

Ranking of Octagonal Fuzzy Numbers for Solving Multi Objective Fuzzy Linear Programming Problem with Simplex Method and Graphical Method

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Abstract

In this paper a ranking procedure based on Hexagonal Fuzzy numbers, is applied to a Multi-objective Linear Programming Problem (MOLPP) with fuzzy coefficients. By this ranking method any multi objective Fuzzy Linear Programming problem (MOFLPP) can be converted in to a crisp value problem to get an optimal solution. This ranking procedure serves as an efficient method wherein a numerical example is also taken and the inference is given.

Keywords

Octagonal fuzzy number, Simplex method, Graphical method, α – level set.

Introduction

Ranking fuzzy number is used in decision-making process in an economic environment. In an organization various activities such as planning, execution, and other process takes place continuously. This requires careful observation of various parameters which are all in uncertain in nature due the competitive business environment globally. In fuzzy environment ranking fuzzy numbers is a very important decision making procedure. The idea of fuzzy set was first proposed by Bellman and Zadeh [1], as a mean of handling uncertainty that is due to imprecision rather than randomness. The concept of fuzzy linear programming (FLP) was first introduced by Tanaka et al. [11, 12] Zimmerman [18] introduced fuzzy linear programming in fuzzy environment. Multi-objective linear programming was introduced by Zelenly [17]. Lai Y.J – Hawng C.L [5]

considered MOLPP with all parameters having a triangular possibility distribution. They used an auxiliary model and it was solved by MOLPP. Zimmerman [18] applied their approach to vector maximum problem by transforming MOFLP problem to a single objective linear programming problem. Qiu – Peng Gu , and Bing – Yuan Cao [7] solved fuzzy linear programming problems based on fuzzy number distance. Tong Shaocheng [13] focused on the fuzzy linear programming with interval numbers. Chanas [2] proposed a fuzzy programming in multi objective linear programming. Verdegay[14] have proposed three methods for solving three models of fuzzy integer linear programming based on the representation theorem and on fuzzy number ranking method . In particular , the most convenient methods are based on the concept of comparison of fuzzy numbers by the use ranking numbers by the use ranking functions.

Preliminaries

Definition-1

If X is a collection of objects denoted generically by X then the fuzzy subset \bar{A} of X is defined as a set of ordered pairs.

$$\bar{A} = \{(x, \mu_A(x)) \mid x \in X\}$$

Where $\mu_A(x)$ is called the membership function for the fuzzy set \bar{A} . The membership function maps each element of X to a membership grade or membership value between 0 and 1.

Definition-2

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist atleast one $x \in X$ such that $\mu_A(x) = 1$.

Definition-3

The family of sets consisting all the subsets of a particular set A referred to as the power set of A and is indicated by $P(A)$.

$$(ie) \mid P(A) \mid = 2^{\mid A \mid}$$

α – Cuts

The α - cut (or) α - level set of fuzzy set \bar{A} is a set consisting of those elements of the universe X whose membership values exceed the threshold level α that is

$$\bar{A}_\alpha = \{ x / \mu_{\bar{A}}(x) \geq \alpha \}$$

Remark

- The membership function of \bar{A} specifies the degree of membership of elements x in fuzzy set \bar{A} (Infact $\mu_{\bar{A}}$ shows the degree that $x \in \bar{A}$).
- A fuzzy set is convex \Leftrightarrow if all its α – cuts are convex.
- The octagonal fuzzy number is convex as their α – cuts are convex sets in the classical sense.

Definition:

A fuzzy number is a normal octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8$ are real numbers and its membership function $\mu_{\bar{A}}(x)$ is given below

$$\mu_{\bar{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ k \left(\frac{x - a_1}{a_2 - a_1} \right) & \text{for } a_1 \leq x \leq a_2 \\ k & \text{for } a_2 \leq x \leq a_3 \\ k + (1-k) \left(\frac{x - a_3}{a_4 - a_3} \right) & \text{for } a_3 \leq x \leq a_4 \\ 1 & \text{for } a_4 \leq x \leq a_5 \\ k + (1-k) \left(\frac{a_6 - x}{a_6 - a_5} \right) & \text{for } a_5 \leq x \leq a_6 \\ k & \text{for } a_6 \leq x \leq a_7 \\ k \left(\frac{a_8 - x}{a_8 - a_7} \right) & \text{for } a_7 \leq x \leq a_8 \\ 0 & \text{for } x \geq a_8 \end{cases}$$

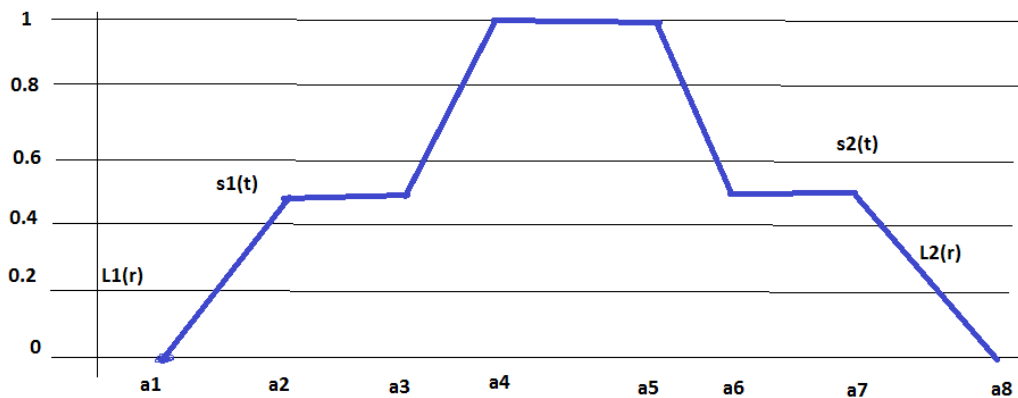


Fig -1 Graphical representation of a octagonal fuzzy number

Ranking of octagonal Fuzzy Numbers:

A number of approaches have been proposed for the ranking of fuzzy numbers. In this paper for a octagonal fuzzy number $\bar{A}_{oc} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$, a ranking method is devised based on the following formula.

$$R(\bar{A}_{oc}) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 3a_7 + 2a_8}{28} \right) \left(\frac{7}{28} \right)$$

Let $\bar{A}_{oc} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{N}_{oc} = (n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8)$ be two octagonal fuzzy numbers then

$$\begin{aligned} \bar{A}_{oc} \Leftrightarrow \tilde{N}_{oc} &\Leftrightarrow R(\bar{A}_{oc}) = R(\tilde{N}_{oc}) \\ \bar{A}_{oc} \geq \tilde{N}_{oc} &\Leftrightarrow R(\bar{A}_{oc}) \geq R(\tilde{N}_{oc}) \\ \bar{A}_{oc} \leq \tilde{N}_{oc} &\Leftrightarrow R(\bar{A}_{oc}) \leq R(\tilde{N}_{oc}) \end{aligned}$$

METHOD OF SOLVING MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING PROBLEM:

In this paper we discuss a multi –objective Fuzzy linear Programming Problem in constraints conditions with fuzzy coefficients.

$$\begin{aligned} &\text{Maximize } z_1 = d_1 y \\ &\text{Minimize } z_2 = d_2 y \\ &\text{Subject to } \bar{A}_H X \leq \bar{e}, X \geq 0 \end{aligned}$$

Where $d_{ij} = (d_{i1}, d_{i2}, \dots, d_{in})$ is an n-dimensional crisp row vector , $\bar{A}_H = \bar{a}_{ij}$ is an $m \times n$ fuzzy matrix , $\bar{e} = (e_1, e_2, e_3, \dots, e_m)^T$ is an m-dimensional fuzzy line vector and $X = (x_1, x_2, x_3, \dots)^T$ is an n-dimensional decision variable vector .

We now consider a bi-objective fuzzy linear programming problem with constraints having fuzzy coefficients is given by

$$\begin{aligned} &\text{Maximize } z_1 = d_{11}x_1 + d_{12}x_2 + \dots + d_{1n}x_n \\ &\text{Minimize } z_2 = d_{21}x_1 + d_{22}x_2 + \dots + d_{2n}x_n \\ &\text{Subject to } \bar{a}_{i1}x_1 + \bar{a}_{i2}x_2 + \bar{a}_{i3}x_3 + \bar{a}_{i4}x_4 + \dots + \bar{a}_{in}x_n \leq \bar{e}_i \\ &x_1, x_2, x_3, \dots, x_n \geq 0, i=1, 2, 3, \dots, m. \text{ where fuzzy numbers are octagonal,} \end{aligned}$$

Where

$$\begin{aligned} \bar{a}_{i1} &= \bar{a}_{i11}, \bar{a}_{i12}, \bar{a}_{i13}, \bar{a}_{i14}, \bar{a}_{i15}, \bar{a}_{i16}, \bar{a}_{i17}, \bar{a}_{i18} \\ \bar{a}_{i2} &= \bar{a}_{i21}, \bar{a}_{i22}, \bar{a}_{i23}, \bar{a}_{i24}, \bar{a}_{i25}, \bar{a}_{i26}, \bar{a}_{i27}, \bar{a}_{i28} \\ &\dots \\ &\dots \\ \bar{a}_{in} &= \bar{a}_{in1}, \bar{a}_{in2}, \bar{a}_{in3}, \bar{a}_{in4}, \bar{a}_{in5}, \bar{a}_{in6}, \bar{a}_{in7}, \bar{a}_{in8} \\ \bar{e} &= \bar{e}_{i1}, \bar{e}_{i2}, \bar{e}_{i3}, \bar{e}_{i4}, \bar{e}_{i5}, \bar{e}_{i6}, \bar{e}_{i7}, \bar{e}_{i8} \end{aligned}$$

By the ranking Algorithm, the above MOFLPP is transformed into a MOLPP is as follows:

$$\begin{aligned} &\text{Maximize } z_1 = d_{11}x_1 + d_{12}x_2 + \dots + d_{1n}x_n \\ &\text{Minimize } z_2 = d_{21}x_1 + d_{22}x_2 + \dots + d_{2n}x_n \end{aligned}$$

Subject to,

$$\begin{aligned} &2(a_{i11}x_1 + a_{i21}x_2 + \dots + a_{in1}x_n) + 3(a_{i12}x_1 + a_{i22}x_2 + \dots + a_{in2}x_n) + \\ &4(a_{i13}x_1 + a_{i23}x_2 + \dots + a_{in3}x_n) + 5(a_{i14}x_1 + a_{i24}x_2 + \dots + a_{in4}x_n) + \\ &5(a_{i15}x_1 + a_{i25}x_2 + \dots + a_{in5}x_5) + 4(a_{i16}x_1 + a_{i26}x_2 + \dots + a_{in6}x_n) + \\ &3(a_{i17}x_1 + a_{i27}x_2 + \dots + a_{in7}x_n) + 2(a_{i18}x_1 + a_{i28}x_2 + \dots + a_{in7}x_n) \\ &\leq 2e_{i1} + 3e_{i2} + 4e_{i3} + 5e_{i4} + 5e_{i5} + 4e_{i6} + 3e_{i7} + 2e_{i8} \\ &x_1, x_2, x_3, \dots, x_n \geq 0, i=1, 2, 3, \dots, m \text{ -----(1)} \end{aligned}$$

Using (1), this can be converted into a single objective problem subject to the constraints with transformed crisp number coefficients and hence solved accordingly.

Similarly, multi-objective problems with more than two objectives can also be solved using the above procedure, here in the very first stage itself the problem is transformed into a crisp problem and afterwards there will be no more fuzziness in the constraints as well as in the problem.

Simplex Method Algorithm

Step 1: Determine a starting basic feasible solution.

Step 2: Select an entering variable using the optimality condition. Stop if there is no entering variable; the last solution is optimal. Else, go to step 3.

Step 3: Select a *leaving variable* using the feasibility condition.

Step 4: Determine the new basic solution by using the appropriate Gauss-Jordan computations. Go to step 2.

Numerical Example

Consider,

$$\text{Max } Z = 60x_1 + 70x_2$$

Subject to

$$\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 \leq \bar{e}_1$$

$$\tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 \leq \bar{e}_2$$

$$x_1, x_2 \geq 0$$

Where

$$\tilde{a}_{11} = (11, 12, 14, 16, 16, 8, 7, 6)$$

$$\tilde{a}_{12} = (8, 7, 5, 4, 4, 3, 2, 1)$$

$$\tilde{a}_{21} = (2, 3, 6, 7, 7, 1, 8, 4)$$

$$\tilde{a}_{22} = (4, 5, 3, 2, 2, 7, 9, 8)$$

$$\bar{e}_1 = (120, 130, 140, 150, 150, 110, 90)$$

$$\bar{e}_2 = (115, 120, 124, 125, 125, 130, 128, 127)$$

Subject to constraints.

$$2(11x_1+8x_2)+3(12x_1+7x_2)+4(14x_1+5x_2)+5(16x_1+4x_2)+5(16x_1+4x_2)+4(8x_1+3x_2)+3(7x_1+2x_2)+2(6x_1+x_2)\leq(120+130+140+150+150+110+90)$$

$$2(2x_1+4x_2)+3(3x_1+5x_2)+4(6x_1+3x_2)+5(7x_1+9x_2)+2(4x_1+8x_2)\leq(115+120+124+125+125+130+128+127)$$

$$\text{Max } Z=60x_1+70x_2$$

Subject to constraints.

$$339x_1+121x_2+s_1=890$$

$$143x_1+126x_2+s_2=994$$

Simplex method

Step:1

The initial basic feasible solution is

| CB | YB | XB | X1 | X2 | S1 | S2 | RATIO |
|---------|----|-----|-----|-----|----|----|-----------|
| 0 | S2 | 890 | 339 | 121 | 1 | 0 | 890/121 ← |
| 0 | S2 | 994 | 143 | 126 | 0 | 0 | 994/126 |
| Zj | | 0 | 0 | 0 | 0 | 0 | |
| Cj | | | 60 | 70 | 0 | 0 | |
| Zj - Cj | = | 0 | -60 | -70 | 0 | 0 | |

STEP:2

Enter x₂ and skip s₂

| CB | YB | XB | X ₁ | X ₂ | S ₁ | S ₂ | RATIO |
|---------|----------------|-----------|----------------|----------------|----------------|----------------|-------|
| 70 | X ₂ | 890/121 | 339/121 | 1* | 1/121 | 0 | 7.35 |
| 0 | S ₁ | -8134/121 | 25411/121 | 0 | 126/121 | 0 | 0 |
| Zj | | 62300/121 | 196 | 70 | 70/121 | 0 | |
| Cj | | | 60 | 70 | | | |
| Zj - Cj | = | 514.8 | 136 | 0 | 0.56 | 0 | |

Maximize Z=514.8at x₁=0;x₂=7.35

Similarliy we can calculate simplex method in Minimize Z= (- Maximize Z)

Therefore minimize $z = -159$ at $x_1 = 2.65$; $x_2 = 0$

GRAPHICAL METHOD

Maximize $Z = 60x_1 + 70x_2$

Subject to constraints

$$339x_1 + 121x_2 \leq 890$$

$$143x_1 + 126x_2 \leq 994$$

Solution

Given that Max $Z = 60x_1 + 70x_2$

Subject to constraints

$$339x_1 + 121x_2 = 890 \text{-----} \rightarrow (1)$$

$$143x_1 + 126x_2 = 994 \text{-----} \rightarrow (2)$$

Put $x_1 = 0$ in eqn 1 $\Rightarrow 121x_2 = 890 \Rightarrow$

| |
|--------------|
| $x_2 = 7.35$ |
|--------------|

A(0, 7.35)

put $x_2 = 0$ in eqn 1 $\Rightarrow 339x_1 = 890 \Rightarrow$

| |
|--------------|
| $x_1 = 2.65$ |
|--------------|

B(2.65, 0)

Put $x_1 = 0$ in eqn 2 $\Rightarrow 126x_2 = 994$

C(0, 7.88)

| |
|--------------|
| $x_2 = 7.88$ |
|--------------|

Put $x_2 = 0$ in eqn 2 $\Rightarrow 143x_1 = 994 \Rightarrow x_1 = 6.95$

D(6.95, 0)

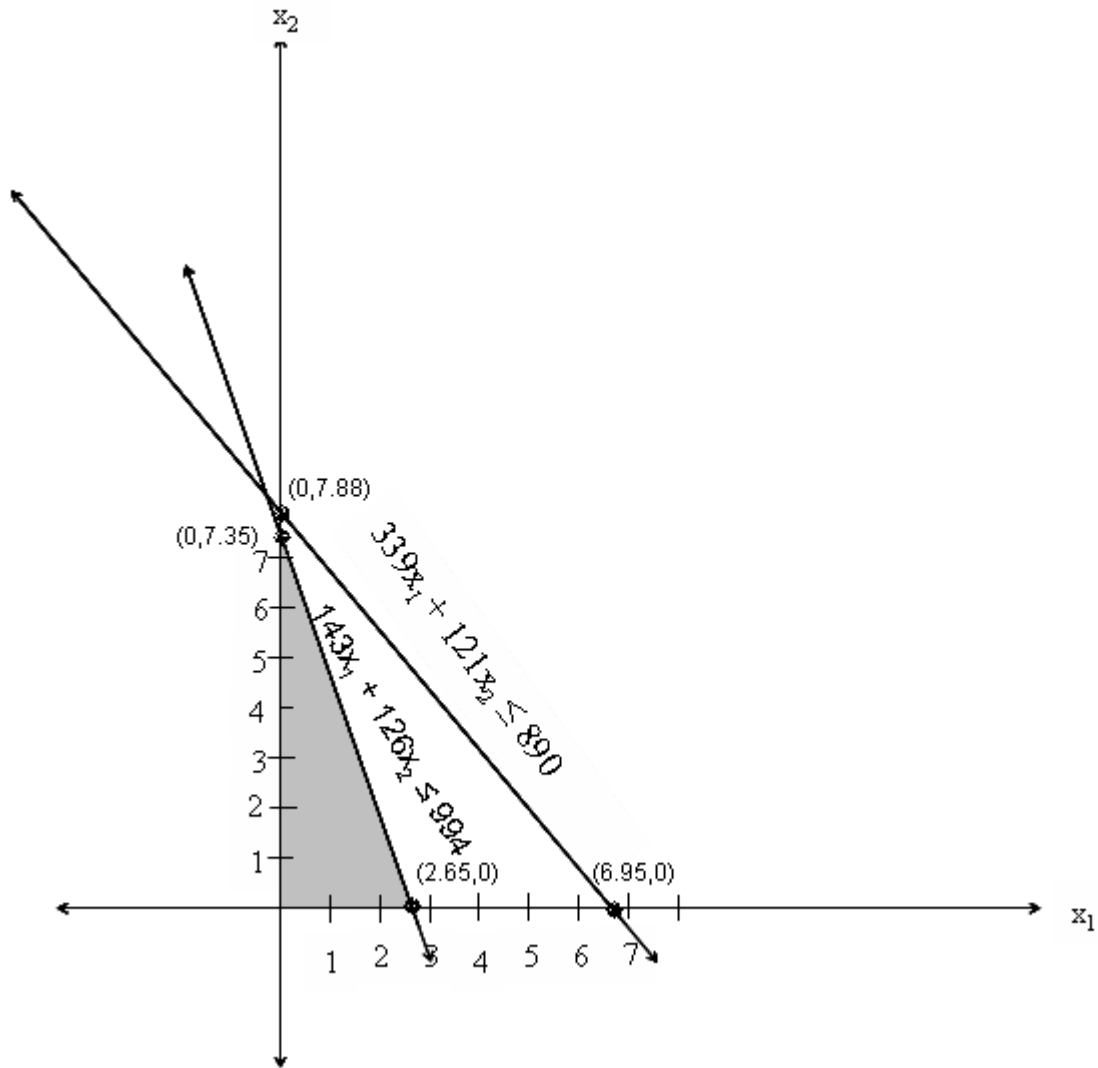


Fig -2 Graphical method

| (x_1, x_2) | Max $z = 60x_1 + 70x_2$ |
|--------------|-------------------------|
| A(0,0) | Max $Z = 0$ |
| B(2.65,0) | Max $Z = 159$ |
| C(0,7.35) | Max $Z = 514.5$ |

Comparison of result obtained by using Simplex method and Graphical method.

| Simplex Method | Graphical Method |
|---|--|
| Maximize $Z = 514.8$ at $X_1 = 0$ and $x_2 = 7.88$ | Maximize $Z = 514.5$ at $X_1 = 0$ and $X_2 = 7.35$ |
| Minimize $Z = 159.10$ at $X_1 = 2.75$ and $X_2 = 0$ | Minimize $Z = 159$ $X_1 = 2.65$ and $X_2 = 0$ |

RESULTS AND DISCUSSIONS:

As per the above table, we have obtained the nearest value from both simplex method and graphical method. But Simplex Method results is accurate value.

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