Numerical Analysis the Equations of Heat and Mass Transfer in Cooling Towers

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Abstract

In the paper, a Merkel simulation is first applied to mass and heat transfer processes, which is then used in almost all analyses of cooling towers. Taking into account thermodynamic properties of water and air and calculating heat transfer between the two, differential relations between enthalpy and temperature changes are then obtained at the core of cooling towers. Using proper numerical methods, the equations are numerically and more exactly when compared with other articles including those of Darvishi and Ashraf Kotb solved. The obtained values are then compared with output values for a real powerhouse (Ramin Power plant, Khouzestan). Finally, some variables including air wet bulb temperature, input air volume, and number of fans are changed and the influence of these parameters is studied on efficiency of cooling towers and output water temperature.

Keywords: cooling tower , numerical analysis , finite difference , romberg method.

1. Introduction

Cooling towers are a very important part of many chemical plants. The primary task of a cooling tower is to reject heat into the atmosphere. They represent a relatively inexpensive and dependable means of removing low-grade heat from cooling water. The make-up water source is used to replenish water lost to evaporation. Hot water from heat exchangers is sent to the cooling tower. The water exits the cooling tower and is sent back to the exchangers or to other units for further cooling. Cooling towers fall into two main categories: Natural draft and Mechanical draft. Natural draft towers use very large concrete chimneys to introduce air through the media. Due to the large size of these towers, they are generally used for water flow rates above 45,000 m3/hr. These types of towers are used only by utility power stations.

Mechanical draft towers utilize large fans to force or suck air through circulated water. The water falls downward over fill surfaces, which help increase the contact time between the water and the air - this helps maximise heat transfer between the two. Cooling rates of Mechanical draft towers depend upon their fan diameter and speed of operation. Since, the mechanical draft cooling towers are much more widely used, the focus is on them in this paper.

A lot of work has been done for modeling cooling towers in the past century. Walker *et al.* (1923) proposed a basic theory of cooling tower operation. Merkel (1925) developed the first practical theory including the differential equations of heat and mass transfer, which has been well received as the basis for most work on cooling tower modeling and analysis (Khan *et al.*, 2003; Elsarrag, 2006; Qureshi and Zubair, 2006; ASHRAE, 2008; Lucas *et al.*, 2009). In Merkel's model, in order to simplify the analysis, the water loss of evaporation is neglected, and the Lewis relation is assumed as unity. These assumptions may cause Merkel's model to underestimates the effective tower volume by 5-15% (Sutherland, 1983). Jaber and Webb (1989) introduced the effectiveness-NTU (number of transfer units) design method for counter-flow cooling towers using Merkel's simplified theory. Sutherland (1983) gave a more rigorous analysis of cooling tower including water loss by evaporation. Braun (1988) and Braun *et al.* (1989) gave a detailed analysis and developed effectiveness models for cooling tower by assuming a linearized air saturation enthalpy and a modified definition of effectiveness using the constant saturation specific heat *Cs*. A modeling framework was developed for estimating the water loss and then validated over a wide range of operating conditions.

Bernier (1994,1995) presented a one-dimensional (1D) analysis of an idealized spray-type tower, whichshowed how the cooling tower performance is
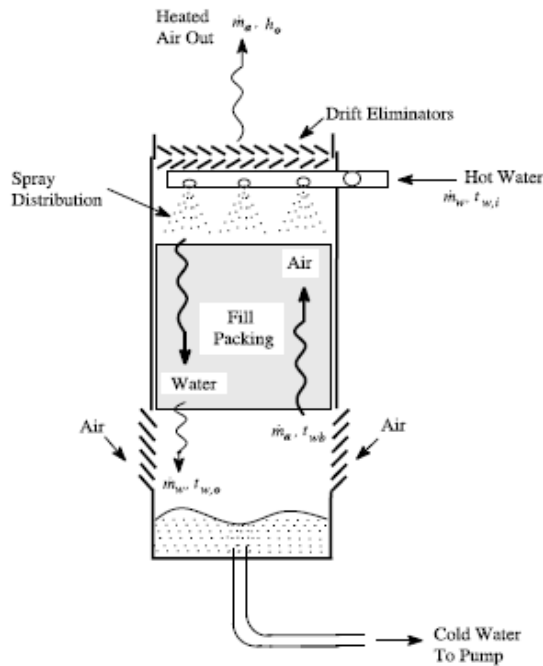

Fig 1: Direct-Contact or Open Evaporative Cooling Tower

affected by the fill height, the water retention time, and the air and water mass flow rates. Fisenko *et al.* (2004) developed a mathematical model of mechanical draft cooling tower, and took into account the radii distribution of the water droplets.

2. Governing Equations

The processes of heat and mass transfer through the counter-flow cooling tower (Figure 2) are mathematically modeled with the finite volume method. The control volume is shown in Figure1 B, C for water and moist air where the flows are in opposite directions. The modeling methodology is governed by the following assumptions.

1. One-dimensional flow.
2. Steady-state and steady flow conditions.
3. Heat and mass transfer are in the direction normal to the water/air flow only.
4. Heat and mass transfer through the tower walls to the surroundings are negligible.
5. Water loss by drift is negligible.

6. The process is isobaric at standard atmospheric value.
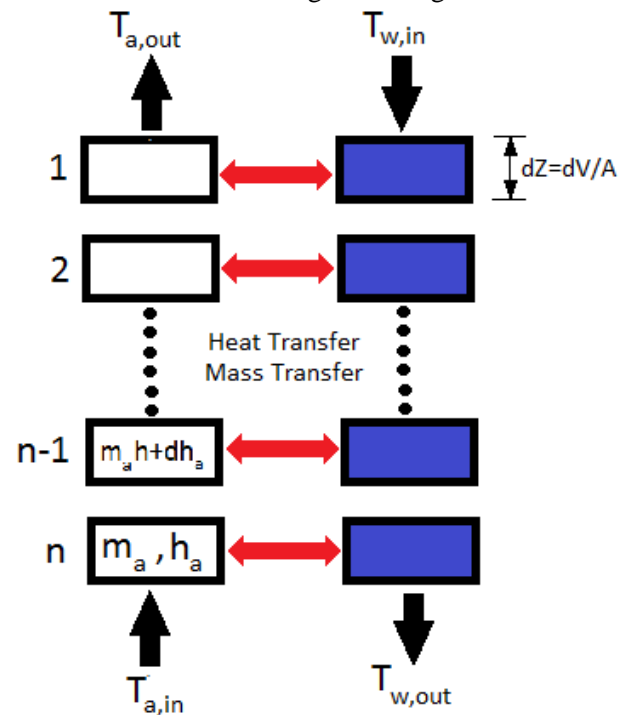7. Potential and kinetic energies are neglected.

Fig 2: Control Volume of Counter Flow Tower

The conservation of mass flow rates for the dry air, moisture content and water through the control volume yields:

$$m_a + w m_a + m_w = m_a + (w + dw)m_a + (m_w + dm_w) \quad (1)$$

m_a : *mass flow rate of dry air*

m_w : *mass flow rate of water*

conservation of mass flow rates for the control volume verifies that; the mass transfer appears in decreasing the water flow rate and increasing the moisture content of the air as a result of evaporation:

$$dm_w = m_a dw \quad (2)$$

Also, the mass transfer flow rate from the water as a result of evaporation into the air is rexpressed by the definition of the mass transfer coefficient and the difference in the concentrations of the moisture content of the air:

$$m_a dw = h_d A_V (w_{sw} - w) dV \quad (3)$$

Since, the control volume is defined as:

$$dV = A dz$$

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$$m_a dw = h_d A_V (w_{sw} - w) A dz$$

$$\frac{dw}{dz} = \frac{h_d A_V}{m_a}(w_{sw} - w)A \qquad (4)$$

By substituting $Ka = h_d A_V$ have:

$$\frac{dw}{dz} = \frac{KaA}{m_a}(w_{sw} - w) \qquad (5)$$

The conservation of energy rates for moist air and water through the control volume yields:

$$m_w h_w + m_a h_a = m_a(h_a + dh_a) + (m_w - dm_w)(h_w - dh_w)$$

Neglecting the second order derivatives $dm_w dh_w \approx 0$

$$m_a dh_a = m_w dh_w + h_w dm_w \qquad (6)$$

By inserting Equation (2) into Equation (6)

$$m_a dh_a = m_w dh_w + h_w m_a dw \qquad (7)$$

$m_a dm_a$ which represents the total rate of heat transferred to the moist air.

$m_w dm_w$ represents the rate of heat transfer from the water to the air and appears as sensible heating .

$h_w m_a dw$ represents the rate of heat transfer from the water to the air and appears as a humidification.

Therefore, one can say that; the rate of heat transferred from the falling water to uprising air is transferred as a result of convection and is associated with mass transfer from water to air:

$$dQ_{tot} = dQ_c + dQ_m$$

$$m_a dh_a = dQ_c + dQ_m$$

While, the rates of heats transfer by convection and evaporation are re-written:

$$m_a dh_a = h_c a_c (T_w - T_a) dV + h_m \rho_a a_c (w_{sw} - w) h_V dV$$

Using both Lewis factor

$$\frac{dh_a}{dz} = \frac{KaA}{m_a}(Le Cp_a (T_w - T_a) + (w_{sw} - w) h_V) \qquad (8)$$

Equation (7) is re-written as follows:

$$dT_W = \frac{m_a}{m_w C_w}(dh_a h_W dw) \qquad (9)$$

$$\frac{dT_W}{dz} = \frac{m_a}{m_w}(\frac{1}{Cp_w}\frac{dh_a}{dz} - T_W \frac{dw}{dz}) \qquad (10)$$

$$\frac{dh_a}{dT_W} = (1 + \frac{Cp_W T_W (w_{s,w} - w)}{(h_{s,w} - h_a - (w_{s,w} - w)Cp_w T_w)}) \qquad (11)$$

$$\frac{dw}{dT_W} = \frac{Cp_W \frac{m_W}{m_a}(w_{s,w} - w)}{(h_{s,w} - h_a - (w_{s,w} - w)Cp_w T)_w} \qquad (12)$$

The set of Equations (2) , (10) , (11) and (12) govern the processes of heat and mass transfer through the counter-flow cooling tower.

Lewis factor is a n indication of the relative rates of heat and mass transfer in an evaporative process. *Bosnjakovic* [2] developed an empirical relation for the Lewis factor for unsaturated air–water vapor systems :

$$Le = 0.865^{0.667}\frac{((w_{sw} + 0.622)/(w + 0.622)) - 1}{\ln((w_{sw} + 0.622)/(w + 0.622))}$$

The cooling tower characteristic, a "degree of difficulty" to cooling is represented by the *Merkel* Equation:

$$\frac{KaV}{m_w} = \int_{T_{w,out}}^{T_{w,in}} \frac{Cp_w dT_w}{h_{s,w} - h_a} \qquad (13)$$

$$\frac{KaV}{m_a} = \int_{h_{a,in}}^{h_{a,out}} \frac{dh_a}{h_{s,w} - h_a} \qquad (14)$$

The *Merkel* Equation primarily says that at any point in the tower, heat and water vapor are transferred into the air due to (approximately) the difference in the enthalpy of the air at the surface of the water and the main stream of the air. Thus, the driving force at any point is the vertical distance between the two operating lines. Therefore, the performance demanded from the cooling tower is the inverse of this difference.

3. Numerical Modeling

To solve the equations (11) and (12) the use of finite differences as well as to calculate integrals of the equation (13) was used of Romberg integration method. The view of solving the problem of method in Figure (2) below.

The mathematical model is numerically formulated based on the explicit scheme. Where the cooling tower is divided into equal numbers of control volumes (i=1), the first control volume (i=n) represents the lower boundary conditions at the air entrance level; the last control (i=1) represents the upper boundary conditions at the air exit level. Therefore, Equations (11) , (12) are respectively numerically formulated as follows.

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Equations 11 and 12 can be shown as follows.

$$\frac{dw}{dT_W} = f(w, h_a, T_W)$$

$$\frac{dh_a}{dT_W} = g(w, h_a, T_W)$$

For ordinary differential equations dw/dT_W and dh_a/dT_W we use the forward finite difference method.

$$\frac{dw}{dT_W} = \frac{w_{n+1} - w_n}{\Delta T_W}$$

$$\frac{dh_a}{dT_W} = \frac{h_{a,n+1} - h_{a,n}}{\Delta T_W}$$

$$w_{n+1} = w_n + [f(w_n, h_{a,n}, T_{W,n})]\Delta T_W$$

$$h_{a,n+1} = h_{a,n} + [g(w_n, h_{a,n}, T_{W,n})]\Delta T_W$$

n=0,1,2,3......,N

If n=1
$$w_2 = w_1 + [f(w_1, h_{a,1}, T_{W,1})]\Delta T_W$$
$$h_{a,2} = h_{a,1} + [g(w_1, h_{a,1}, T_{W,1})]\Delta T_W$$

If n=2
$$w_3 = w_2 + [f(w_2, h_{a,2}, T_{W,2})]\Delta T_W$$
$$h_{a,3} = h_{a,2} + [g(w_2, h_{a,2}, T_{W,2})]\Delta T_W$$

If n=N
$$w_N = w_{n-1} + [f(wn-1_2, h_{a,n-1}, T_{W,n-1})]\Delta T_W$$

$$h_{a,N} = h_{a,n-1} + [g(w_{n-1}, h_{a,n-1}, T_{W,n-1})]\Delta T_W$$

The integral in the equation (13) is solved by Romberg integration. The number of tower characteristic is calculated by:

$$\frac{KaV}{m_w} = \int_{T_{w,out}}^{T_{w,in}} \frac{Cp_w dT_w}{h_{s,w} - h_a} = Cp_w \int_{T_{w,out}}^{T_{w,in}} \frac{dT_w}{dh}$$

$$\Delta y_1 = T_{w,in} - T_{w,out}$$
$$\Delta y_2 = (T_{w,in} - T_{w,out})/2$$
$$\Delta y_3 = (T_{w,in} - T_{w,out})/4$$
$$\Delta y_4 = (T_{w,in} - T_{w,out})/8$$

$$R_{1,1} = \frac{(T_{w,in} - T_{w,out})}{2}\left[\frac{1}{h_{a,out}} + \frac{1}{h_{a,in}}\right]$$

$$R_{2,1} = \frac{1}{2}\left[R_{1,1} + \frac{1}{\Delta y_1 h_{a,T_{out+\Delta y_2}}}\right]$$

$$R_{3,1} = \frac{1}{2}\left[R_{2,1} + \frac{1}{\Delta y_2 h_{a,T_{out+\Delta y_3}}} + \frac{1}{\Delta y_2 h_{a,T_{out+3\Delta y_3}}}\right]$$

$$R_{4,1} = \frac{1}{2}[R_{3,1} + \frac{1}{\Delta y_3 h_{a,T_{out+\Delta y_4}}} + \frac{1}{\Delta y_3 h_{a,T_{out+3\Delta y_4}}} + \frac{1}{\Delta y_3 h_{a,T_{out+5\Delta y_4}}}$$
$$+ \frac{1}{\Delta y_3 h_{a,T_{out+8\Delta y_4}}}]$$

$$R_{2,2} = R_{2,1} + \frac{R_{2,1} - R_{1,1}}{3}$$

$$R_{3,2} = R_{3,1} + \frac{R_{3,1} - R_{2,1}}{3}$$

$$R_{4,2} = R_{4,1} + \frac{R_{4,1} - R_{3,1}}{3}$$

$$R_{3,3} = R_{3,2} + \frac{R_{3,2} - R_{2,2}}{3}$$

$$R_{4,3} = R_{4,2} + \frac{R_{4,2} - R_{3,2}}{3}$$

$$R_{4,4} = R_{4,3} + \frac{R_{4,3} - R_{3,3}}{3}$$

$$\Rightarrow \frac{KaV}{m_W} = Cp_W * R_{4,4}$$

```
========================================
======================= INPUT =========
========================================
Twout=30c |Twin=39.4c |Tdb=30c |Twb=20.6c |Mw=899.72kg/s |Ma=802.5kg/s
========================================
============= ROMBERG INTEGRETION =======
========================================
R11= 0.10116
R21= 0.05096      R22= 0.03422
R31= 0.02732      R32= 0.01944      R33= 0.01451
R41= 0.01954      R42= 0.01694      R43= 0.01611      R44= 0.01665

     KAV/Mw=0.623377
========================================
```

Table1: Tower Characteristic is calculated by Romberg integration

4. The Proposed Model

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Basically, the problem of cooling tower into the general plan:

1-Design

$$T_{Win}, T_{W,out}, T_{wb}, T_{db}, m_W, m_a, h, A, P_{atm} \Rightarrow V(volume\ )$$

2-Ratings

$$T_{Win}, T_{wb}, T_{db}, m_W, m_a, h, A, P_{atm}, V \Rightarrow T_{W,out}(Out\ \ Temp\ )$$

Having information on the design performance of a cooling tower or the tower capacity is obtained. The ratings of the Tower and information on the conditions of entry and exit of air and water conditions entrance to the tower, the tower outlet water temperature is obtained.

Assess the accuracy of the answers

To evaluate the accuracy of the answers obtained from the are used from articles Sutherland [9] and

```
--------------------------------------------------------------
------------  Curve for Simpson & Sherwood dete  -------------
------------------------  INPUT DETA  ------------------------
case1 | Twout=23.88c | Twin=31.22c | Tdb=37.05c | Twb=21.11c | Mw=0.754kg/s | Ma=1.158kg/s |

case2 | Twout=24.22  | Twin=28.72c | Tdb=29c    | Twb=21.11c | Mw=1.259kg/s | Ma=1.187kg/s |

case3 | Twout=29.33c | Twin=38.78c | Tdb=35c    | Twb=26.67c | Mw=1.008kg/s | Ma=1.265kg/s |

case3 | Twout=29.33c | Twin=38.78c | Tdb=35c    | Twb=26.67c | Mw=1.008kg/s | Ma=1.250kg/s |
--------------------------------------------------------------
```

Table2: Experimental data provided by Simpson and Sherwood

Jamal Rahman Khan and Zubair [12]. The formula used for experimental NTU in [12] for:
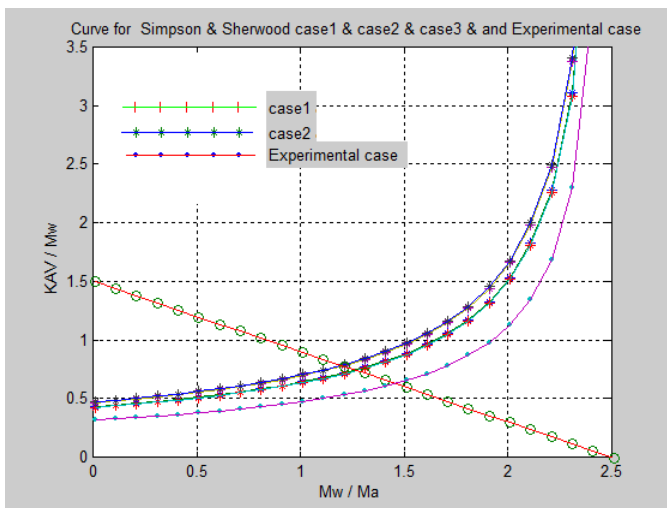
$$NTU_{\exp} = \frac{KaV}{m_W}$$

Fig 1: Tower Characteristic for design conditions in Ramin thermal power plant

$$\varepsilon = \frac{h_{a,out} - h_{a,in}}{h_{s,w} - h_{a,in}} \qquad Effectiveness$$

```
======  KA=3.025(Kg/m3*s)  from paper Zubair  =======

=========================  OUTPUT  ==================
====== EXP:Experimental ===  MT:MATMER === MTP:MATPRES=======

    NTU(Exp) |V(EXP)|NTU(MT)|V(MT)|ERROR(MT)| NTU(MTP)|V(MTP)|ERROR(MTP)
case1 1.2589 0.3138 1.8357   0.457   0.4564    1.2312  0.471    0.5011
case2 0.9150 0.3803 1.5748   0.655   0.7199    1.8291  0.712    0.8696
case3 1.1616 0.3871 1.7532   0.582   0.5036    1.4301  0.598    0.5450
case4 1.1506 0.3834 1.7575   0.585   0.5259    1.4581  0.602    0.5702

=======  ERROR=|V(EXP) -V|/V(EXP)  ===================
```

Tabel3: NTU Values and Volumes from the paper Zubair and Comparison with Matmer and Matpres Models

Changed Air and Water Temperatures on Tower Performance

The influence of changed thermodynamic features of water and air and some other influential factors in cooling mechanisms of cooling tower are investigated and discussed in this section.
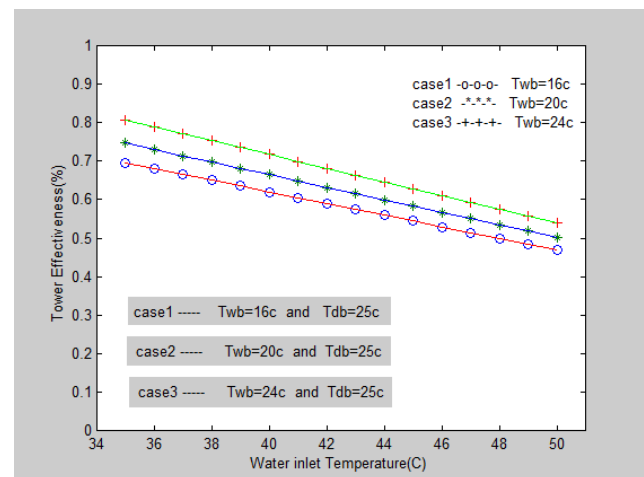

Fig 2: Effects of inlet water temperature and temperature on tower effectiveness

shows different efficiencies of the tower for different input water temperatures and different mass flow rate. As is evident from the figure, when input water temperature increases from 35 to 45 degrees, tower efficiency comes lower and lower. Mass ratio of water to air is fixed here while the input temperature increases. That is why output air cannot reach saturated temperature of input water. This deceasing trend is almost the same for different mass ratios.

The Influence of Changed Mass Ratio on Tower Performance

In towers with forced flows, changed mass flow rate can occur due to either changed water flow rate or

changed air flow rate, which itself is resulted by changed number, speed, or angle of fans. Changed air mass flow rate has a direct influence on tower performance and output water temperature.
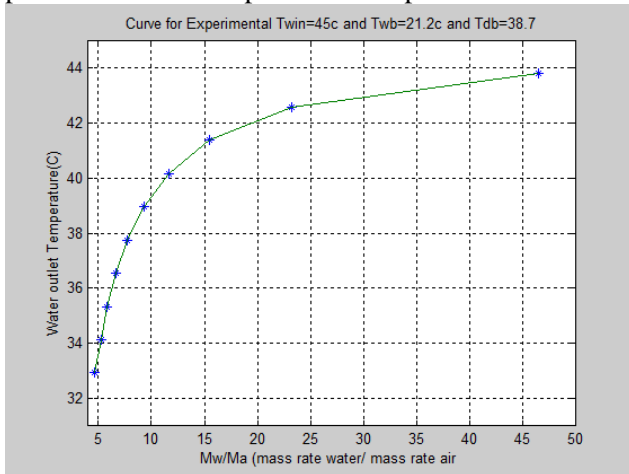

Fig 3: Effects of inlet water flow rate and temperature Water outlet

Here, with regard to the figure, the influence of mass ratio rate is discussed on tower performance at aforementioned temperatures: 45-degree input water, 21.2-degree wet environment, and 38.7-degree environment.
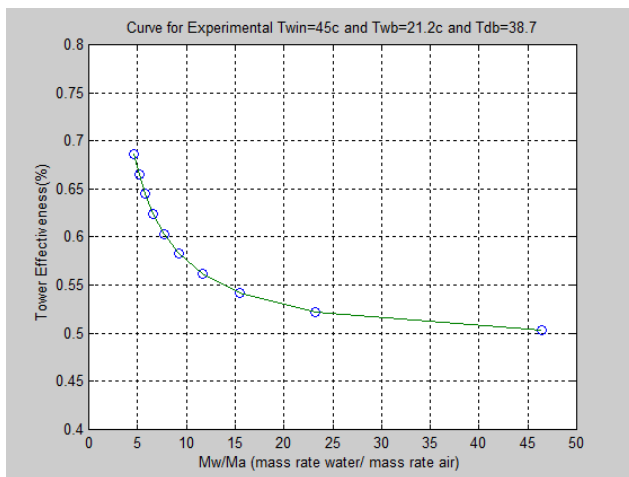

Fig 4: Effects of inlet mass ratio rate and tower effectiveness

With respect to obtained results of the figure and increased output water temperature with increased mass flow rate, and taking into account the fact that tower efficiency is dependent to output water temperature, it is expected that as output water

temperature goes up, a decrease be seen in tower efficiency. As is evident from the figure, tower efficiency comes down as mass ratio rate increases, which was not un-expected.

5. Conclusion

1- A comparison of the proposed model with other existing models shows that the present model is accurate enough for investigation of cooling towers performance; it, furthermore, has less simplifying assumptions.

2-Experimental results and results of numerical simulations reveal the fact that by having one single point of tower performance and calculation of its properties, output water temperature can be estimated with proper accuracy at different environmental conditions.

3- Due to cooling towers' performance, water temperature comes down as water moves toward lower points of towers. This decreasing trend is a bit different for different mass ratios such that for lower mass ratios, maximum changes of water temperature occur at tower tops and as water comes down, changes become less.

4- When other variables are fixed, as wet bulb temperature increases, an increase is observed in output water temperature. The environmental variable which is responsible for internal cooling mechanism of towers is, therefore, wet bulb temperature of input air, rather than changed partial humidity of the environment. This parameter is uncontrollable.

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