International Journal of Scientific Engineering and Applied Science (IJSEAS) - Volume-1, Issue-7,October 2015
ISSN: 2395-3470
www.ijseas.com

Tree Related Hetro-Cordial Graphs

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Abstract

Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that each edge uv is assigned the label 0 if f(u)=f(v) or 1 if f(u) ≠ f(v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial Graphs(HeCG). In this paper, we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, $<K_{1,n} : n>$, $K_{1,2n}$ X K_2 are Hetro-Cordial Graphs.

Keywords- Hetro-Cordial Labeling, Hetro-Cordial Graphs.

2000 Mathematics Subject classification 05C78.

I.INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e={uv} of vertices in E is called edges or a lineof G. In this paper , we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, $<K_{1,n} : n>$, $K_{1,2n}$ X K_2 are Hetro-Cordial Graphss. For graph theory terminology, we follow [2]

II.PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a Graph G with vertex set V is a bijection from V to {0,1} such that each edge uv is assigned the label 0 if f(u)=f(v) or 1 if f(u) ≠ f(v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial Graphs(HCG). In this paper, we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, Subdivided star$<K_{1,n} : n>$, Book $K_{1,2n}$ X K_2 are Hetro-Cordial Graphs.

Definition: 2.1

A bipartite graph is a graph whose vertex set V(G) can be partitioned into two subsets V_1 and V_2 such that every edge of G has one end in V_1 and the other end in V_2 ; (V_1,V_2) is called a bipartition of G.If further, every vertex of V_1 is joined to all the vertices of V_2, then G is called a complete bipartite graph. The complete bipartite graph with bipartition (V_1, V_2) such that $|V_1|=m$ and $|V_2|=n$ is denoted by $K_{m,n}$. A complete bipartite graph $K_{1,n}$ (or) $K_{n,1}$ (or) S_n is called a star.

Definition: 2.2

Subdivided star $<K_{1,n}:n>$ is a graph obtained as one point union of n paths of path length 2.

Definition: 2.3

$K_{1,1,n}$ is a graph obtained by attaching root of a star $K_{1,n}$ at one end of P_2 and other end of $P_2$is joined with each pendant vertex of $K_{1,n}$.

Definition: 2.4

The product G_1 x G_2 of two graphs G_1 and G_2 is defined to be the graph whose vertex set is V_1 x V_2 and two vertices u= (u_1, u_2) and v= (v_1 ,v_2) in V=V_1xV_2 are adjacent in G_1 x G_2 if either u_1=v_1 and u_2 is adjacent to v_2 or u_2=v_2 and u_1 is adjacent to v_1. $K_{1,2n}$ X K_2 is called a Book.

III. MAIN RESULTS

Theorem:3.1

Star $K_{1,n}$ is Hetro-Cordial Graph.

Proof:

Let $V(K_{1,n}) =\{[u,u_i : 1 \leq i \leq n]\}$ and

$E(K_{1,n}) = \{[(uu_i) \ : 1 \leq i \leq n]\}$.

Define $f : V(K_{1,n}) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(u) = 0$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

Here, $v_f(0) = v_f(1)$ for all $n \equiv 1 \bmod 2$,

$v_f(0) = v_f(1)+1$ for all $n \equiv 0 \bmod 2$,

$e_f(1) = e_f(0)+1$ for all $n \equiv 1 \bmod 2$ and

$e_f(0) = e_f(1)$ for all $n \equiv 0 \bmod 2$.

Therefore, Star $K_{1,n}$ satisfies the conditions $| v_f(0) - v_f(1) | \leq 1$ and

$| e_f(0) - e_f(1) | \leq 1$.

Hence, Star $K_{1,n}$ is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $K_{1,5}$ and $K_{1,6}$ are shown in the following fig 3.2 and fig 3.3 respectively.

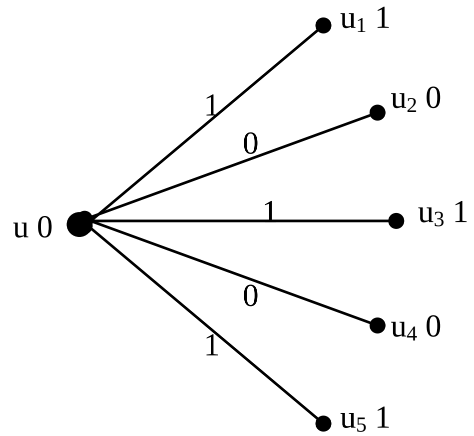


fig 3.2: $K_{1,5}$

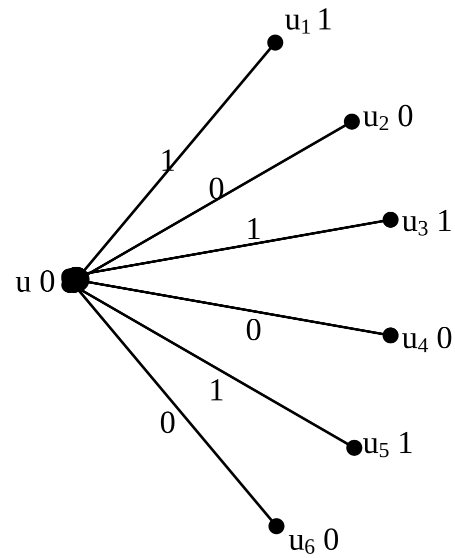


fig 3.3: $K_{1,6}$

Theorem:3.4

$K_{1,1,n}$ is Hetro-Cordial Graph.

Proof:

International Journal of Scientific Engineering and Applied Science (IJSEAS) - Volume-1, Issue-7,October 2015
ISSN: 2395-3470
www.ijseas.com

Let $V(K_{1,1,n}) = \{[u_i : 1 \le i \le n], [v_1, v_2]\}$ and

$E(K_{1,1,n}) = \{[(v_1u_i) \cup (v_2u_i) : 1 \le i \le n]\}$.

Define $f : V(K_{1,1,n}) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(v_1) = 1$

$f(v_2) = 0$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

The induced edge labeling are,

$f^*[(v_1v_2)] = 1$

$$f^*[(v_1u_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f^*[(v_2u_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

Here, $v_f(1) = v_f(0)+1$ for all $n \equiv 1 \bmod 2$,

$v_f(1) = v_f(0)$ for all $n \equiv 0 \bmod 2$ and

$e_f(1) = e_f(0)+1$ for all n.

Therefore, $K_{1,1,n}$ satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, $K_{1,1,n}$ is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $K_{1,1,2}$ and $K_{1,1,3}$ are shown in the following fig 3.5 and fig 3.6 respectively.

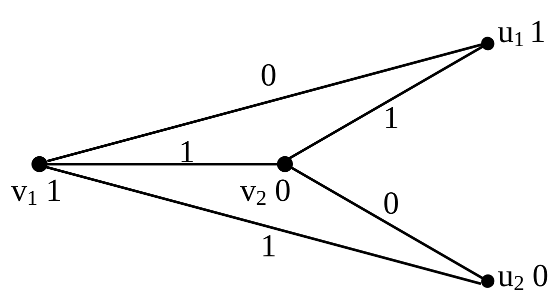


fig 3.5: $K_{1,1,2}$

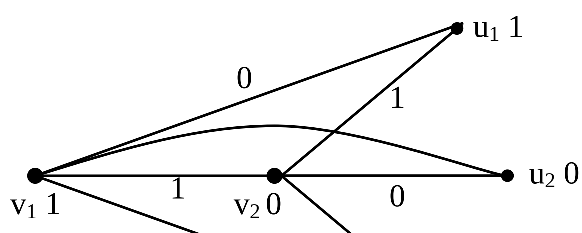


fig 3.6: $K_{1,1,3}$

Theorem:3.7

Subdivided star $\langle K_{1,n} : n\rangle$ is Hetro-Cordial Graph.

Proof:

Let $V(\langle K_{1,n} : n\rangle) = \{[u, u_i, v_i : 1 \le i \le n]\}$ and

$E(\langle K_{1,n} : n\rangle) = \{[(uu_i) \cup (u_iv_i) : 1 \le i \le n]\}$.

Define $f : V(\langle K_{1,n} : n\rangle) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(u) = 0$

$f(u_i) = 0 \qquad 1 \le i \le n$

$f(u_i) = 1 \qquad 1 \le i \le n$

The induced edge labeling are,

$f^*[(uu_i)] = 0 \qquad 1 \le i \le n$

$f^*[(u_iv_i)] = 1 \qquad 1 \le i \le n$

Here, $v_f(0) = v_f(1)+1$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, Subdivided star $\langle K_{1,n} : n\rangle$ satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

International Journal of Scientific Engineering and Applied Science (IJSEAS) - Volume-1, Issue-7,October 2015
ISSN: 2395-3470
www.ijseas.com

Hence, Subdivided star $<K_{1,n} : n>$ is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $<K_{1,3} : 3>$ and $<K_{1,4} : 4>$ are shown in the following fig 3.8 and fig 3.9 respectively.

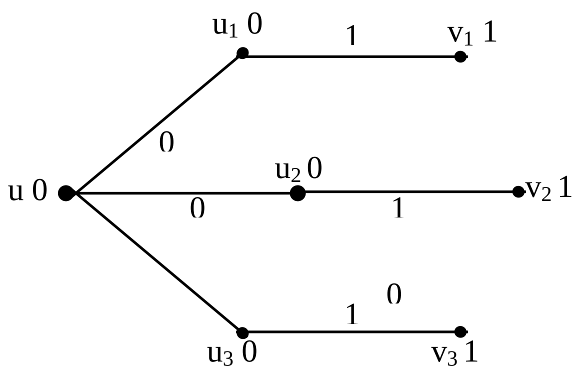


fig 3.8: $<K_{1,3} : 3>$

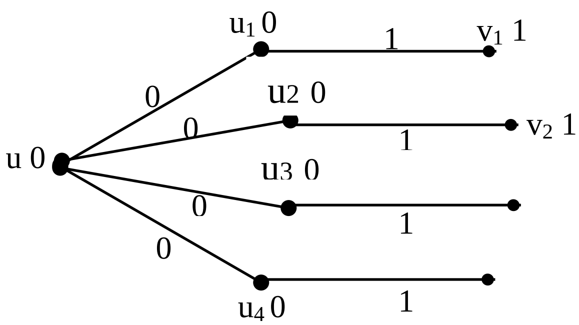


fig 3.9: $<K_{1,4} : 4>$

Theorem:3.10

$K_{1,2n} XK_2$ is a Hetro-Cordial Graph.

Proof:

Let $V(G) = \{[u,v],[u_i,v_i : 1 \leq i \leq 2n]\}$ and $E(G) = \{[(uv)] \cup [(uu_i) \cup (vv_i) \cup (u_iv_i) :$

$1 \leq i \leq 2n]\}$.

Define $f : V(G) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(u) = 0$

$f(v) = 1$

$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases}$ $1 \leq i \leq n$

$f(u_i) = 0 \quad n+1 \leq i \leq 2n$

$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ $1 \leq i \leq n$

$f(v_i) = 1 \quad n+1 \leq i \leq 2n$

The induced edge labeling are,

$f^*[(uv)] = 0$

$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ $1 \leq i \leq n$

$f^*[(uu_i)] = 1 \quad n+1 \leq i \leq 2n$

$f^*[(vv_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ $1 \leq i \leq n$

$f^*[(vv_i)] = 1 \quad n+1 \leq i \leq 2n$

$f^*[(u_iv_i)] = 0 \quad 1 \leq i \leq 2n$

Here, $v_f(0) = vR_f(1)$ for all n,

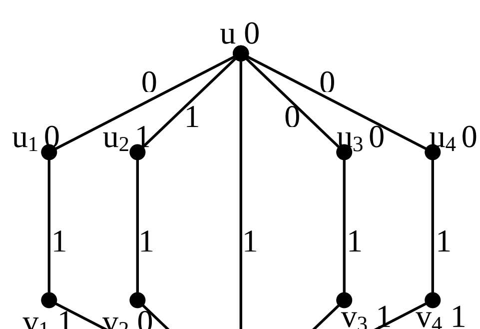
and $e_f(0) = e_f(1)+1$ for $n \equiv 1 \bmod 2$

$e_f(1) = e_f(0)+1$ for $n \equiv 0 \bmod 2$.

Therefore, $K_{1,2n} XK_2$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence , $K_{1,2n} XK_2$ is a Hetro-Cordial Graph.

For example, the Hetro-Cordial labeling of $K_{1,4} XK_2$ and $K_{1,6} XK_2$ are shown in the following fig 3.11 and fig 3.12 respectively.



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ISSN: 2395-3470
www.ijseas.com

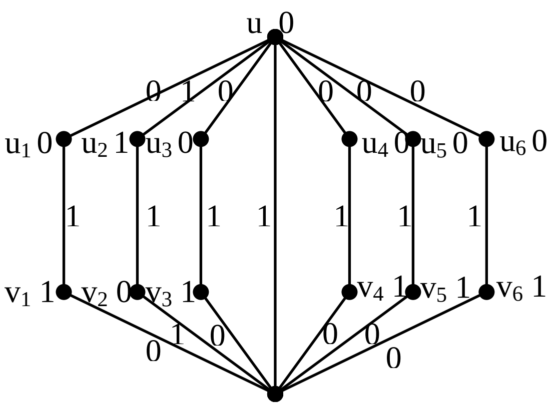

fig 3.12: $K_{1,6} \times K_2$

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