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Path Related Relaxed Cordial Graphs

Dr. A. Nellai Murugan and R. Megala
Department of Mathematics
V.O.Chidambaram College
Tuticorin 628 008

Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Relaxed Cordial Labeling of a Graph G with vertex set V is a bijection from V to {-1 , 0, 1} such that each edge uv is assigned the label 1if |f (u) + f (v) | = 1or 0 if | f (u) + f (v) | = 0 with the condition that the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Relaxed Cordial Labeling (RCL) is called Relaxed Cordial Graph (RCG). In this paper, we proved that path related graphs Path P_n, Comp $P_n \Theta K_1$, Fan P_n+K_1, Doublefan P_n+2K_1 are relaxed Cordial Graphs.

Keywords–Fan, Comp, Doublefan, Ladder, Relaxed Cordial Graph, Relaxed Cordial Labeling.

2000 Mathematics Subject classification 05C78.

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called edges or a line of G. In this paper, we proved that path related graphs Path P_n, Comp $P_n \Theta K_1$, Fan P_n+K_1, Doublefan P_n+2K_1 are Relaxed Cordial Graphs. For graph theory terminology, we follow [2].

II. PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Homo Cordial Labeling of a Graph G with vertex set V is a bijection from from V to {-1 , 0, 1} such that each edge uv is assigned the label 1if |f (u) + f (v) | = 1or 0 if | f (u) + f (v) | = 0 with the condition that the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Relaxed Cordial Labeling (RCL) is called Relaxed Cordial Graph (RCG). In this paper, we proved that path related graphs Path P_n, Comp $P_n \Theta K_1$, Fan P_n+K_1, Doublefan P_n+2K_1 are relaxed Cordial Graphs.

Definition:2.1

$P_n$is a path of length n-1.

Definition:2.2

The join of G_1 and G_2 is the graph $G=G_1+G_2$ with vertex set $V=V_1 \cup V_2$ and edge set $E=E_1 \cup E_2 \cup$ {UV: $u \in V_1$, $v \in V_2$}. The graph P_n+K_1 is called a Fan and P_n+2K_1 is called the Doublefan.

Definition: 2.3

In a pair of path P_n, i^{th} vertex of a path P_n is joined with $i+1^{th}$ vertex of a path P_n . It is denoted by Z-(P_n).

Definition:2.4

The Middle graph M(G) of a graph G is a graph whose vertex set is V(G) ∪ E(G) and in which two vertices are adjacent if either they are adjacent edges in G or one is vertex of G and other is an edge incident with it.

Definition:2.5

Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G to the neighbours of the corresponding vertex v in G_2, that is for every vertex u in G_1 there exists v in G_2 such that N(u)=N(v). The resulting graph is known as shadow graph and it is denoted by $D_2(G)$.

Definition: 2.6

[P_n:S_3] is a graph obtained from a path P_n by joining every vertex of a path to a root of a star S_3 by an edge.

Definition:2.6

The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th}point of G_1 to every point in the i^{th}copy of G_2. The graph $P_n \Theta K_1$ is called a comb.

III. MAIN RESULTS

Theorem : 2.1

Path P_n(n≥ 3) is Relaxed Cordial Graph .

Proof:

Let the graph be a P_n

Let $V(P_n) = \{ [u_i : 1 \leq i \leq n]\}$

$E(P_n) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \}$

Define $f : V(P_n) \to \{-1, 0, 1\}$

The vertex labeling are

$$f(u_i) = \begin{cases} 1 & i \equiv 3 (mod\ 4) \\ 0 & i \equiv 1 (mod 4) \\ -1 & i \equiv 0\ (mod\ 2) \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$$f^* (u_i u_{i+1}) = \begin{cases} 1 & i \equiv 0,1 (mod\ 4) \\ 0 & i \equiv 2,3 (mod\ 4) \end{cases} \quad 1 \leq i \leq n-1$$

$$|e_f(0) - e_f(1)| = \begin{cases} 0 & if\ n\ is\ odd \\ 1 & if\ n\ is\ even \end{cases}$$

Therefore, Path $P_n$satisfies the condition

$|e_f(0) - e_f(1)| \leq 1$

Hence ,P_n is Relaxed Cordial.

For example, the RelxedCardial labeling of P_7 is shown in figure 1

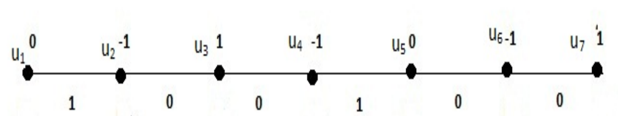


figure1:P_n (n≥ 3)

Theorem : 2.2

Fan $P_n + K_1$ is Relaxed Cordial Graph.

Proof:

Let G be a Fan $P_n + K_1$

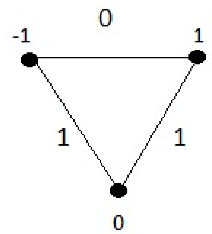
Let $V(G) = \{u, [u_i : 1 \leq i \leq n]\}$

$E(G) = \{[(uu_i) : 1 \leq i \leq n] \cup [(u_i u_{i+1}) : 1 \leq i \leq n]\}$

Define $f : V(G) \to \{-1, 0, 1\}$

Case : 1

When n=2



case : 2 n≥3

The vertex labeling are

f(u) = 0

$$f(u_i) = \begin{cases} 0 & i \equiv 1, n \\ 1 & i \equiv 1(mod\ 2) \\ -1 & i \equiv 0 (mod 2) \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$$f^* (uu_i) = \begin{cases} 0 & i = 1\ ,\ n \\ 1 & 2 \leq i \leq n-1 \end{cases}$$

$$f^* (u_i u_{i+1})] = \begin{cases} 0 & 2 \leq i \leq n-2 \\ 1 & i = 1\ , n-1 \end{cases}$$

$e_f(0) + 1 = e_f(1)$

Therefore it satisfies the condition $| e_f(0) – e_f(1)| \leq 1$

Hence , Fan $P_n + k_1$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of $P_7 + k_1$ is shown in the figure 2

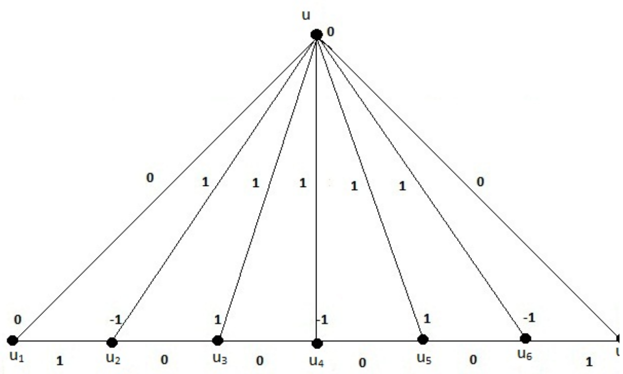


figure2 : Fan p_7+k_1

Theorem :2.3

Z - (P_n) is Relaxed Cordial Graph .

Proof:

Let the graph G be Z - (P_n)

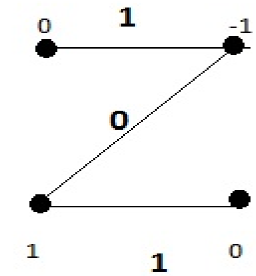
Let $V(G) = \{ [u_i : 1 \leq i \leq n] , [v_i : 1 \leq i \leq n] \}$

Let $E(G) = \{ [(u_iu_{i+1}) : 1 \leq i \leq n-1] \cup [(v_iv_{i+1}) : 1 \leq i \leq n-1] \cup [(u_iv_{i+1}) : 1 \leq i \leq n-1]\}$

Define $f : V(G) \to \{-1, 0, 1\}$

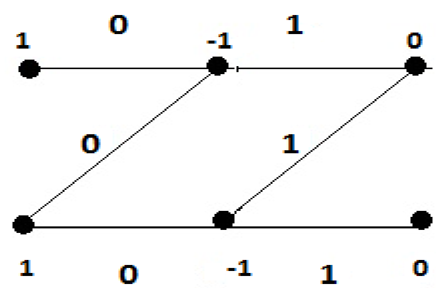
Case : 1

When n=2,



Case: 2

When n=3,



Case :3

When n > 3,

The vertex labeling are

$f(u_i) = 0 \quad 1 \leq i \leq n$

$$f(v_i) = \begin{cases} 1 & i \equiv 2 \pmod 4 \\ 0 & i \equiv 1,3 \pmod 4 \\ -1 & i \equiv 0 \pmod 4 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$f^* (u_iu_{i+1}) = 0 \quad 1 \leq i \leq n-1$

$f^* (v_iv_{i+1}) = 1 \quad 1 \leq i \leq n-1$

$$f^*(u_iv_{i+1}) = \begin{cases} 1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$|e_f(0) – e_f(1)| = \begin{cases} 0 \text{ if } n \text{ is odd} \\ 1 \text{ if } n \text{ is even} \end{cases}$$

therefore, the graph G satisfies the condition

$|e_f(0) – e_f(1)| \leq 1$.

Hence Z - (P_n) is Relaxed Cordial.

For example, the Relaxed Cordial labeling of Z - (P_8) is shown in figure 3

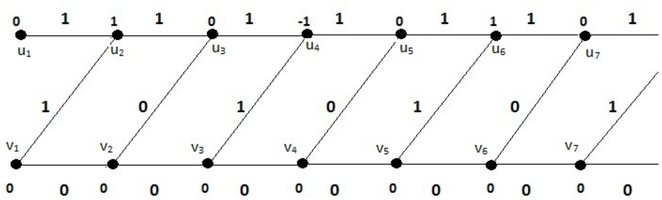


Figure 3 : $Z(p_8)$

Theorem : 2.4

$M(P_n)$ $n \geq 3$ is Relaxed Cordial Graph .

Let the graph G be $M(P_n)$

Let $V(G) = \{ [u_i : 1 \leq i \leq n] , [v_i : 1\leq i \leq n-1]\}$

Let $E(G) = \{ [(u_iu_{i+1}) : 1 \leq i \leq n-1] U[(v_iv_{i+1}) : 1 \leq i \leq n-2]U [(u_iv_i) : 1 \leq i \leq n-1] U[(u_{i+1}v_i) : 1 \leq i \leq n-1]\}$

Define f : V(G) -> {-1, 0, 1}

The vertex labeling are

$f(u_i) = 0 \quad 1 \leq i \leq n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1 \ (mod\ 2) \\ -1 & i \equiv 0 \ (mod\ 2) \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 0 \qquad 1 \leq i \leq n-1$

$f^* (v_iv_{i+1}) = 0 \qquad 1 \leq i \leq n-2$

$f^*(u_iv_i) = 1 \qquad 1\leq i \leq n-1$

$f^*(u_{i+1}v_i) = 1 \qquad 1\leq i \leq n-1$

$e_f(0) +1 = e_f(1)$

Therefore, the graph G satisfies the condition

$|e_f(0) – e_f(1)| \leq 1$.

Hence , $M(p_n)$ is relaxed cordial Graph .

For example, the Relaxed Cordial labeling of $M(P_7)$ is shown in figure4

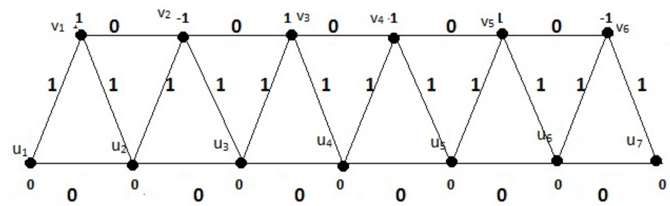


figure 4 : $M(P_7)$

Theorem : 2.5

$D_2(P_n)$ ($n \geq 2$) is Relaxed Cordial Graph .

Proof:

Let Graph G be $D_2(P_n)$

Let $V(G)= \{[u_i : 1 \leq i \leq n] , [v_i : 1\leq i \leq n]\}$

Let $E(G) = \{ [(u_iu_{i+1}) : 1\leq i \leq n-1] U [(v_iv_{i+1}) : 1\leq i \leq n-1] U [(u_iv_{i+1}) : 1\leq i \leq n-1] U [(u_{i+1}v_i) : 1\leq i \leq n-1]\}$

Define f : V(G) -> { -1, 0, 1}

The vertex labeling are

$f(u_i) = 0 \quad 1\leq i \leq n$

$$f(v_i) = \begin{cases} 1 & i \equiv 1 \ (mod\ 2) \\ -1 & i \equiv 0 \ (mod\ 2) \end{cases} \quad 1 \leq i \leq n$$

The induce edge labeling are

$f^*(u_iu_{i+1}) = 0 \quad 1 \leq i \leq n-1$

$f^* (v_iv_{i+1}) = 0 \quad 1 \leq i \leq n-1$

$f^*(u_iv_i) = 1 \qquad 1\leq i \leq n-1$

$f^*(u_{i+1}v_i) = 1 \qquad 1\leq i \leq n-1$

$e_f(0) +1 = e_f(1)$

Therefore , the graph G satisfies the condition

$|e_f(0) - e_f(1)| \leq 1$

Hence , $D_2(P_n)$ is Relaxed Cordial Graph.

For example , the Relaxed Cordial labeling of $D_2(P_8)$ is shown in the figure 5

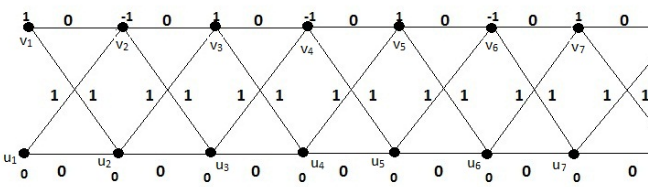


figure 5 :$D_2(P_8)$

Theorem : 2.6

$[P_n : S_3]$ ($n \geq 2$) is Relaxed Cordial Graph .

Proof:

Let G be $[P_n : S_3]$

Let $V(G) = \{[u_i : 1 \leq i \leq n], [v_i : 1 \leq i \leq n], [v_{i1}, v_{i2}, v_{i3} : 1 \leq i \leq n]\}$

Let $E(G) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \cup [(v_i v_{ij}) : 1 \leq i \leq n , 1 \leq j \leq 3]\}]$

Define f : V(G) -> {-1, 0, 1}

The vertex labeling are

$f(u_i) = \begin{cases} -1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \leq i \leq n$

$f(v_i) = 0 \quad 1 \leq i \leq n$

$f(v_{ij}) = \begin{cases} 0 & i \equiv 1 \pmod 2 \\ 1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \leq i \leq n , 1 \leq j \leq 3$

The induced edge labeling are

$f^*(u_i u_{i+1}) = 1 \quad 1 \leq i \leq n-1$

$f^*(u_i v_i) = \begin{cases} 1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases}$

$f^*(v_i v_{i1}) = 0 \quad 1 \leq i \leq n$

$f^*(v_i u_{i1}) = 1 \quad 1 \leq i \leq n$

$f^*(v_i v_{i3}) = 0 \quad 1 \leq i \leq n$

And $|e_f(0) - e_f(1)| = \begin{cases} 0 \text{ if } n \text{ is odd} \\ 1 \text{ if } n \text{ is even} \end{cases}$

therefore , it satisfies the condition

$|e_f(0) - e_f(1)| \leq 1.$

Hence , the graph $[P_n : S_3]$ is Relaxed Cordial Graph.

For example , the Relaxed Cordial labeling of $[P_5 : S_3]$ is shown in the figure 6

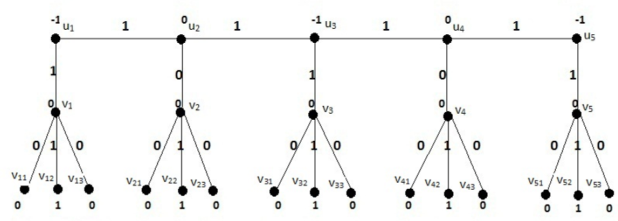


Figure 6 : $[P_5 : S_3]$

Theorem : 2.7

Comb $P_n \odot k_1$ is Relaxed Cordial Graph.

Proof:

Let the graph G be $P_n \odot k_1$

Let $V(G) = \{[u_i: 1 \leq i \leq n] , [v_I : 1 \leq i \leq n]\}$

Let $E(G) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i): 1 \leq i \leq n]\}$

Define f : V(G) -> { -1, 0, 1}

The vertex labeling are

$f(u_I) = 0 \quad 1 \leq i \leq n$

$f(v_i) = \begin{cases} 1 & i \equiv 1 \pmod 2 \\ -1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 0 \quad 1 \le i \le n-1$

$f^*(u_iv_i) = 1 \quad 1 \le i \le n$

And $e_f(0) + 1 = e_f(1)$

Therefore , the graph $P_n \odot k_1$ satisfies the condition

$|e_f(0) - e_f(1)| \le 1$.

Hence ,$P_n \odot k_1$ is Relaxed Cordial Graph.

For example , the Relaxed Cordial labeling of $P_7 \odot k_1$ is shown in the figure 7

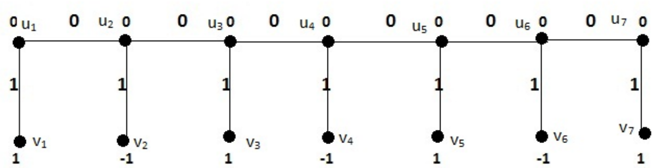


Figure 7 :$P_7 \odot k_1$

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