

Tree Related Hetro-Cordial Graphs

Dr. A. Nellai Murugan, V. Selva Vidhya and M. Mariasingam

Department of Mathematics

V.O.Chidambaram College

Tuticorin 628 008

s

Abstract

Let $G = (V, E)$ be a graph with p vertices and q edges. A Hetro-Cordial labeling of a graph G with vertex set V is a bijection from V to $\{0, 1\}$ such that each edge uv is assigned the label 0 if $f(u)=f(v)$ or 1 if $f(u) \neq f(v)$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial Graphs(HeCG). In this paper, we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, $\langle K_{1,n} : n \rangle$, $K_{1,2n} \times K_2$ are Hetro-Cordial Graphs.

Keywords- Hetro-Cordial Labeling, Hetro-Cordial Graphs.

2000 Mathematics Subject classification 05C78.

I.INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair $e=\{uv\}$ of vertices in E is called edges or a lineof G . In this paper , we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, $\langle K_{1,n} : n \rangle$, $K_{1,2n} \times K_2$ are Hetro-Cordial Graphss. For graph theory terminology, we follow [2]

II.PRELIMINARIES

Let $G = (V, E)$ be a graph with p vertices and q edges. A Hetro-Cordial labeling of a Graph G with vertex set V is a bijection from V to $\{0, 1\}$ such that each edge uv is assigned the label 0 if

$f(u)=f(v)$ or 1 if $f(u) \neq f(v)$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial Graphs(HCG). In this paper, we proved that tree related graphs Star $K_{1,n}$, $K_{1,1,n}$, Subdivided star $\langle K_{1,n} : n \rangle$, Book $K_{1,2n} \times K_2$ are Hetro-Cordial Graphs.

Definition: 2.1

A bipartite graph is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every edge of G has one end in V_1 and the other end in V_2 ; (V_1, V_2) is called a bipartition of G . If further, every vertex of V_1 is joined to all the vertices of V_2 , then G is called a complete bipartite graph. The complete bipartite graph with bipartition (V_1, V_2) such that $|V_1|=m$ and $|V_2|=n$ is denoted by $K_{m, n}$. A complete bipartite graph $K_{1,n}$ (or) $K_{n,1}$ (or) S_n is called a star.

Definition: 2.2

Subdivided star $\langle K_{1,n} : n \rangle$ is a graph obtained as one point union of n paths of path length 2.

Definition: 2.3

$K_{1,1,n}$ is a graph obtained by attaching root of a star $K_{1,n}$ at one end of P_2 and other end of P_2 is joined with each pendant vertex of $K_{1,n}$.

Definition: 2.4

The product $G_1 \times G_2$ of two graphs G_1 and G_2 is defined to be the graph whose vertex set is $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$ are adjacent in $G_1 \times G_2$ if either $u_1 = v_1$ and u_2 is adjacent to v_2 or $u_2 = v_2$ and u_1 is adjacent to v_1 . $K_{1,2n} \times K_2$ is called a Book.

III. MAIN RESULTS

Theorem:3.1

Star $K_{1,n}$ is Hetro-Cordial Graph.

Proof:

Let $V(K_{1,n}) = \{[u, u_i : 1 \leq i \leq n]\}$ and

$E(K_{1,n}) = \{[(u, u_i) : 1 \leq i \leq n]\}$.

Define $f : V(K_{1,n}) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u) = 0$$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(u, u_i)] = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

Here, $v_f(0) = v_f(1)$ for all $n \equiv 1 \pmod 2$,

$v_f(0) = v_f(1) + 1$ for all $n \equiv 0 \pmod 2$,

$e_f(1) = e_f(0) + 1$ for all $n \equiv 1 \pmod 2$

and

$e_f(0) = e_f(1)$ for all $n \equiv 0 \pmod 2$.

Therefore, Star $K_{1,n}$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and

$$|e_f(0) - e_f(1)| \leq 1.$$

Hence, Star $K_{1,n}$ is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $K_{1,5}$ and $K_{1,6}$ are shown in the following fig 3.2 and fig 3.3 respectively.

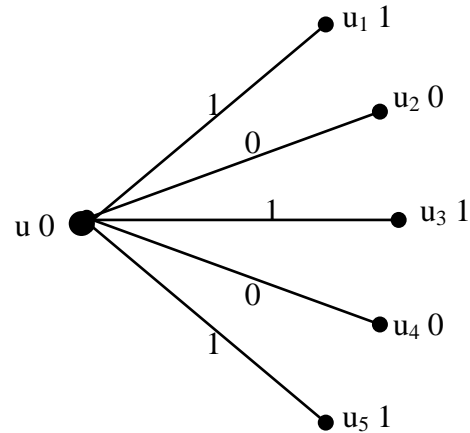


fig 3.2: $K_{1,5}$

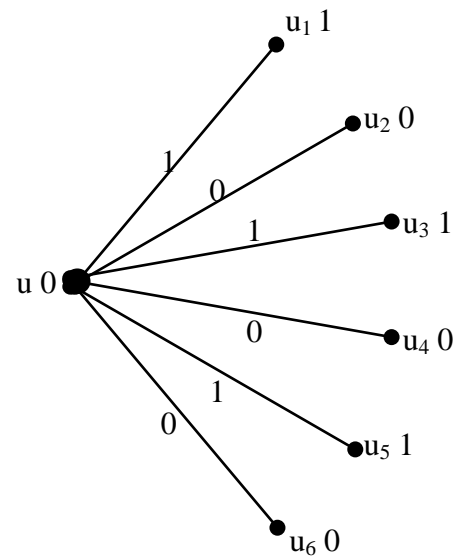


fig 3.3: $K_{1,6}$

Theorem:3.4

$K_{1,1,n}$ is Hetro-Cordial Graph.

Proof:

Let $V(K_{1,1,n}) = \{[u_i : 1 \leq i \leq n], [v_1, v_2]\}$ and

$$E(K_{1,1,n}) = \{[(v_1 u_i) \cup (v_2 u_i) : 1 \leq i \leq n]\}.$$

Define $f : V(K_{1,1,n}) \rightarrow \{0,1\}$.

1

The vertex labeling are ,

$$f(v_1) = 1$$

$$f(v_2) = 0$$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(v_1 v_2)] = 1$$

$$f^*[(v_1 u_i)] = \begin{cases} 0 & i \equiv 1 \pmod 2 \\ 1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(v_2 u_i)] = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

Here, $v_f(1) = v_f(0) + 1$ for all $n \equiv 1 \pmod 2$,

$$v_f(1) = v_f(0) \text{ for all } n \equiv 0 \pmod 2 \text{ and}$$

$$e_f(1) = e_f(0) + 1 \text{ for all } n.$$

Therefore, $K_{1,1,n}$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $K_{1,1,n}$ is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $K_{1,1,2}$ and $K_{1,1,3}$ are shown in the following fig 3.5 and fig 3.6 respectively.

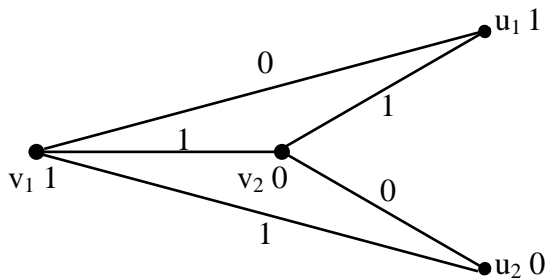


fig 3.5: $K_{1,1,2}$

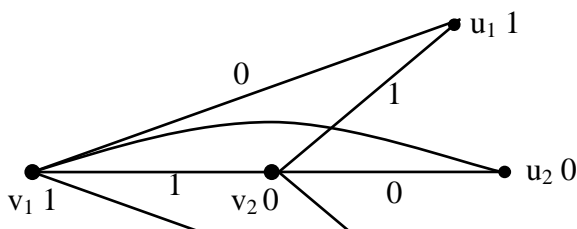


fig 3.6: $K_{1,1,3}$

Theorem:3.7

Subdivided star $\langle K_{1,n} : n \rangle$ is Hetro-Cordial Graph.

Proof:

Let $V(\langle K_{1,n} : n \rangle) = \{[u, u_i, v_i : 1 \leq i \leq n]\}$ and

$$E(\langle K_{1,n} : n \rangle) = \{[(u u_i) \cup (u_i v_i) : 1 \leq i \leq n]\}.$$

Define $f : V(\langle K_{1,n} : n \rangle) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u) = 0$$

$$f(u_i) = 0 \quad 1 \leq i \leq n$$

$$f(v_i) = 1 \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(u u_i)] = 0 \quad 1 \leq i \leq n$$

$$f^*[(u_i v_i)] = 1 \quad 1 \leq i \leq n$$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$$e_f(0) = e_f(1) \text{ for all } n.$$

Therefore, Subdivided star $\langle K_{1,n} : n \rangle$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, Subdivided star $\langle K_{1,n} : n \rangle$ is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $\langle K_{1,3} : 3 \rangle$ and $\langle K_{1,4} : 4 \rangle$ are shown in the following fig 3.8 and fig 3.9 respectively.

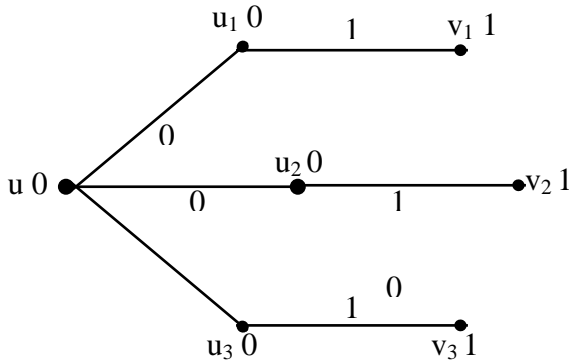


fig 3.8: $\langle K_{1,3} : 3 \rangle$

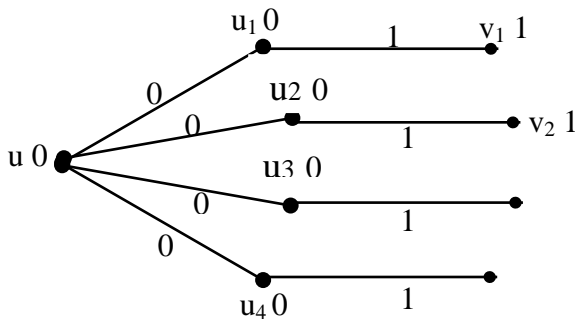


fig 3.9: $\langle K_{1,4} : 4 \rangle$

Theorem:3.10

$K_{1,2n} XK_2$ is a Hetro-Cordial Graph.

Proof:

Let $V(G) = \{[u,v], [u_i, v_i] : 1 \leq i \leq 2n\}$
 and $E(G) = \{[(uv)] \cup [(uu_i) \cup (vv_i) \cup (u_i v_i) : 1 \leq i \leq 2n]\}$.

Define $f : V(G) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u) = 0$$

$$f(v) = 1$$

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \pmod 2 \\ 1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(u_i) = 0 \quad n+1 \leq i \leq 2n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i) = 1 \quad n+1 \leq i \leq 2n$$

The induced edge labeling are,

$$f^*[(uv)] = 0$$

$$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(uu_i)] = 1 \quad n+1 \leq i \leq 2n$$

$$f^*[(vv_i)] = \begin{cases} 0 & i \equiv 0 \pmod 2 \\ 1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(vv_i)] = 1 \quad n+1 \leq i \leq 2n$$

$$f^*[(u_i v_i)] = 0 \quad 1 \leq i \leq 2n$$

Here, $\forall \beta \in \mathbb{Q} = vR_f(1)$ for all n,

$$e_f(0) = e_f(1) + 1 \quad \text{for } n \equiv 1 \pmod 2$$

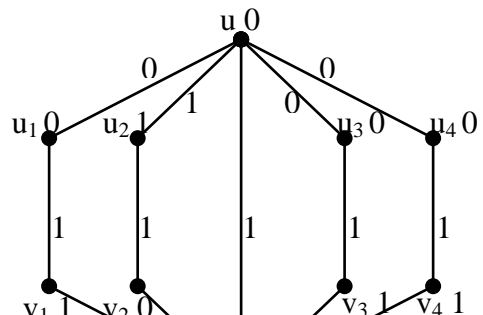
and

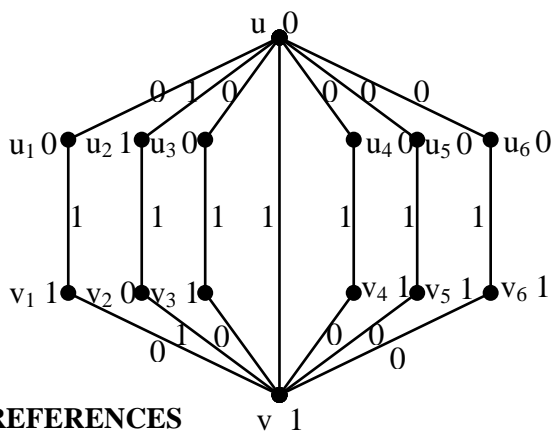
$$e_f(1) = e_f(0) + 1 \quad \text{for } n \equiv 0 \pmod 2.$$

Therefore, $K_{1,2n} XK_2$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $K_{1,2n} XK_2$ is a Hetro-Cordial Graph.

For example, the Hetro-Cordial labeling of $K_{1,4} XK_2$ and $K_{1,6} XK_2$ are shown in the following fig 3.11 and fig 3.12 respectively.





IV. REFERENCES

1. Gallian. J.A., *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics 6(2001)#DS6.
2. Harary, F. (1969), *Graph Theory*, Addison – Wesley Publishing Company Inc, USA.
3. A.Nellai Murugan (September 2011), *Studies in Graph theory- Some Labeling Problems in Graphs and Related topics*, Ph.D Thesis.
4. A.Nellai Murugan and V.Baby Suganya, Cordial labeling of path related splitted graphs, Indian Journal of Applied Research ISSN 2249 –555X, Vol.4, Issue 3, Mar. 2014, ISSN 2249 – 555X , PP 1-8. I .F . 2.1652
5. A.Nellai Murugan and M. Taj Nisha, A study on divisor cordial labelling of star attached paths and cycles, Indian Journal of Research ISSN 2250 –1991, Vol.3, Issue 3, Mar. 2014, PP 12-17. I .F . 1.6714.

6. A.Nellai Murugan and V.Brinda Devi, A study on path related divisor cordial graphs International Journal of Scientific Research, ISSN 2277–8179, Vol.3, Issue 4, April. 2014, PP 286 - 291. I .F . 1.8651.
7. A.Nellai Murugan and A Meenakshi Sundari, On Cordial Graphs International Journal of Scientific Research, ISSN 2277–8179, Vol.3, Issue 7 ,July. 2014, PP 54-55. I .F . 1.8651
8. A.Nellai Murugan and A Meenakshi Sundari, Results on Cycle related product cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968, Vol.I, Issue 5 ,July. 2014, PP 462-467. IF 0.611
9. A.Nellai Murugan and P.Iyadurai Selvaraj, Cycle and Armed Cup cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968, Vol.I, Issue 5 ,July. 2014, PP 478-485. IF 0.611
10. A.Nellai Murugan and G.Esther, Some Results on Mean Cordial Labelling , International Journal of Mathematics Trends and Technology ,ISSN 2231-5373, Volume 11, Number 2, July 2014, PP 97-101.
11. A.Nellai Murugan and A Meenakshi Sundari, Path related product cordial graphs, International Journal of Innovation in Science and Mathematics , ISSN 2347-9051, Vol 2., Issue 4 ,July 2014, PP 381-383
12. A.Nellai Murugan and P. Iyadurai Selvaraj, *Path Related Cup Cordial graphs*, Indian Journal of Applied Research, ISSN 2249 –555X, Vol.4, Issue 8, August. 2014, PP 433-436.
13. A.Nellai Murugan , G.Devakiriba and S.Navaneethakrishnan, Star Attached Divisor cordial graphs, International Journal of Innovative Science,

- Engineering & Technology , ISSN 2348-7968, Vol.I, Issue 6 ,August. 2014, PP 165-171.
14. A.Nellai Murugan and G. Devakiriba, Cycle Related Divisor Cordial Graphs, International Journal of Mathematics Trends and Technology , ISSN 2231-5373, Volume 12, Number 1, August 2014, PP 34-43.
 15. A.Nellai Murugan and V.Baby Suganya, A study on cordial labeling of Splitting Graphs of star Attached C_3 and $(2k+1)C_3$ ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 142 -147. I.F 6.531
 16. A.Nellai Murugan and V.Brinda Devi , A study on Star Related Divisor cordial Graphs ,ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 169-172. I.F 6.531
 17. A.Nellai Murugan and M. Taj Nisha, A study on Divisor Cordial Labeling Star Attached Path Related Graphs, ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 173-178. I.F 6.531 .
 18. .A .Nellai Murugan and V .Sripratha, Mean Square Cordial Labelling , International Journal of Innovative Research & Studies, ISSN 2319-9725 ,Volume 3, Issue 10 Number 2 ,October 2014, PP 262-277.
 19. A.Nellai Murugan and G. Esther , Path Related Mean Cordial Graphs , Journal of Global Research in Mathematical Archive , ISSN 2320 5822 , Volume 02, Number 3, March 2014, PP 74-86.
 20. A. Nellai Murugan and A. Meenakshi Sundari , Some Special Product Cordial Graphs, Proceeding of the UGC Sponsored National Conference on Advances in Fuzzy Algebra , Fuzzy Topology and Fuzzay Graphs , Journal ENRICH , ISSN 2319-6394, January 2015, PP 129-141.
 21. L. Pandiselvi ,S.Navaneethakrishan and A. Nellai Murugan ,Fibonacci divisor Cordial Cycle Related Graphs, Proceeding of the UGC Sponsored National Conference on Advances in Fuzzy Algebra , Fuzzy Topology and Fuzzay Graphs , Journal ENRICH , ISSN 2319-6394, January 2015, PP 142-150.