Path Related Relaxed Cordial Graphs

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I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called edges or a line of G. In this paper, we proved that path related graphs Path P_n, Comp P_nOK_1, Fan P_n+K_1, Doublefan P_n+2K_1 are relaxed Cordial Graphs. For graph theory terminology, we follow [2].

II. PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Relaxed Cordial Labeling of a Graph G with vertex set V is a bijection from V to {-1 , 0, 1} such that each edge uv is assigned the label 1 if |f (u) + f (v) | = 1 or 0 if | f (u) + f (v) | = 0 with the condition that the number of edges labeled with 0 and the number of edges labeled with 1 differ by an atmost 1.

The graph that admits a Relaxed Cordial Labeling (RCL) is called Relaxed Cordial Graph (RCG). In this paper, we proved that path related graphs Path P_n, Comp P_nOK_1, Fan P_n+K_1, Doublefan P_n+2K_1 are relaxed Cordial Graphs.

Definition: 2.1

P_n is a path of length n-1.

Definition: 2.2

The join of G_1 and G_2 is the graph G=G_1+G_2 with vertex set V=V_1∪V_2 and edge set E=E_1∪E_2∪\{UV: u∈V_1, v∈V_2\}. The graph P_n+K_1 is called a Fan and P_n+2K_1 is called the Doublefan.

Definition: 2.3

In a pair of path P_n, i-th vertex of a path P_n is joined with i+1-th vertex of a path P_n. It is denoted by Z-(P_n).

Definition: 2.4

The Middle graph M(G) of a graph G is a graph whose vertex set is V(G) U E(G) and in which two vertices are adjacent if either they are adjacent edges in G or one is vertex of G and other is an edge incident with it.

Definition: 2.5

Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G to the neighbours of the corresponding vertex v in G_2, that is for every vertex u in G_1 there exists v in G_2 such that N(u)=N(v). The resulting graph is known as shadow graph and it is denoted by D_2(G).

Definition: 2.6

[P_n:S_3] is a graph obtained from a path P_n by joining every vertex of a path to a root of a star S_3 by an edge.

Definition: 2.7
The corona $G_1 \odot G_2$ of two graphs $G_1$ and $G_2$ is defined as the graph $G$ obtained by taking one copy of $G_1$ (which has $P_r$ points) and $P_r$ copies of $G_2$ and joining the $i^{th}$ point of $G_1$ to every point in the $i^{th}$ copy of $G_2$. The graph $P_r \odot K_1$ is called a comb.

III. MAIN RESULTS

Theorem: 2.1

Path $P_n (n \geq 3)$ is Relaxed Cordial Graph.

Proof:

Let the graph be a $P_n$.

Let $V(P_n) = \{ u_i : 1 \leq i \leq n \}$

Let $E(P_n) = \{ (u_i, u_{i+1}) : 1 \leq i \leq n-1 \}$

Define $f : V(P_n) \rightarrow \{-1, 0, 1\}$

Case: 1

When $n=2$

The vertex labeling are

$f(u) = 0$

$f(u_i) = \begin{cases} 1 & i \equiv 3 \pmod{4} \\ 0 & i \equiv 1 \pmod{4} \\ -1 & i \equiv 0 \pmod{2} \end{cases} 1 \leq i \leq n$

The induced edge labeling are

$f^*(uu_i) = \begin{cases} 1 & i \equiv 0,1 \pmod{4} \\ 0 & i \equiv 2,3 \pmod{4} \end{cases} 1 \leq i \leq n-1$

$|e_f(0) - e_f(1)| = \begin{cases} 0 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases}$

Therefore, Path $P_n$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Hence, $P_n$ is Relaxed Cordial.

For example, the Relaxed Cordial labeling of $P_7$ is shown in figure 1

Theorem: 2.2

Fan $P_n + K_1$ is Relaxed Cordial Graph.

Proof:

Let $G$ be a Fan $P_n + K_1$

Let $V(G) = \{ u, [u_i : 1 \leq i \leq n] \}$

Let $E(G) = \{ [(uu_i) : 1 \leq i \leq n] \cup [(u_i, u_{i+1}) : 1 \leq i \leq n] \}$

Define $f : V(G) \rightarrow \{-1, 0, 1\}$

Case: 1

When $n=2$

Case: 2 $n \geq 3$

The vertex labeling are

$f(u) = 0$

$f(u_i) = \begin{cases} 0 & i \equiv 1 \pmod{2} \\ 1 & i \equiv 1 \pmod{2} \end{cases} 1 \leq i \leq n$

The induced edge labeling are

$f^*(uu_i) = \begin{cases} 0 & i \equiv 1, n \\ 1 & 2 \leq i \leq n-1 \\ -1 & i \equiv 0 \pmod{2} \end{cases}$

$|e_f(0) - e_f(1)| = \begin{cases} 0 & 2 \leq i \leq n-2 \\ 1 & i = 1, n-1 \end{cases}$

$e_f(0) + 1 = e_f(1)$
Therefore it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Hence, $\text{Fan } P_n + k_1$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of $P_7 + k_1$ is shown in the figure 2.

\textbf{Theorem : 2.3}

$Z - (P_n)$ is Relaxed Cordial Graph.

\textbf{Proof:}

Let the graph $G$ be $Z - (P_n)$

Let $V(G) = \{ u_i : 1 \leq i \leq n \}, \{ v_i : 1 \leq i \leq n \}$

Let $E(G) = \{ [(u_i, u_{i+1}) : 1 \leq i \leq n-1] \cup [(v_i, v_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i, v_{i+1}) : 1 \leq i \leq n-1] \}$

Define $f : V(G) \rightarrow \{-1, 0, 1\}$

\textbf{Case : 1}

When $n=2$,

\textbf{Case : 2}

When $n=3$,

\textbf{Case : 3}

When $n > 3$,

The vertex labeling are

$f(u_i) = 0 \quad 1 \leq i \leq n$

$f(v_i) = \begin{cases} 
1 & i \equiv 2 \text{(mod 4)} \\
0 & i \equiv 1,3 \text{(mod 4)} \\
-1 & i \equiv 0 \text{(mod 4)}
\end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are

$f'(u_i, u_{i+1}) = 0 \quad 1 \leq i \leq n-1$

$f'(v_i, v_{i+1}) = 1 \quad 1 \leq i \leq n-1$

$f'(u_i, v_{i+1}) = \begin{cases} 
1 & i \equiv 1 \text{(mod 2)} \\
0 & i \equiv 0 \text{(mod 2)}
\end{cases} \quad 1 \leq i \leq n$

$|e_f(0) - e_f(1)| = \begin{cases} 
0 & \text{if } n \text{ is odd} \\
1 & \text{if } n \text{ is even}
\end{cases}$

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Let $E(G) = \{(u_i, u_{i+1}) : 1 \leq i \leq n-1\}$, where $u_0 = v_0$ and $u_n = v_n$. Therefore, the graph $G$ satisfies the condition

$$|e_i(0) - e_i(1)| \leq 1.$$ 

Hence $Z - (P_n)$ is Relaxed Cordial.

For example, the Relaxed Cordial labeling of $Z - (P_8)$ is shown in figure 3.

$$\text{Figure 3 : } Z(p_8)$$

**Theorem 2.4**

$M(P_n)$ $n \geq 3$ is Relaxed Cordial Graph.

Let the graph $G$ be $M(P_n)$

Let $V(G) = \{(u_i : 1 \leq i \leq n), (v_i : 1 \leq i \leq n-1)\}$

Let $E(G) = \{(u_i, u_{i+1}) : 1 \leq i \leq n-1\} \cup \{(v_i, v_{i+1}) : 1 \leq i \leq n-2\} \cup \{(u_i, v_i) : 1 \leq i \leq n-1\}$

Define $f : V(G) \rightarrow \{-1, 0, 1\}$

The vertex labeling are

$$f(u_i) = \begin{cases} 
1 & i \equiv 1 \pmod{2} \\
-1 & i \equiv 0 \pmod{2}
\end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$$f'(u_i, u_{i+1}) = \begin{cases} 
0 & 1 \leq i \leq n-1
\end{cases}$$

$$f'(v_i, v_{i+1}) = \begin{cases} 
0 & 1 \leq i \leq n-2
\end{cases}$$

$$f'(u_i, v_i) = 1 \quad 1 \leq i \leq n-1$$

$$f'(u_{i+1}, v_i) = 1 \quad 1 \leq i \leq n-1$$

$$e_i(0) + 1 = e_i(1)$$

Therefore, the graph $G$ satisfies the condition

$$|e_i(0) - e_i(1)| \leq 1.$$ 

Hence $M(p_n)$ is relaxed cordial Graph.

For example, the Relaxed Cordial labeling of $M(P_7)$ is shown in figure 4.

$$\text{figure 4 : } M(p_7)$$

**Theorem 2.5**

$D_2(P_n)$ ($n \geq 2$) is Relaxed Cordial Graph.

Proof:

Let Graph $G$ be $D_2(P_n)$

Let $V(G) = \{(u_i : 1 \leq i \leq n), (v_i : 1 \leq i \leq n)\}$

Let $E(G) = \{(u_i, u_{i+1}) : 1 \leq i \leq n-1\} \cup \{(v_i, v_{i+1}) : 1 \leq i \leq n-1\} \cup \{(u_i, v_i) : 1 \leq i \leq n-1\}$

Define $f : V(G) \rightarrow \{-1, 0, 1\}$

The vertex labeling are

$$f(u_i) = \begin{cases} 
1 & i \equiv 1 \pmod{2} \\
-1 & i \equiv 0 \pmod{2}
\end{cases} \quad 1 \leq i \leq n$$

The induce edge labeling are

$$f'(u_i, u_{i+1}) = \begin{cases} 
0 & 1 \leq i \leq n-1
\end{cases}$$

$$f'(v_i, v_{i+1}) = \begin{cases} 
0 & 1 \leq i \leq n-1
\end{cases}$$

$$f'(u_i, v_i) = 1 \quad 1 \leq i \leq n-1$$

$$f'(u_{i+1}, v_i) = 1 \quad 1 \leq i \leq n-1$$

$$e_i(0) + 1 = e_i(1)$$

Therefore, the graph $G$ satisfies the condition

$$|e_i(0) - e_i(1)| \leq 1.$$ 

Hence, $D_2(p_n)$ is relaxed cordial Graph.
Therefore, the graph $G$ satisfies the condition

$$|e_0 - e_1| \leq 1$$

Hence, $D_2(P_n)$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of $D_2(P_8)$ is shown in the figure 5.

**Theorem 2.6**

$[P_n : S_3]$ $(n \geq 2)$ is Relaxed Cordial Graph.

Proof:

Let $G$ be $[P_n : S_3]$

Let $V(G) = \{[u_i : 1 \leq i \leq n], [v_i : 1 \leq i \leq n], [v_{ij} : 1 \leq i \leq n, 1 \leq j \leq 3]\}$

Let $E(G) = \{[(u_i, u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i, v_i) : 1 \leq i \leq n] \cup [(v_{ij}, v_{i,j+1}) : 1 \leq i \leq n, 1 \leq j \leq 3]\}$

Define $f^r : V(G) \rightarrow \{-1, 0, 1\}$

The vertex labeling are

$$f^r(u_i) = \begin{cases} -1 & i \equiv 1 \pmod{2} \\
0 & i \equiv 0 \pmod{2} \quad 1 \leq i \leq n
\end{cases}$$

$$f^r(v_i) = 0 \quad 1 \leq i \leq n$$

$$f^r(v_{ij}) = \begin{cases} 0 & i \equiv 1 \pmod{2} \\
1 & i \equiv 0 \pmod{2} \quad 1 \leq i \leq n, 1 \leq j \leq 3
\end{cases}$$

The induced edge labeling are

$$f^e(u_i, u_{i+1}) = 1 \quad 1 \leq i \leq n - 1$$

$$f^e(u_i, v_i) = \begin{cases} 1 & i \equiv 1 \pmod{2} \\
0 & i \equiv 0 \pmod{2}
\end{cases}$$

$$f^e(v_i, v_{i,j}) = \begin{cases} 0 & i \text{ is odd} \\
1 & i \text{ is even}
\end{cases}$$

And $|e_0 - e_1| = \begin{cases} 0 & \text{if } n \text{ is odd} \\
1 & \text{if } n \text{ is even}
\end{cases}$

Therefore, it satisfies the condition

$$|e_0 - e_1| \leq 1.$$ 

Hence, the graph $[P_n : S_3]$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of $[P_5 : S_3]$ is shown in the figure 6.

**Theorem 2.7**

Comb $P_n \odot k_1$ is Relaxed Cordial Graph.

Proof:

Let the graph $G$ be $P_n \odot k_1$

Let $V(G) = \{[u_i : 1 \leq i \leq n], [v_1 : 1 \leq i \leq n]\}$

Let $E(G) = \{[(u_i, u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i, v_1) : 1 \leq i \leq n-1] \cup [(v_1, v_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 3]\}$

Define $f : V(G) \rightarrow \{-1, 0, 1\}$

The vertex labeling are

$$f(u_i) = 0 \quad 1 \leq i \leq n$$

$$f(v_1) = \begin{cases} 1 & i \equiv 1 \pmod{2} \\
-1 & i \equiv 0 \pmod{2} \quad 1 \leq i \leq n
\end{cases}$$

The induced edge labeling are
\[ f^* (u_i u_{i+1}) = 0 \quad 1 \leq i \leq n-1 \]
\[ f^* (u_i v_i) = 1 \quad 1 \leq i \leq n \]

And \( e_i(0) +1 = e_i(1) \)

Therefore, the graph \( P_n \bigodot k_1 \) satisfies the condition
\[ |e_i(0) - e_i(1)| \leq 1. \]

Hence, \( P_n \bigodot k_1 \) is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of \( P_7 \bigodot k_1 \) is shown in the figure 7

![Figure 7: \( P_7 \bigodot k_1 \)](image)

### 4. References


