E_{\boldsymbol{v}_{\text{em}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)\\&=2V_{\text{one}}\left(1-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right).\end{aligned}\quad(80)$$

Furthermore, we must consider the magnetic energy ($E_{\text{magnetic}}(\Delta E_{\text{unit}},0)\left(=E_{\text{magnetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\right)$) for the induced magnetic field ΔB_{unit} as a consequence of the applied electric field ΔE_{unit},

$$\begin{aligned}E_{\text{magnetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})&=E_{\text{magnetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=E_{B_{\text{induced}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}).\end{aligned}\quad(81)$$

The total energy level for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}})\right\rangle$ state can be estimated as

$$\begin{aligned}&E_{\text{total}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=E_{\text{electronic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})+E_{\text{magnetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}})+E_{B_{\text{induced}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}).\end{aligned}\quad(82)$$

We can consider from Eqs. (79)–(82) that the energy level for the excited normal metallic $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}})\right\rangle$ state is $-2V_{\text{one}}$ with the kinetic energy of the supercurrent $2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)$ and the energy of the induced magnetic field $E_{B_{\boldsymbol{k}_{\text{HOCO}}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$, and thus the total energy for the bosonic excited normal metallic $\left|\boldsymbol{k}_{\text{HOCO}}(T)((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}})\right\rangle$ state is $-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}})+E_{B_{\boldsymbol{k}_{\text{HOCO}}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$. In

other words, the energy for the initially applied electric field ΔE_{unit} is converted to the kinetic energy of the supercurrent $2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)$ and the energy of the induced magnetic field $E_{B_{\text{induced}}}(\Delta E_{\text{unit}},0)$.

The $E_{\text{electronic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ value for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state can be estimated as

$$\begin{aligned}&E_{\text{electronic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=-2V_{\text{one}}f_{\text{Bose},0}(0)+E_{\boldsymbol{p}_{\text{canonical}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}}).\end{aligned}\quad(83)$$

where the $E_{\boldsymbol{p}_{\text{canonical}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$ value denotes the kinetic energy of the normal current, and is estimated as

$$\begin{aligned}&E_{\boldsymbol{p}_{\text{canonical}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)\\&=2V_{\text{one}}\left(1-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)\end{aligned}\quad(84)$$

Furthermore, we must consider the magnetic energy ($E_{\text{magnetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$) as a consequence of the induced spin magnetic energy $E_{\sigma_{\text{spin,HOMO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$,

$$\begin{aligned}&E_{\text{magnetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=E_{\sigma_{\text{spin,HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}).\end{aligned}\quad(85)$$

The total energy level for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state can be estimated as

$$\begin{aligned}&E_{\text{total}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=E_{\text{electronic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})+E_{\text{magnetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})\\&=-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}})+E_{\sigma_{\text{spin,HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}).\end{aligned}\quad(86)$$

We can consider from Eqs. (83)–(86) that the energy level for the excited normal metallic $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state is $-2V_{\text{one}}$ with the kinetic energy of the normal current $2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)$ and the energy of the induced magnetic field $E_{B_{\boldsymbol{k}_{\text{HOCO}}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$, and thus the total energy for the bosonic excited normal metallic $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((\Delta B_{\text{unit}},0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});\sigma_{\text{spin}};\boldsymbol{p}_{\text{canonical}}\right)\right\rangle$ state is $-2V_{\text{one}}f_{\text{Bose},0}(\Delta E_{\text{unit}})+E_{\sigma_{\text{spin,HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$. In other words, the energy for the initially applied electric field ΔE_{unit} is converted to the kinetic energy of the normal current $2V_{\text{one}}\left(f_{\text{Bose},0}(0)-f_{\text{Bose},0}(\Delta E_{\text{unit}})\right)$ and the energy of the induced spin magnetic moment $E_{\sigma_{\text{spin,HOCO}}}(\Delta E_{\text{unit}},0)$.

On the other hand, such excited states with currents can be immediately destroyed because of the unstable opened-shell electronic states, and the induced current and magnetic field can be immediately destroyed, and the electronic state goes back to the original ground bosonic metallic state for $j=0$. Therefore, we can consider that the energy for the initially applied electric field ΔE_{unit} is converted to photon emission energy and the Joule's heats.

4.4 Ampère's Law in the Two-Electrons Systems in Superconductivity

Similar discussions can be made in the two-electrons systems in superconductivity (Fig. 6). Because of the very large stabilization energy ($2V_{\text{kin,Fermi},\boldsymbol{k}_{\text{HOCO}}\sigma}(0)\approx 70$ eV) for the Bose–Einstein condensation ($\boldsymbol{p}_{\text{canonical}}=0$; $V_{\text{kin,Bose},\boldsymbol{k}_{\text{HOCO}}\sigma}(0)=0$ eV), the electric and magnetic momentum of a bosonic Cooper pair cannot be changed but the magnetic field can be induced soon after the electric field is applied. Therefore, the electronic state becomes $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0\right)\right\rangle$ (Fig. 6 (b)). On the other hand, such excited bosonic supercurrent states with the induced magnetic fields $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0\right)\right\rangle$ can be immediately destroyed because the applied electric field penetrates into the superconducting specimen, and the electronic state becomes another bosonic excited supercurrent state for $j=0$ ($\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$) (Fig. 6 (c)). In the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$ state, the supercurrent can be induced, and thus there is kinetic energy ($E_{\text{kinetic}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}})$). That is, the expelling energy of the initially applied electric field ΔE_{unit} for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0\right)\right\rangle$ state is converted to the kinetic energy of the supercurrent for the $\left|\boldsymbol{k}_{\text{HOCO}}(T)\left((0,0);(\Delta E_{\text{unit}},\Delta E_{\text{unit}});B_{\text{induced}};\boldsymbol{v}_{\text{em}}\right)\right\rangle$ state.

Both the supercurrent ($\boldsymbol{v}_{\mathrm{em},\boldsymbol{k}_{\mathrm{HOCO}}}(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}})\neq 0$) and the magnetic field ($B_{\mathrm{induced},\boldsymbol{k}_{\mathrm{HOCO}}}(\Delta E_{\mathrm{unit}},0)\neq 0$) can be induced under the condition of the closed-shell electronic structure with zero spin magnetic field and zero canonical momentum in a bosonic Cooper pair ($\sigma_{\mathrm{spin}}=0$; $\boldsymbol{p}_{\mathrm{canonical}}=0$). Such excited bosonic states with supercurrents $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)\left((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}}\right)\right\rangle$ cannot be destroyed because of the stable closed-shell electronic states, and thus the induced supercurrent and magnetic field cannot be dissipated. That is, the excited bosonic superconducting $\left|\boldsymbol{k}_{\mathrm{HOCO}}(T)\left((0,0);(\Delta E_{\mathrm{unit}},\Delta E_{\mathrm{unit}});B_{\mathrm{induced}};\boldsymbol{v}_{\mathrm{em}}\right)\right\rangle$ state is very stable and do not try to go back to the original ground bosonic superconducting state for $j=0$, and the induced electrical current and the induced magnetic field cannot be dissipated. This is the origin of the Ampère's law observed in the superconductivity.

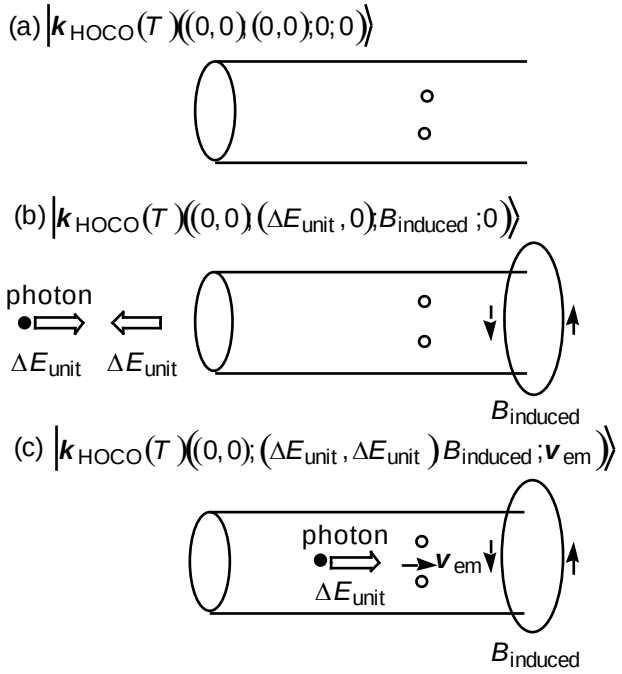

Fig. 6. The electronic states between $j=0$ and $j=1$ in superconductivity.

4.5 Ampère's Law in an Electron Traveling in Vacuum

Let us next consider the Ampère's law in vacuum (Fig. 7). We consider that there is a reference electron in metal. If very strong electric field is applied to a reference electron, this electron behaves according to the Ampère's law in metal (Fig. 7 (a), (b)), as discussed in the previous section. On the other hand, once the electron moves away from the metal, and goes into the vacuum, there is no reason (electrical resistivity and Stern–Gerlach effect, etc.) that the electron moving is dissipated. Therefore, the electron moves in vacuum with electrical momentum ($\boldsymbol{v}_{\mathrm{em}}\neq 0$) and the induced magnetic field ($B_{\mathrm{induced}}\neq 0$) (Fig. 7 (c), (d)). This is the Ampère's law observed in the traveling charged particles such as electrons in vacuum.

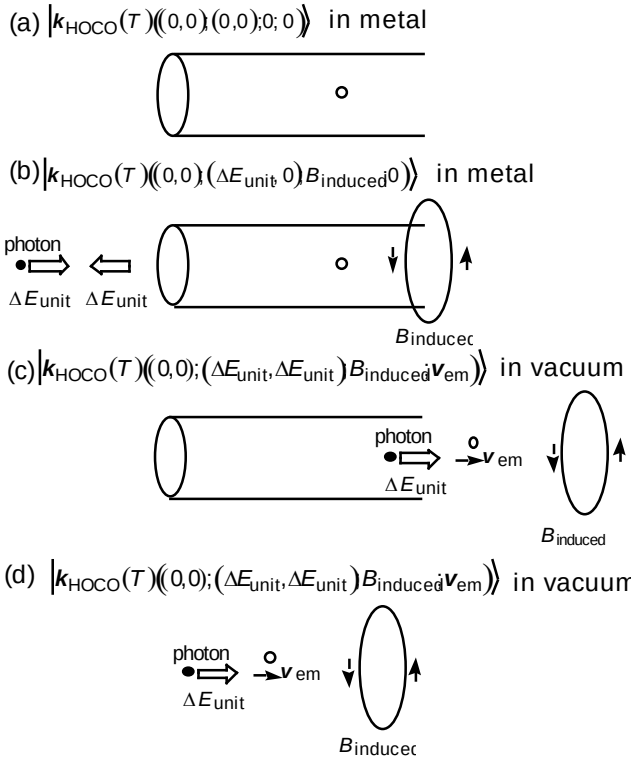

Fig. 7. The electronic states between $j=0$ and $j=1$ from normal metal to vacuum.

4.6 Reconsideration of the Ampère's Law

According to the conventional empirical Ampère's law, it has been considered that any moving charged particle itself creates a magnetic field. On the other hand, according to our theory, the magnetic field can be induced in order that the photon becomes massive (that is, electric field is expelled from the specimen) by absorbing Nambu–Goldstone boson formed by the fluctuation of the electronic state pairing of an electron, because of the very large stabilization energy ($V_{\mathrm{kin,Fermi},\boldsymbol{k}_{\mathrm{LUCO}}\sigma}(0)\approx 35$ eV) for the Bose–Einstein condensation ($\boldsymbol{p}_{\mathrm{canonical}}=0$; $V_{\mathrm{kin,Bose},\boldsymbol{k}_{\mathrm{LUCO}}\sigma}(0)=0$ eV), and the Stern–Gerlach effect. The initial electronic state tries not to change the electronic structure ($\boldsymbol{p}_{\mathrm{canonical}}=0$) by induction of the magnetic field. After that, the photon becomes massless (electric field can penetrate into the specimen), and thus the electrical current can be induced. Therefore, the induced electrical current as well as the magnetic field can be observed. This is the reason why we can observe that

any moving charged particle seems to create a magnetic field, as explained by the Ampère's law. On the other hand, such electrical currents as well as the induced magnetic fields can be immediately disappeared because of the unstable opened-shell electronic states. And at the same time, photon is emitted from an electron and this is the origin of the electrical resistivity. This process can be continuously repeated while the external applied electric field continues to be applied. That is, according to our theory, the electrical current (i.e., moving charged particle) itself is not directly related to the creation of the magnetic field. The induced magnetic field in the Ampère's law is realized because the bosonic electronic state tries not to change the electronic structure ($\boldsymbol{p}_{\text{canonical}} = 0$ and $\boldsymbol{v}_{\text{em}} = 0$) by inducing the magnetic field B_{induced}. If an electron were not in the bosonic state, the applied electric field would immediately penetrate into the specimen as soon as the electric field is applied, and any induced magnetic momentum (the induced magnetic field) even in the normal metals, expected from the Ampère's law, would not be observed. This bosonic electron is closely related to the concepts of the Higgs boson.

5. Concluding Remarks

We discussed the origin of the vibronic and Jahn–Teller stabilization for the opened-shell randomly moving independent one electron in view of the second-order processes in the vibronic and electron–phonon interactions. Furthermore, we discussed the relationships between the vibronic and Jahn–Teller stabilization energies, and the Joule's heats as a consequence of electrical resistivity, as a function of the external applied electric field, originating from the nondissipative second- and dissipative first-order processes in the vibronic interactions, respectively.

The total energies originating from both the second- and first-order processes are constant, and these values are the electron–phonon coupling constants calculated in the previous calculations and discussions. However, the roles of the first- and second-order processes have not been clearly distinguished in the previous researches.

According to our calculations, a phonon emitted by an electron is received by the same electron, and as a consequence of this phonon exchange between two electronic states with opposite momentum and spins (i.e., nondissipative phonon exchange), this randomly moving independent one electron is stabilized in energy. This is the vibronic and Jahn–Teller stabilization energies originating from the second-order processes. On the other hand, in the first-order processes, a phonon emitted by an electron is not absorbed by any electron (i.e., dissipative phonon emission). This is the origin of the electrical resistivity and Joule's heats. That is, momentum of an electron is changed in the first-order processes, but not changed in the second-order processes.

Electrons and molecular vibrations are more clearly treated as waves and particles, respectively, in this study, than those in the previous researches. By considering the wave characteristics of electrons and the direction of the motion of an electron, we clearly defined the wave function for randomly moving one electron (Eq. (1)), which has not been clearly defined. By using this wave function, the roles of the first- and second-order processes in the vibronic interactions can be clearly distinguished in this study. We suggest that an electron behaves as if it occupies these two electronic states orbitals with opposite momentum and spins, the characteristics of which change from the bonding interactions to the nonbonding interactions, and even to the antibonding interactions with an increase in the external applied electric field. The singlet and doublet electronic states become less and more dominant, respectively, with an increase in the external applied electric field. The vibronic stabilization energies for the second- and first-order processes decrease and increase, respectively, with an increase in the external applied electric field.

We also elucidate the mechanism of the Ampère's law (experimental rule discovered in 1820) in normal metallic and superconducting states [10], on the basis of the theory suggested in our previous researches [1–7]. Because of the very large stabilization energy ($V_{\text{kin,Fermi},\boldsymbol{k}_{\text{HOCO}}\sigma}(0) \approx 35$ eV) for the Bose–Einstein condensation ($\boldsymbol{p}_{\text{canonical}} = 0$; $V_{\text{kin,Bose},\boldsymbol{k}_{\text{HOCO}}\sigma}(0) = 0$ eV) [1–7], the electric and magnetic momentum of a bosonic electronic state pairing of an electron cannot be changed but the magnetic field can be induced soon after the electric field is induced. Therefore, the electronic state becomes $\left|\boldsymbol{k}_{\text{HOCO}}(T)((0,0);(\Delta E_{\text{unit}},0);B_{\text{induced}};0)\right\rangle$. Both the supercurrent ($\boldsymbol{v}_{\text{em},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},\Delta E_{\text{unit}}) \neq 0$) and the magnetic field ($B_{\text{induced},\boldsymbol{k}_{\text{HOCO}}}(\Delta E_{\text{unit}},0) \neq 0$) can be induced under the condition of the opened-shell electronic structure with zero spin magnetic momentum and zero canonical momentum in a bosonic electronic state pairing of an electron ($\sigma_{\text{spin}} = 0$; $\boldsymbol{p}_{\text{canonical}} = 0$). This is the origin of the Ampère's law.

According to the conventional empirical Ampère's law, it has been considered that any moving charged particle itself creates a magnetic field. On the other hand, according to our theory, the magnetic field can be induced in order that the photon becomes massive (that is, electric field is expelled from the specimen) by absorbing Nambu–Goldstone boson formed by the fluctuation of the electronic state pairing of an electron, because of the very large stabilization energy ($V_{\text{kin,Fermi},\boldsymbol{k}_{\text{LUCO}}\sigma}(0) \approx 35$ eV)

for the Bose–Einstein condensation ($\boldsymbol{p}_{\text{canonical}} = 0$; $V_{\text{kin, Bose}, \boldsymbol{k}_{\text{LUCO}}\sigma}(0) = 0$ eV), and the Stern–Gerlach effect. The initial electronic state tries not to change the electronic structure ($\boldsymbol{p}_{\text{canonical}} = 0$) by induction of the magnetic field. After that, the photon becomes massless (electric field can penetrate into the specimen), and thus the electrical current can be induced. Therefore, the induced electrical current as well as the magnetic field can be observed. This is the reason why we can observe that any moving charged particle seems to create a magnetic field, as explained by the Ampère's law. On the other hand, according to our theory, the electrical current (i.e., moving charged particle) itself is not directly related to the creation of the magnetic field. The induced magnetic field in the Ampère's law is realized because the bosonic electronic state tries not to change the electronic structure ($\boldsymbol{p}_{\text{canonical}} = 0$ and $\boldsymbol{v}_{\text{em}} = 0$) by inducing the magnetic field B_{induced}. If an electron were not in the bosonic state, the applied electric field would immediately penetrate into the specimen as soon as the electric field is applied, and any induced magnetic momentum (the induced magnetic field) even in the normal metals, expected from the Ampère's law, would not be observed. This bosonic electron is closely related to the concepts of the Higgs boson.

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Author Profile

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