

# Dufour Effects on Unsteady MHD Free Convection and Mass Transfer Flow Past Through a Porous Medium in a Slip Regime with Heat Source/Sink

K.Sharmilaa<sup>1</sup>, S.Kaleeswari<sup>2</sup>

<sup>1,2</sup>PSGR Krishnammal College for Women, Coimbatore, India.

## Abstract

The problem of Dufour effects on unsteady MHD free convection and mass transfer flow past through a porous medium in a slip regime with heat source/sink has been examined in detail in this paper. A magnetic field of uniform strength is assumed to be applied transversely to the direction of the main flow. Perturbation technique is applied to transform the non-linear coupled governing partial differential equations in dimensionless form into a system of ordinary differential equations. The equations are solved analytically and the solutions for the velocity, temperature and concentration fields are obtained. The effects of various flow parameters on velocity, temperature and concentration fields are presented graphically.

**Key Words:** *Thermal radiation, Porous medium, Free convection, MHD and skin-friction.*

## 1. Introduction

Free convection flow of magnetohydrodynamic fluid has attracted many researchers in view of its numerous applications in geophysics, astrophysics, meteorology, aerodynamics, magnetohydrodynamic power generators and pumps, boundary layer control energy generators, accelerators, aerodynamics heating, polymer technology, petroleum industry, purification of crude oil, and in material processing such as extrusion, metal forming, continuous casting wire, and glass fibre drawing. Further, the convective flow through porous medium has applications in the field of chemical engineering for filtration and purification processes. In petroleum technology, it is used to study the movement of natural gas oil and water through oil channels or reservoirs, and in the field of agriculture engineering to study the underground water resources. Ostrach [1], the

initiator of the study of convection flow, made a technical note on the similarity solution of transient free convection flow past a semi-infinite vertical plate by an integral method. Study of fluid flow in porous medium is based upon the empirically determined Darcy's law. Such flows are considered to be useful in diminishing the free convection, which would otherwise occur intensely on a vertical heated surface. Study of flow problems through porous medium is heavily based on Darcy's experimental law [2]. Wooding [3] and Brinkman [4,5] have modified Darcy's law, which are used by many authors on study of convective flow in porous media.

A few representative fields of interest in which combined heat and mass transfer plays an important role are designing of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution. In this context, Soundalgekar [7] extended his own problem of Soundalgekar [6] to mass transfer effects. Callahan and Marner [8] considered the transient free convection flow past a semi-infinite vertical plate with mass transfer. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical plate was studied by Soundalgekar and Wavre [9]. Verma and Srivastava [10] proposed the effect of magnetic field on unsteady blood flow through a narrow tube. Huges and Young [11] gave an excellent summary of applications. MHD free convective flow of an electrically conducting

fluid between two heated parallel plates in the presence of induced magnetic field by using analytical solution was carried out by Sharma [12]. Chakrabarti and Gupta [13] explained the heat transfer effect on hydromagnetic flow over a stretching sheet. Convective heat transfer effect on MHD flow past a continuously moving plate embedded in a non-Darcian porous medium has been proposed by Abo-Eldahab and El-Gendy [14].

Radiation is a process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environmental process. For example, heating and cooling chambers, evaporation from large open water reservoirs, astrophysical flows etc. Radiative heat and mass transfer play an important role in space related technology. The effect of radiation on various convective flows under different conditions have been studied by many researchers including Hussain and Thakar [15], Ahmed and Sarmah [16], Rajesh and Varma [17], Pal and Mondal [18], Samad and Rahman [19], Karthikeyan *et al.* [20], Das *et al.* [21] and Pal *et al.* [22].

The study of heat source/sink effects on heat transfer is very important because its effects are crucial in controlling the heat transfer. Radiation effects on unsteady MHD flow through a porous medium with variable temperature in presence of heat source/sink is studied by Vijaya Kumar *et al.* [23].

Vijaya sekhar and Viswanadh reddy [24] have obtained the analytical solution for the effects of heat sink and chemical reaction on MHD free convective oscillatory flow past a porous plate with viscous dissipation. Gireesh Kumar and Satyanarayana [25] studied the heat and mass transfer effects on unsteady MHD free convective walter's memory flow with constant suction in presence of heat sink.

The present work is concerned with the Dufour effects on unsteady MHD free convection and mass transfer flow past through a porous medium in a slip regime with heat source/sink. Perturbation technique is applied to convert the governing non-linear partial differential equations into a system of ordinary differential equations, which are solved analytically.

## 2. Mathematical Analysis

We consider a two-dimensional unsteady flow of a laminar, incompressible, electrical conducting and heat absorbing fluid past a semi-infinite vertical porous plate embedded in a uniform porous medium. We introduce the coordinate system  $(\bar{x}, \bar{y}, \bar{z})$  with  $X$  axis is chosen along the  $Y$  axis perpendicular to it and directed in the fluid region and  $Z$  axis is along the width of the plate as shown in the Fig. 1.

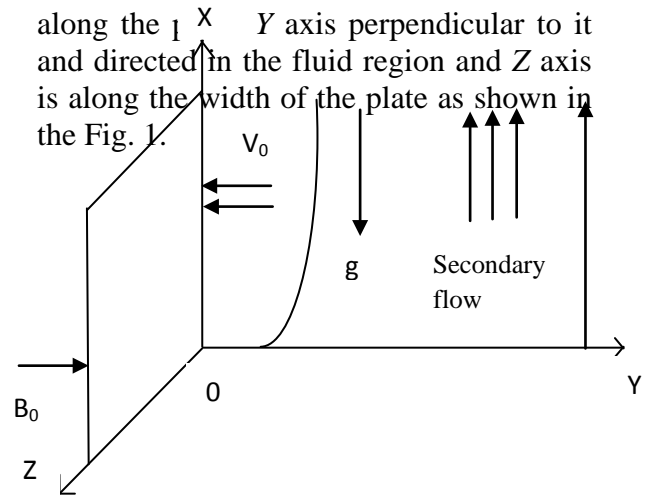


Fig. 1 Physical model of the problem.

The induced magnetic field is neglected under the assumption that the magnetic Reynolds number is small. It is assumed that there is no applied voltage which implies the absence of any electrical field. The radiative heat flux in the  $X$  direction is considered negligible in comparison to that in  $Y$  direction. The governing equations for this study are based on the conservation of mass, linear momentum, energy and species concentration. Taking in to consideration the assumptions made above, these equations in Cartesian frame of reference are given by equation of

$$\text{Continuity: } \frac{\partial \bar{v}}{\partial y} = 0 \tag{2.1}$$

Momentum equation:

$$\begin{aligned} \frac{\partial \bar{u}}{\partial t} + \bar{v} \frac{\partial \bar{u}}{\partial y} = & -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + g\beta(\bar{T} - \bar{T}_\infty) + g\beta^*(\bar{C} - \bar{C}_\infty) \\ & + \nu \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \bar{u} - \frac{\nu}{K^*} \bar{u} \end{aligned} \tag{2.2}$$

Energy equation:

$$\begin{aligned} \frac{\partial \bar{T}}{\partial t} + \bar{v} \frac{\partial \bar{T}}{\partial y} = & \frac{K}{\rho C_p} \frac{\partial^2 \bar{T}}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r^*}{\partial y} + \frac{D_M k_T}{c_s c_p} \frac{\partial^2 \bar{C}}{\partial y^2} \\ & - \frac{Q_0(\bar{T} - \bar{T}_\infty)}{\rho C_p} \end{aligned} \tag{2.3}$$

Species equation:

$$\frac{\partial \bar{C}}{\partial t} + \bar{v} \frac{\partial \bar{C}}{\partial y} = D \frac{\partial^2 \bar{C}}{\partial y^2} - \bar{K}_c (\bar{C} - \bar{C}_\infty) \tag{2.4}$$

Where  $\bar{x}$ ,  $\bar{y}$  and  $\bar{t}$  are the dimensional distances along and perpendicular to the plate and dimensional time, respectively.  $\bar{u}$  and  $\bar{v}$  are the components of the dimensional velocities along  $\bar{x}$  and  $\bar{y}$  respectively.  $\rho$  is the density of the medium,  $g$  is the acceleration due to gravity,  $\nu$  is the kinematic viscosity,  $\sigma$  is the fluid electrical conductivity,  $B_0$  is the magnetic induction,  $K^*$  is the permeability of the porous medium,  $\beta$  is the coefficient of thermal expansion,  $\bar{\beta}$  is the coefficient of mass expansion,  $\bar{T}$  is the dimensional temperature of the fluid near the plate,  $\bar{T}_\infty$  is the dimensional free stream temperature,  $\bar{C}$  is the dimensional concentration of the fluid near the plate,  $\bar{C}_\infty$  is the dimensional free stream concentration,  $k$  is the thermal conductivity of the fluid,  $q_r^*$  is the radiative heat flux, the term  $Q_0(\bar{T} - \bar{T}_\infty)$  is assumed to be amount of heat generated or absorbed per unit volume,  $Q_0$  is constant, which may take on either

positive or negative values. When plate temperature  $\bar{T}$  exceeds the free stream temperature  $\bar{T}_\infty$ , the source term  $Q_0 > 0$  and heat sink when  $Q_0 < 0$ ,  $\mu$  is the fluid viscosity,  $C_p$  is the specific heat at constant pressure,  $D_m$  is the coefficient of mass diffusivity and  $D$  is the is the molecular diffusivity,  $K_c$  is chemical reaction parameter.

Cogley et al. [26] showed that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the

$$\text{following form: } \frac{\partial q_r^*}{\partial y} = 4(\bar{T} - \bar{T}_\infty) I^* \tag{2.5}$$

where  $I^* = \int K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial \bar{T}} d\lambda$ ,  $K_{\lambda w}$  is the absorption coefficient at the wall and  $e_{b\lambda}$  is the Planck's function. Under the assumption, the appropriate boundary conditions for velocity involving slip flow, temperature and concentration fields are given by

$$\begin{aligned} \bar{u} = \bar{u}_{slip} = \bar{h} \frac{\partial \bar{u}}{\partial y}, \bar{T} = \bar{T}_w + \varepsilon(\bar{T}_w - \bar{T}_\infty) e^{n^* t^*}, \\ \bar{C} = \bar{C}_w + \varepsilon(\bar{C}_w - \bar{C}_\infty) e^{n^* t^*} \quad \text{at } \bar{y} = 0 \end{aligned} \tag{2.6}$$

$$\bar{u} \rightarrow \bar{U}_\infty = U_0(1 + \varepsilon e^{n^* t^*}), \bar{T} \rightarrow \bar{T}_\infty, \bar{C} \rightarrow \bar{C}_\infty \text{ as } \bar{y} \rightarrow \infty \tag{2.7}$$

where  $\bar{T}_w$  and  $\bar{C}_w$  are the dimensional temperature and species concentration at the wall respectively and  $\bar{h}$  is the characteristic dimension of the flow fluid. The suction velocity normal to the plate is a function of time only, it can be taken in the exponential form as

$$\bar{v} = -V_0(1 + \varepsilon A e^{n^* t^*}) \tag{2.8}$$

where  $A$  is a real positive constant,  $\varepsilon$  and  $\varepsilon A$  are small quantities less than unity and  $V_0$  is a scale of suction velocity which is a non-zero

positive constant. In the free stream, from Eq. (2) we get

$$-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} = \frac{d\bar{U}_\infty}{d\bar{t}} + \frac{\sigma B_0^2}{\rho} \bar{U}_\infty + \frac{\nu}{K^*} \bar{U}_\infty \quad (2.9)$$

Now we introduce the dimensionless variables as follows

$$\begin{aligned} u &= \frac{\bar{u}}{U_0}, \quad v = \frac{\bar{v}}{V_0}, \quad y = \frac{\bar{y}V_0}{\nu}, \quad U_\infty = \frac{\bar{U}_\infty}{U_0}, \quad t = \frac{\bar{t}V_0^2}{\nu}, \\ \theta &= \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \quad \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, \quad n = \frac{n^* \nu}{V_0^2}, \quad K = \frac{K^* V_0^2}{\nu^2}, \\ Pr &= \frac{\mu C_p}{k}, \quad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \quad Gr = \frac{\nu \beta g (\bar{T}_w - \bar{T}_\infty)}{U_0 V_0^2}, \\ Gm &= \frac{\nu \beta^* g (\bar{C}_w - \bar{C}_\infty)}{U_0 V_0^2}, \quad Q = \frac{Q_0 \nu}{\rho C_p V_0^2}, \quad R = \frac{4\nu I^*}{\rho C_p V_0^2}, \\ h &= \frac{V_0 \bar{h}}{\nu}, \quad Sc = \frac{\nu}{D}, \quad Du = \frac{DmDt(C_w - C_\infty)}{CsCpv(T_w - T_\infty)}, \\ Kc &= \frac{\bar{K}c\nu}{V_0^2} \end{aligned} \quad (2.10)$$

where  $Pr$  is the Prandtl number,  $M$  is the magnetic field parameter,  $Gr$  is the Grashof number for heat transfer,  $Gm$  is the Grashof number for mass transfer,  $Q$  is the heat sink parameter,  $\alpha$  is the permeability parameter,  $\theta$  is the non dimensional temperature,  $\phi$  is the non dimensional concentration,  $R$  is the radiation parameter,  $Du$  is the Dufour number,  $h$  is the rarefaction parameter,  $So$  is the Soret number and  $Sc$  is the Schmidt number.

In view of equations (2.8) to (2.10) the governing equations (2.2), (2.3) and (2.4) reduce the following non-dimensional form:

$$\begin{aligned} \frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} &= \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi \\ &+ N(U_\infty - u) \end{aligned} \quad (2.11)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - R\theta + Du \frac{\partial^2 \phi}{\partial y^2} - Q\theta$$

$$(2.12)$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K_c \phi$$

$$(2.13)$$

$$\text{where } N = M + \frac{1}{K}$$

The boundary conditions (2.6) and (2.7) in the dimensionless form can be written as,

$$\begin{aligned} u &= u_{slip} = h \frac{\partial u}{\partial y}, \quad \theta = 1 + \varepsilon e^{nt}, \quad \phi = 1 + \varepsilon e^{nt}, \quad \text{at} \\ &y = 0 \end{aligned}$$

$$(2.14)$$

$$u \rightarrow U_\infty = 1 + \varepsilon e^{nt}, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0, \quad \text{as } y \rightarrow \infty \quad (2.15)$$

### 3. Solution of the Problem

Equations (2.11) to (2.13) are coupled non-linear partial differential equations and these can be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. These can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the plate as

$$u = u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) \quad (3.1)$$

$$\theta = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) \quad (3.2)$$

$$\phi = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2) \quad (3.3)$$

Substituting (3.1) to (3.3) in equations (2.11) to (2.13) and equating the harmonic and non harmonic terms and neglecting the coefficient of  $O(\varepsilon^2)$  we get the following pairs of equations for  $(u_0, \theta_0, \phi_0)$  and  $(u_1, \theta_1, \phi_1)$ .

$$u_0'' + u_0' - Nu_0 = -N - Gr\theta_0 - Gm\phi_0 \quad (3.4)$$

$$u_1'' + u_1' - (N + n)u_1 = -Au_0' - Gr\theta_1 - Gm\phi_1 - (N + n) \quad (3.5)$$

$$\theta_0'' + Pr\theta_0' - Pr(R + Q)\theta_0 = -DuPr\phi_0'' \quad (3.6)$$

$$\theta_1'' + Pr\theta_1' - Pr(R + Q + n)\theta_1 = -APr\theta_0' - DuPr\phi_1'' \quad (3.7)$$

$$\phi_0'' + Sc\phi_0' - ScKc\phi_0 = 0 \quad (3.8)$$

$$\phi_1'' + Sc\phi_1' - Sc(Kc + n)\phi_1 = -ASc\phi_0' \quad (3.9)$$

where the primes denote the differentiation with respect to  $y$ .

The corresponding boundary conditions can be written as

$$u_0 = hu_0', u_1 = hu_1', \theta_0 = 1, \theta_1 = 1, \phi_0 = 1, \phi_1 = 1 \text{ at } y = 0 \quad (3.10)$$

$$u_0 = 1, u_1 = 1, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, \phi_0 \rightarrow 0, \phi_1 \rightarrow 0 \text{ as } y \rightarrow \infty \quad (3.11)$$

The solutions of equations (3.4) to (3.9) which satisfy the boundary conditions (3.10) and (3.11) are given by

$$u_0 = 1 + K_{13}e^{-m_5y} + K_{10}e^{-m_3y} + K_{11}e^{-m_1y} + K_{12}e^{-m_1y} \quad (3.12)$$

$$u_1 = 1 + K_{27}e^{-m_6y} + K_{14}e^{-m_5y} + K_{15}e^{-m_3y} + K_{16}e^{-m_1y} + K_{17}e^{-m_1y} - K_{18}e^{-m_4y} - K_{19}e^{-m_3y} - K_{20}e^{-m_1y} + K_{21}e^{-m_2y} + K_{22}e^{-m_1y} - K_{23}e^{-m_2y} - K_{24}e^{-m_1y} \quad (3.13)$$

$$\theta_0 = K_4e^{-m_3y} + K_3e^{-m_1y} \quad (3.14)$$

$$\theta_1 = K_9e^{-m_4y} + K_5e^{-m_3y} + K_6e^{-m_1y} - K_7e^{-m_2y} - K_8e^{-m_1y} \quad (3.15)$$

$$\phi_0 = e^{-m_1y} \quad (3.16)$$

$$\phi_1 = K_2e^{-m_2y} + K_1e^{-m_1y} \quad (3.17)$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(y, t) = 1 + K_{13}e^{-m_5y} + K_{10}e^{-m_3y} + K_{11}e^{-m_1y} + K_{12}e^{-m_1y} + \varepsilon e^{nt} (1 + K_{27}e^{-m_6y} + K_{14}e^{-m_5y} + K_{15}e^{-m_3y} + K_{16}e^{-m_1y} + K_{17}e^{-m_1y} - K_{18}e^{-m_4y} - K_{19}e^{-m_3y} - K_{20}e^{-m_1y} + K_{21}e^{-m_2y} + K_{22}e^{-m_1y} - K_{23}e^{-m_2y} - K_{24}e^{-m_1y})$$

$$\theta(y, t) = K_4e^{-m_3y} + K_3e^{-m_1y} + \varepsilon e^{nt} (K_9e^{-m_4y} + K_5e^{-m_3y} + K_6e^{-m_1y} - K_7e^{-m_2y} - K_8e^{-m_1y})$$

$$\phi(y, t) = e^{-m_1y} + \varepsilon e^{nt} (K_2e^{-m_2y} + K_1e^{-m_1y})$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin-friction) is given by

$$\tau_w^* = \mu \left( \frac{\partial \bar{u}}{\partial y} \right)_{y=0}$$

and in dimensionless form, we obtain

$$C_f = \frac{\tau_w^*}{\rho U_0 V_0} = \left( \frac{\partial u}{\partial y} \right)_{y=0} = u'(0) = \left( \frac{\partial u_0}{\partial y} + \varepsilon e^{nt} \frac{\partial u_1}{\partial y} \right)_{y=0} = -m_5K_{13} - m_3K_{10} - m_1K_{11} - m_1K_{12} + \varepsilon e^{nt} (-m_6K_{27} - m_5K_{14} - m_3K_{15} - m_1K_{16} - m_1K_{17} + m_4K_{18} + m_3K_{19} + m_1K_{20} - m_2K_{21} - m_1K_{22} + m_2K_{23} + m_1K_{24})$$

Knowing the temperature field, it is interesting to study the non-dimensional form of the rate of heat transfer in terms of Nusselt number at the plate is given by:

$$N_u = - \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = - \left( \frac{\partial \theta_0}{\partial y} + \varepsilon e^{nt} \frac{\partial \theta_1}{\partial y} \right)_{y=0} = -(-m_3K_4 - m_1K_3 + \varepsilon e^{nt} (-m_4K_9 - m_3K_5 - m_1K_6 + m_2K_7 + m_1K_8))$$

Knowing the concentration field, it is interesting to study the non-dimensional form

of the rate of mass transfer in terms of Sherwood number at the plate is given by:

$$S_h = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -\left(\frac{\partial \phi_0}{\partial y} + \varepsilon e^{nt} \frac{\partial \phi_1}{\partial y}\right)_{y=0}$$

$$= -(-m_1 + \varepsilon e^{nt} (-m_2 K_2 - m_1 K_1))$$

#### 4. Results and Discussion

In order to get physical insight into the problem, we have carried out numerical calculations for non-dimensional velocity field, temperature field, concentration field, coefficient of skin friction  $C_f$  at the plate, the rate of heat transfer in terms of Nusselt number  $N_u$  and the rate of mass transfer in terms of Sherwood number  $S_h$  by assigning specific values to the different values to the parameters involved in the problem, viz., Magnetic field parameter  $M$ , Grashof number for heat transfer  $Gr$ , Grashof number for mass transfer  $Gm$ , permeability parameter  $K$ , rarefaction parameter  $h$ , heat sink parameter  $Q$ , Prandtl number  $Pr$ , Radiation parameter  $R$ , Schmidt number  $Sc$  and Dufour number  $Du$ , time  $t$ . In the present study, the following default parametric values are adopted.  $Gr = 6.0$ ,  $Gm = 4.0$ ,  $M = 3.0$ ,  $K = 1.0$ ,  $n = 0.1$ ,  $A = 1.0$ ,  $t = 1.0$ ,  $Pr = 0.71$ ,  $R = 1.0$ ,  $Q = 1.0$ ,  $Sc = 0.6$ ,  $Du = 1.0$ ,  $h = 0.3$  and  $\varepsilon = 0.2$ . All graphs therefore correspond to these unless specifically indicated on the appropriate graph.

It is observed from Fig. 2 that an increase in Grashof number for heat transfer  $Gr$  leads to a rise in the values of velocity  $u$  due to enhancement in buoyancy force.

The plot of velocity profile for different values of Grashof number for mass transfer  $Gm$  is given in Fig. 3 It is observed that velocity increase for the increasing values of Grashof number for mass transfer  $Gm$ .

The changes in velocity profile due to different permeability of the porous medium are plotted in Fig. 4. This figure shows that the velocity profiles increases rapidly as increases permeability of porous medium  $K$ .

Fig. 5 plots the velocity profiles against for different magnetic field parameter  $M$ . this illustrates that

velocity decreases as the existence of magnetic field becomes stronger. This conclusion agrees with the fact that magnetic field exerts retarding force on the free-convection flow.

The effect of the Radiation parameter  $R$  on the dimensionless velocity  $u$  is shown in Fig. 6 shows that velocity component decreases with an increase in the radiation parameter  $R$ .

Fig. 7 illustrates the dimensionless velocity  $u$  for different values of the Prandtl number  $Pr$ . The analytical results show that the effect of increasing values of Prandtl number results in a decreasing velocity.

Fig. 8 depicts the dimensionless velocity component  $u$  profiles for different values of heat sink parameter  $Q$ . It is noticed that an increase in the heat sink parameter  $Q$  results in decrease in the dimensionless velocity component  $u$  within the boundary layer.

The influence of the Schmidt number  $Sc$  on velocity profiles are plotted in Fig. 9. The Schmidt number  $Sc$  embodies the ratio of the momentum to the mass diffusivity. It is noticed that as the Schmidt number  $Sc$  increases the velocity decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

Fig. 10 illustrates the velocity profiles for different values of the Dufour number  $Du$ . The analytical results show that the effect of increasing values of Dufour number results in a decreasing velocity.

Fig. 11 indicated that the velocity component  $u$  for different values of rarefaction parameter  $h$ . The velocity  $u$  increases as the rarefaction parameter  $h$  is increased indicating the fact that slips at the surface accelerates the fluid motion.

From Fig. 12, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer.

Fig. 13 shows the variation of temperature profiles with respect to the radiation parameter  $R$ . from this figure, it is observed that as

temperature increases for the increasing values of radiation parameter  $R$ . This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

The influence of the parameter heat sink on dimensionless temperature profiles  $\theta$  is plotted in Fig. 14. It is noticed that dimensionless temperature decreases with an increase in heat sink parameter  $Q$ .

The temperature profiles for different values of Schmidt number  $Sc$  are plotted in Fig. 15. The analytical results show that the effect of increasing Schmidt number decreases the temperature profile.

Fig. 16 shows the variation of temperature profiles with respect to the chemical reaction  $Kc$ . From this figure, it is observed that as temperature decreases for the increasing values of chemical reaction  $Kc$

Fig. 17 illustrates the temperature profiles for different values of the Dufour number  $Du$ . The analytical results show that the effect of increasing values of Dufour number results in a decreasing temperature.

The concentration profiles for different values of Schmidt number  $Sc$  are plotted in Fig. 18. The analytical results show that the effect of increasing Schmidt number decreases the concentration profile.

For various values of chemical reaction  $Kc$ , the concentration profile is plotted in Fig. 19. Clearly as  $Kc$  increases, the dimensionless concentration decreases.

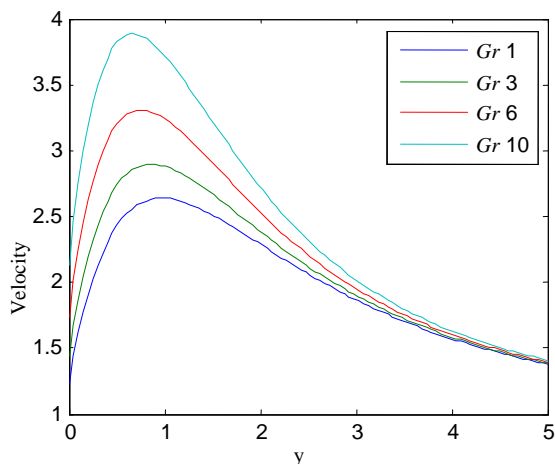


Fig . 2. Velocity Profiles for different values of  $Gr$

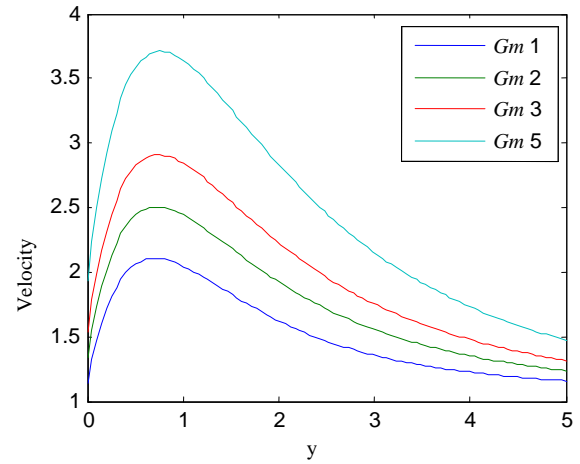


Fig . 3. Velocity Profiles for different values of  $Gm$

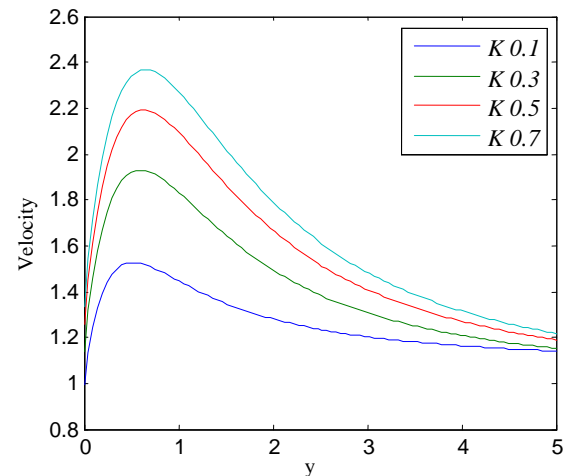


Fig . 4. Velocity Profiles for different values of  $K$

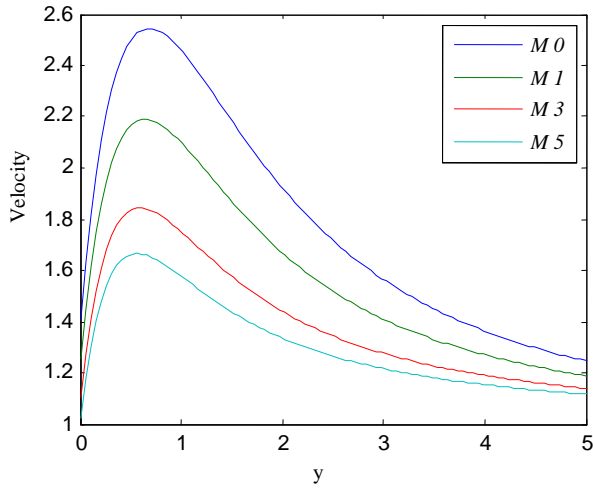


Fig . 5. Velocity Profiles for different values of  $M$

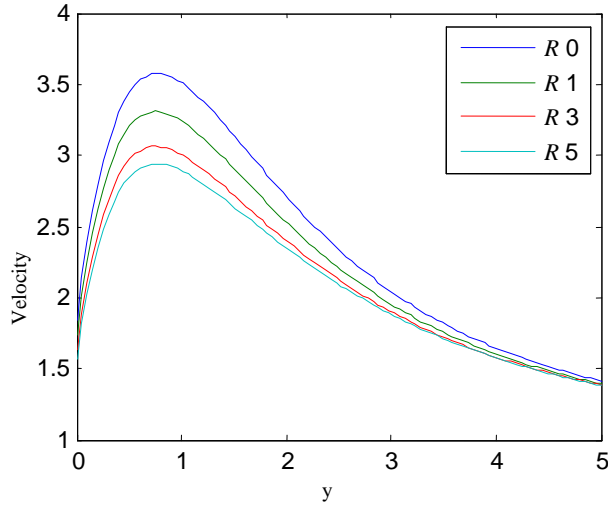


Fig . 6. Velocity Profiles for different values of  $R$

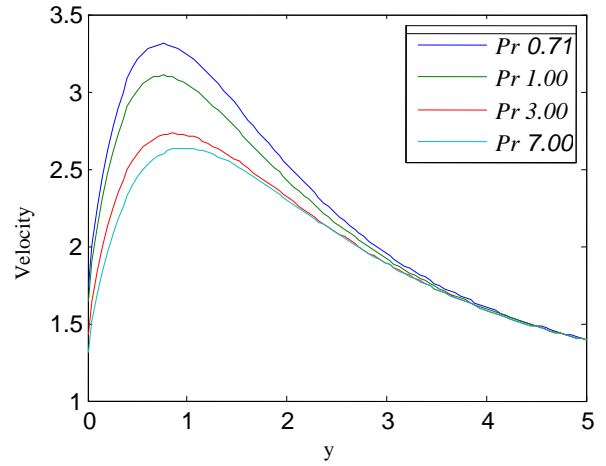


Fig . 7. Velocity Profiles for different values of  $Pr$

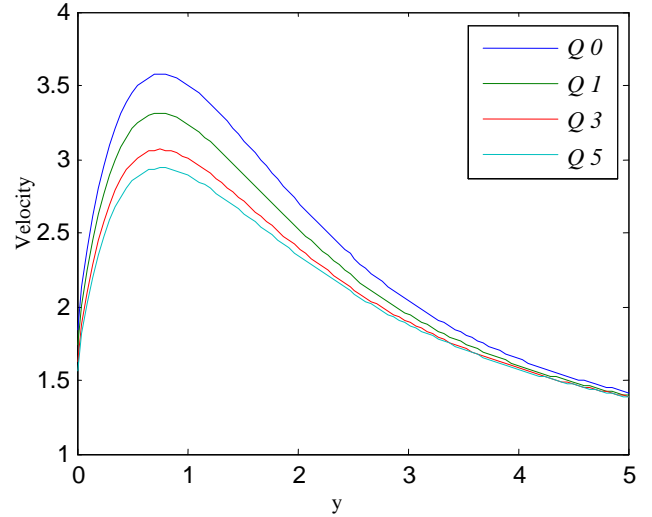


Fig .8. Velocity Profiles for different values of  $Q$



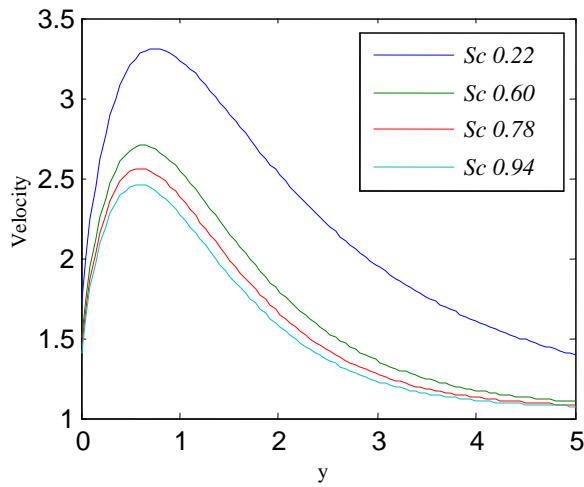


Fig .9. Velocity Profiles for different values of  $Sc$

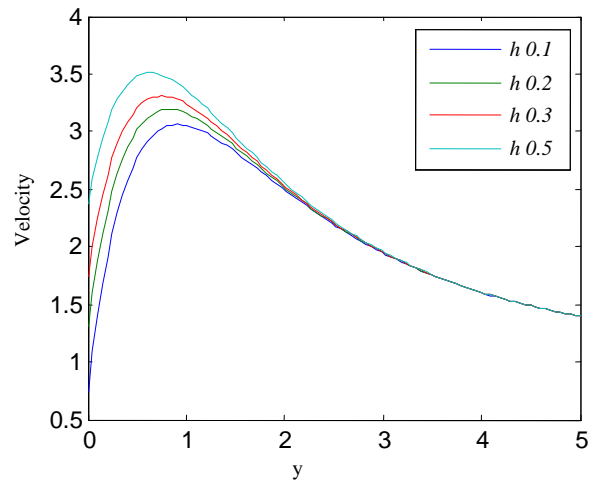


Fig .11. Velocity Profiles for different values of  $h$

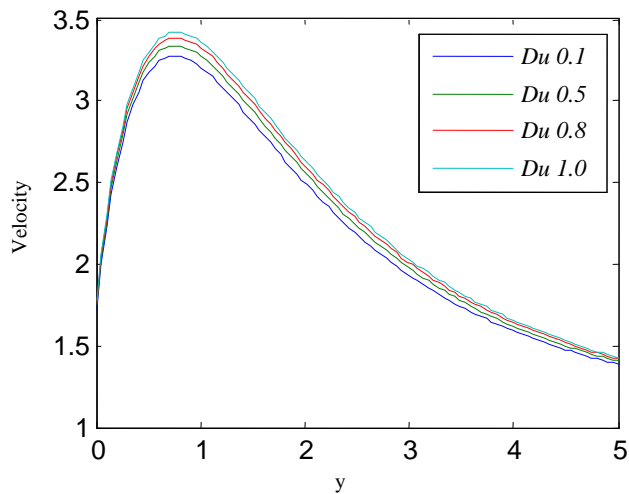


Fig .10. Velocity Profiles for different values of  $Du$

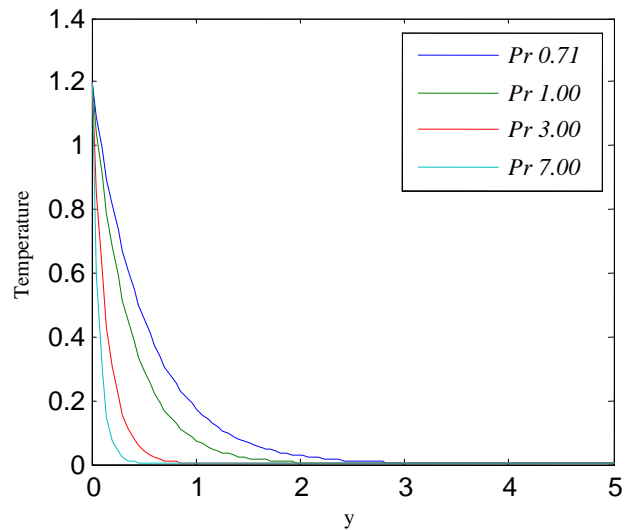


Fig 12. Temperature Profiles for different values of  $Pr$

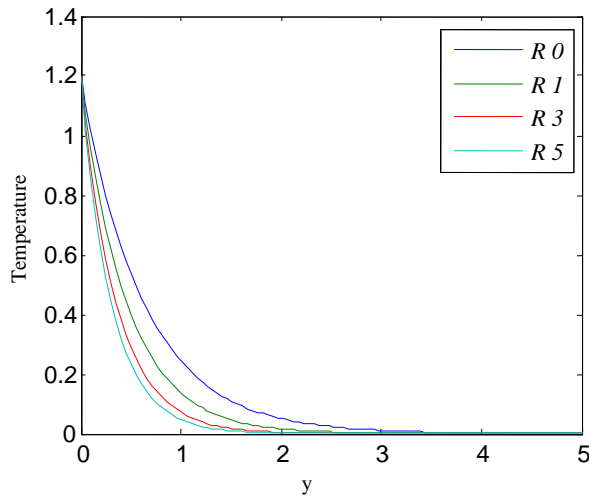


Fig .13 Temperature Profiles for different values of  $R$

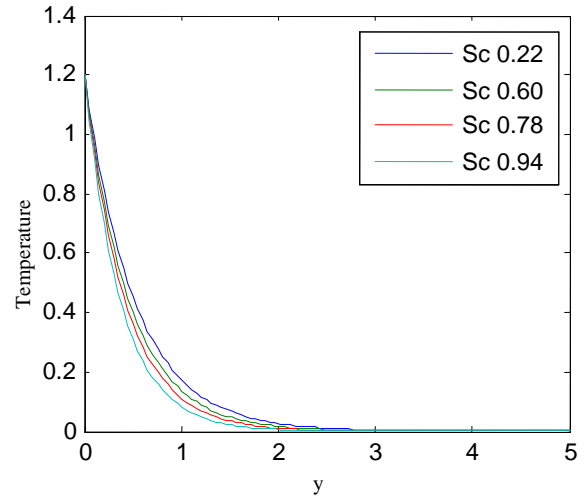


Fig 15 Temperature Profiles for different values of  $Sc$

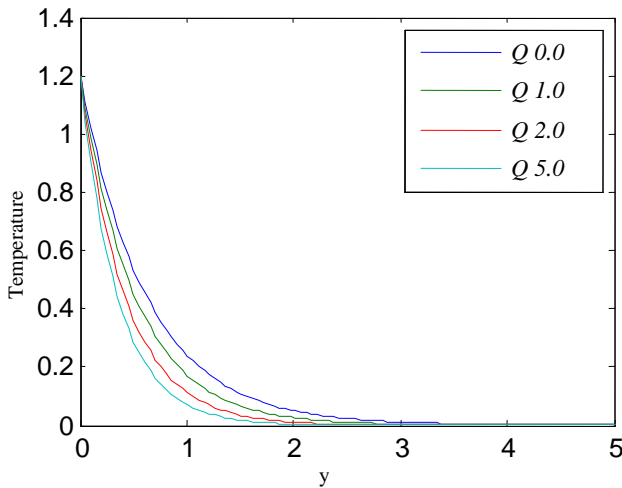


Fig .14 Temperature Profiles for different values of  $Q$

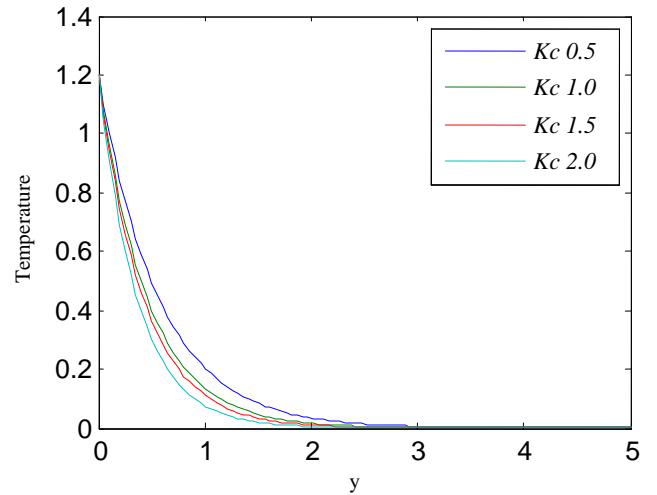


Fig 16 Temperature Profiles for various values of  $Kc$

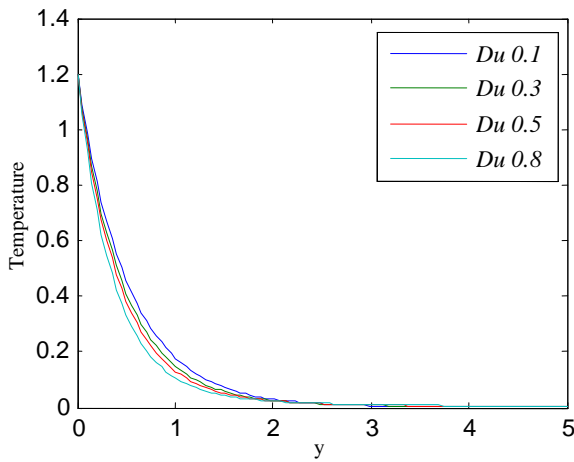


Fig 17 Temperature Profiles for various values of  $Du$

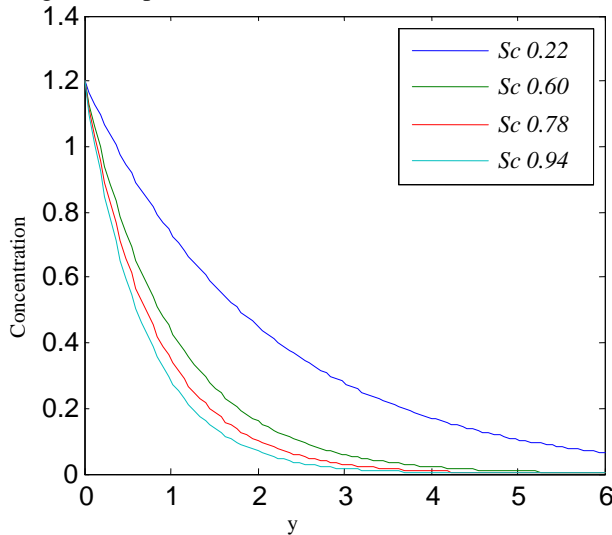


Fig 18 Concentration Profiles for various values of  $Sc$

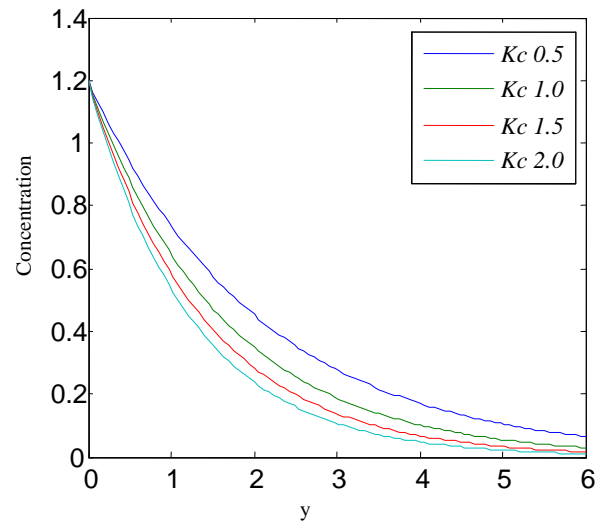


Fig 19 Concentration Profiles for various values of  $Kc$

## 5. Conclusion

In this study, we examined the Dufour effects on unsteady MHD free convection and mass transfer flow past through a porous medium in a slip regime with heat source/sink. The leading governing equations are solved analytically by perturbation method. We present the results to illustrate the flow characteristics for the velocity, temperature and concentration and show how the flow fields are influenced by the material parameters of the flow problem. We can conclude from these results that

1. An increase in  $Gr$ ,  $Gm$ ,  $K$ , and  $h$  increases the velocity field, while an increase in  $M$ ,  $R$ ,  $Pr$ ,  $Q$ ,  $Sc$  and  $Du$  decreases the velocity field.
2. An increase in  $R$  increases the temperature distribution, while an increase in  $Pr$ ,  $Q$ ,  $Sc$ ,  $Kc$  and  $Du$  decrease the temperature distribution.
3. An increase in  $Sc$  decreases the concentration distribution, while an increase in  $Kc$  decreases the concentration distribution.

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### Appendix

$$m_1 = \frac{Sc + \sqrt{Sc^2 + 4ScKc}}{2}$$

$$m_2 = \frac{Sc + \sqrt{Sc^2 + 4Sc(Kc + n)}}{2}$$

$$m_3 = \frac{Pr + \sqrt{Pr^2 + 4Pr(R + Q)}}{2}$$

$$m_4 = \frac{\text{Pr} + \sqrt{\text{Pr}^2 + 4\text{Pr}(R+Q+n)}}{2} \quad m_5 = \frac{1 + \sqrt{1+4N}}{2}$$

$$m_6 = \frac{1 + \sqrt{1+4(N+n)}}{2}$$

$$K_1 = \frac{A\text{Sc}m_1}{m_1^2 - \text{Sc}m_1 - \text{Sc}(Kc+n)} \quad K_2 = 1 - K_1$$

$$K_3 = \frac{-Du\text{Pr}m_1^2}{m_1^2 - \text{Pr}m_1 - \text{Pr}(R+Q)} \quad K_4 = 1 - K_3$$

$$K_5 = \frac{A\text{Pr}K_4m_3}{m_3^2 - \text{Pr}m_3 - (R+Q+n)}$$

$$K_6 = \frac{A\text{Pr}K_3m_1}{m_1^2 - \text{Pr}m_1 - (R+Q+n)}$$

$$K_7 = \frac{Du\text{Pr}K_2m_2^2}{m_2^2 - \text{Pr}m_2 - (R+Q+n)}$$

$$K_8 = \frac{Du\text{Pr}K_1m_1^2}{m_1^2 - \text{Pr}m_1 - (R+Q+n)}$$

$$K_9 = 1 - K_5 - K_6 + K_7 + K_8 \quad K_{10} = \frac{-GrK_4}{m_3^2 - m_3 - N}$$

$$K_{11} = \frac{-GrK_3}{m_1^2 - m_1 - N} \quad K_{12} = \frac{-Gm}{m_1^2 - m_1 - N}$$

$$K_{13} = \frac{1 + K_{10} + K_{11} + K_{12} + hm_3K_{10} + hm_1K_{11} + hm_1K_{12}}{-hm_5 - 1}$$

$$K_{14} = \frac{Am_5K_{13}}{m_5^2 - m_5 - (N+n)} \quad K_{15} = \frac{Am_3K_{10}}{m_3^2 - m_3 - (N+n)}$$

$$K_{16} = \frac{Am_1K_{11}}{m_1^2 - m_1 - (N+n)} \quad K_{17} = \frac{Am_1K_{12}}{m_1^2 - m_1 - (N+n)}$$

$$K_{18} = \frac{GrK_9}{m_4^2 - m_4 - (N+n)} \quad K_{19} = \frac{GrK_5}{m_3^2 - m_3 - (N+n)}$$

$$K_{20} = \frac{GrK_6}{m_1^2 - m_1 - (N+n)} \quad K_{21} = \frac{GrK_7}{m_2^2 - m_2 - (N+n)}$$

$$K_{22} = \frac{GrK_8}{m_1^2 - m_1 - (N+n)} \quad K_{23} = \frac{GmK_2}{m_2^2 - m_2 - (N+n)}$$

$$K_{24} = \frac{GmK_1}{m_1^2 - m_1 - (N+n)}$$

$$K_{25} = 1 + K_{14} + K_{15} + K_{16} + K_{17} - K_{18} - K_{19} - K_{20} + K_{21} + K_{22}$$

$$K_{26} = -hm_5K_{14} - hm_3K_{15} - hm_1K_{16} - hm_1K_{17} + hm_4K_{18} + hm_1K_{19} - hm_1K_{20} + hm_1K_{21} + hm_1K_{22}$$

$$K_{27} = \frac{K_{25} - K_{26}}{-hm_6 - 1}$$