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CHEMICALLY REACTING FLUID ON UNSTEADY MHD OSCILLATORY SLIP FLOW IN A PLANER CHANNEL WITH VARYING TEMPERATURE AND CONCENTRATION IN THE PRESENCE OF SUCTION/INJECTION.

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ABSTRACT

This paper examined the problem of unsteady MHD mixed convictive oscillatory flow of an electrically conducting optically thin fluid through a planer channel filled with saturated porous medium. The effect of buoyancy, heat source, thermal radiation and chemical reaction of the fluid were taken into considerations with slip boundary condition, varying temperature and concentration. The closed-form analytical solutions are obtained for the momentum, energy and concentration equations. It was discovered that the velocity increases with increase in ω, and Gr. While the velocity decreases with increase in M, Gm, K and γ. The temperature increases with increase in R, α and $d2$. The concentration increases with increase in Sc, while it decreases with increase in $d1$ and K_r. The effect of skin friction, the rate of heat and mass transfer coefficients at the walls are shown in the tables.

Key words: chemical reaction, MHD, oscillatory flow, planer channel.

1. INTRODUCTION

Oscillatory flows has known to result in higher rates of heat and mass transfer, many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors. Cramer, K. R. and Pai, S. I. [1] taken transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible Muthucumaraswamy et al. [2] have studied the effect of homogenous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. Sharma [3] investigate the effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slip flow regime when suction velocity oscillates in time. Chaudhary and Jha [4] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Anjalidevi et al. [5] have examined the effect of chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy et al. [6] have investigated the effect of thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate in the presence of first order chemical reaction. Moreover, Al-Odat and Al-Azab [7] studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order. The effect of radiation on the heat and fluid flow over an unsteady stretching surface has been analyzed by El-Aziz [8]. Singh et. al. [9] studied the heat transfer over stretching surface in porous media with transverse magnetic field. Singh et. al. [10] and [11] also investigated MHD oblique stagnation-point flow towards a stretching sheet with heat transfer for steady and unsteady cases. Elbashbeshy et. al. [12] investigated the effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface. Ahmed Sahin studied influence of chemical reaction on transient MHD free Convective flow over a vertical plate.

Recently, the chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandasamy et al. [13]. The opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by Angirasa et al.[14]. Ahmed [15] investigates the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Ahmed Sahin [16] studied the Magneto hydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate in a slip-flow regime. V. SriHari Babu and G. V. Ramana Reddy [17] analyzed the Mass transfer effects on MHD mixed convective flow from a vertical surface with Ohmic heating and viscous dissipation. Satya Sagar Saxena and G. K. Dubey [18] studied the effects of MHD free convection heat and mass transfer flow of visco-elastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime. Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion was analysed by Satya Sagar Saxena and G. K. Dubey [19]. Sudeerbabu et al [20] analyzed the radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation. Unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation is studied by Seethamahalakshmi et al [21].

To the best of the author's knowledge, studies pertaining to oscillatory flow investigations in a planer channel with variable temperature and concentration have not received much attention. Therefore, the main goal here is to study the chemical reaction effects on unsteady MHD oscillatory slip flow in an optically thin fluid through a planer channel in the presence of a temperature-dependent heat source. The closed form solutions for velocity, temperature, skin friction, concentration, Nusselt number, and Sherwood number are presented. The effects of pertinent parameters on fluid flow of heat and mass transfer characteristics are studied in detail and presented graphically and discussed qualitatively.

2. FORMATION OF THE PROBLEM

We consider the unsteady mixed convection, two dimensional slip flow of an electrically conducting, heat generating, optically thin and chemically reacting oscillatory fluid flow in a planer channel filled with porous medium in the presence of thermal radiation with temperature and concentration variation. Take a Cartesian coordinate system (x', y') where $x'-$axis is taken along the flow and $y'-$axis is taken normal to the flow direction. A uniform transverse magnetic field of magnitude B_0 is applied in the presence of thermal and Solutal buoyancy effects in the direction of $y'-$axis. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + v\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_1') + g\beta^*(C' - C_1') \tag{2}$$

$$-\frac{v}{K'}u' - \frac{\sigma B_0^2}{\rho}u' - v'\frac{\partial u'}{\partial y'}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho}\frac{\partial q_r'}{\partial y'} + Q\frac{(T'-T_1')}{\rho C_p} - v'\frac{\partial T'}{\partial y'} \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r'(C'^{-}C_1') - v'\frac{\partial C'}{\partial y'} \quad (4)$$

Where $v' = -v_o(1+\epsilon e^{i\omega t})$ is suction velocity.

The appropriate boundary conditions of the problem are

$$u' = L_1\frac{\partial u'}{\partial y'}, T' = T_1' + \delta_T^*\frac{\partial T'}{\partial y'}, C' = C_1' + \delta_C^*\frac{\partial C'}{\partial y'} \quad at\ y' = 0 \quad (5)$$

$$u' = 0, T' = T_2' + \delta_T^*\frac{\partial T'}{\partial y'}, C' = C_2' + \delta_C^*\frac{\partial C'}{\partial y'} \quad at\ y' = d \quad (6)$$

where u', v' - the velocity components in the x', y' directions respectively, v - the kinematics viscosity, k- the thermal conductivity, β - the coefficient of volume expansion due to temperature, β^* - the coefficient of volume expansion due to concentration, ρ- the density, σ- the electrical conductivity of the fluid, g- the acceleration due to gravity, T'- the temperature, T_1' - wall temperature of the fluid, q_r - the radiation heat flux, C'-the concentration, C_1' - wall concentration of the fluid and K_r'- chemical reaction parameter,L_1 − mean free path,C_p − specific heat at a constant pressure and D- mass diffusivity.

The radiative heat flux (Cogley *et al.* [22]) is given by

$$\frac{\partial q_r'}{\partial y'} = 4(T_1' - T)I', \text{Where } I' = \int_0^\infty K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T}d\lambda \quad (7)$$

Where $K_{\lambda w}$-radiation absorption coefficient at the wall, $e_{b\lambda}$-Planck's function.

Introducing the following non-dimensional quantities

$$x = \frac{x'}{d}, y = \frac{y'}{d}, P = \frac{dP'}{\mu u_o}, u = \frac{u'}{u_o}, \theta = \frac{T'-T_1'}{T_2-T_1}, \phi = \frac{C'-C_1'}{C_2-C_1}, t = \frac{u_o t'}{d}, Re = \frac{u_o d}{v}, \gamma = \frac{K'}{d^2}, M = \frac{\sigma B_0^2 d^2}{\mu}, Gr = \frac{g\beta(T_2-T_1)d^2}{v u_o}, Gc = \frac{g\beta^*(C_2-C_1)d^2}{v u_o}, R = \frac{4I'd^2}{k}, Pe = \frac{\rho C_p u_o d}{k},, Sc = \frac{D}{u_o d}, K_r = \frac{K_r'}{u_o}, d_2 = \frac{\delta_T^*}{d}, d_1 = \frac{\delta_C^*}{d}, k' = kd^2$$

In view of the above dimensionless variables, the basic field equations (2) to (4) can be expressed in non-dimensionless form as

$$Re\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \lambda_1\frac{\partial u}{\partial y} + Gr\theta + Gc\phi - (M + \frac{1}{K})u \quad (9)$$

$$Pe\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + Pe\,\lambda_2\frac{\partial \theta}{\partial y} + (C_p R + \alpha)\theta \quad (10)$$

$$\frac{\partial \phi}{\partial t} = Sc\frac{\partial^2 \phi}{\partial y^2} + \lambda_3\frac{\partial \phi}{\partial y} - K_r\phi \quad (11)$$

The corresponding boundary conditions for t>0 are transformed to:

$$u = \gamma\frac{\partial u}{\partial y}, \theta = d_2\frac{\partial \theta}{\partial y}, \phi = d_1\frac{\partial \phi}{\partial y} \text{ at } y = 0 \quad (12)$$

$$u = 0, \theta = 1 + d_2\frac{\partial \theta}{\partial y}, \phi = 1 + d_1\frac{\partial \phi}{\partial y} \text{ at } y = 1$$

where $P, Re, M, K, Pe, R, Sc, \lambda, d_1, d_2, Gr, Gc$ and γ are pressure, Reynolds number, magnetic parameter, permeability parameter, Peclet number, thermal radiation parameter, Schmidt number, real constant, volumetric concentration expansion, volumetric thermal expansion, thermal Grashof number, Solutal Grashof number and slip parameter respectively.

3. SOLUTION OF THE PROBLEM

In order to solve equations (9) - (11) with respect to the boundary conditions (12) for purely oscillatory flow, let us take

$$u(y, t) = u_o(y)e^{i\omega t} \quad (13)$$

$$\theta(y, t) = \theta_o(y)e^{i\omega t} \quad (14)$$

$$\phi(y, t) = \phi_o(y)e^{i\omega t} \quad (15)$$

$$-\frac{\partial P}{\partial x} = e^{i\omega t} \quad (16)$$

Substituting the Equations (13)-(16) in Equations (9)-(12), we obtain:

$$u_0''(y) + a_1 u_0'(y) - a_2 u_0 = -(\lambda + Gr\theta_0 + Gc\phi_0) \qquad (17)$$

$$\theta_0''(y) + a_5\theta_0' + a_6\theta_0 = 0 \qquad (18)$$

$$\phi_0''(y) + a_3 - a_4\emptyset_0 = 0 \qquad (19)$$

Where prime denotes ordinary differentiation with respect to y.

The corresponding boundary conditions can be written as

$$u_o = \gamma u_o', \theta_o = d_2\theta_o', \phi_o = d_1\phi_o' \text{ At y=0} \qquad (20)$$

$$u_o = 0, \theta_o = 1 + d_2\theta_o', \phi_o = d_1\phi_o' \text{ At y=1}$$

Solving equations (17) – (19) under the boundary conditions (20), we obtain the velocity, temperature and concentration distribution in the boundary layer as:

$$u_0(y,t) = A_1 e^{m_1 y} + A_2 e^{m_2 y} + K_1 + K_2 e^{m_5 y} + K_3 e^{m_6 y} + K_4 e^{m_3 y} + K_5 e^{m_4 y}$$

$$\theta_0(y,t) = A_5 e^{m_5 y} + A_6 e^{m_6 y}$$

$$\emptyset_0(y,t) = A_3 e^{m_3 y} + A_4 e^{m_4 y}$$

Where

$K_1 = \frac{\lambda_1}{a_2}, K_2 = \frac{-GrA_5}{m_5^2 + a_1 m_5 - a_2}, K_3 = \frac{-GrA_6}{m_6^2 + a_1 m_6 - a_2}, K_4 = \frac{-GrA_3}{m_3^2 + a_1 m_3 - a_2}, K_5 = \frac{-GrA_4}{m_4^2 + a_1 m_4 - a_2}, A_1 = \frac{K_{14}}{K_{12}} - \frac{K_{13}}{K_{12}} K_{18}, A_2 = K_{18}, A_3 = \frac{-k_{20}\sin n\pi}{k_{19}}$

$A_4 = \sin n\pi, A_5 = \frac{k_7}{k_7 k_8 - k_6 k_9}, A_6 = \frac{-k_6}{k_7 k_8 - k_6 k_9}, m_1 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2}}{2}, m_2 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2}, m_3 = \frac{-a_3 + \sqrt{a_3^2 - 4a_4}}{2}, m_4 = \frac{-a_3 - \sqrt{a_3^2 - 4a_4}}{2}, m_5 = \frac{-a_5 + \sqrt{a_5^2 - 4a_6}}{2}, m_6 = \frac{-a_5 - \sqrt{a_5^2 - 4a_6}}{2}, a_1 = \lambda_1, a_2 = M +$

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$\frac{1}{k} + iwRe, a_3 = \frac{\lambda_3}{Sc}, a_4 = \frac{(K_r+iw)}{Sc}, a_5 = Pe\,\lambda_2\, iw, a_6 = R + \alpha - Peiw\,, k_6 = 1 - d_2 m_5\,, k_7 = 1 - d_2 m_6, k_8 = (e^{m_5} - m_5 e^{m_5}), k_9 = (e^{m_6} - m_6 e^{m_6}), K_{10} = K_1 + K_2 + K_3 + K_4 + K_5, K_{11} = \gamma(K_2 m_5 + K_3 m_6 + K_4 m_3 + K_5 m_4)\quad, K_{12} = \quad (1 - \gamma m_1), \quad K_{13} = (1 - \gamma m_2)\,, K_{14} = K_{11} - K_{10}, K_{15} = K_1 + K_2 e^{m_5} + K_3 e^{m_6} + K_4 e^{m_3} + K_5 e^{m_4}, K_{16} = e^{m_1}\,, k_{17} = e^{m_2}, K_{18} = \frac{K_{14}K_{16}+K_{12}K_{15}}{K_{13}K_{16}-K_{12}K_{17}}\,;$

The shear stress, the coefficient of the rate of heat transfer and the rate of mass transfer at any point in the fluid can be characterized by

$$\tau^* = -\mu\frac{\partial u}{\partial y}\,, Nu^* = -k\frac{\partial T}{\partial y}, Sh^* = -D\frac{\partial C}{\partial y},$$

In dimensional form

$$\tau = \frac{\tau^* d}{\mu u_0} = -\frac{\partial u}{\partial y}\,, Nu = -\frac{Nu^* d}{x(T_1 - T_0)} = -\frac{\partial \theta}{\partial y}\,, Sh = -\frac{Sh^* d}{C_1 - C_0} = -\frac{\partial \phi}{\partial y}$$

The skin-friction (τ), the Nusselt (Nu) and the Sherwood number (Sh) at the walls y =0 and y =1 are given by

$$\tau_0 = -\frac{\partial u}{\partial y}\Big|_{y=0} \quad , \quad \tau_1 = -\frac{\partial u}{\partial y}\Big|_{y=1}\ ,$$

$$Nu_0 = -\frac{\partial \theta}{\partial y}\Big|_{y=0} \quad , \quad Nu_1 = -\frac{\partial \theta}{\partial y}\Big|_{y=1}\,,$$

$$Sh_0 = -\frac{\partial \phi}{\partial y}\Big|_{y=0} \quad , \quad Sh_1 = -\frac{\partial \phi}{\partial y}\Big|_{y=1}\,.$$

DISCUSSION OF RESULTS

To study the effect of heat and mass transfer in an MHD oscillatory flow in a planer channel with varying temperature and concentration in the presence of suction and injection, the velocity u,temperature θ and the species concentration ϕ profiles are depicted graphically against y for different values of different parameters; magnetic parameter m, frequency of oscillation ω, thermal Grashof number Gr,modified Grashof number Gc, permeability parameter K, Slip parameter γ, Thermal radiation parameter R, Heat source parameter α, Volumemetric concentration expansion $d1$, Volumemetric temperature expansion $d2$, Schmid number Sc, Chemical radiation parameter Kr. The graphs are plotted using MATLAB where only the real parts of the equation were considered. Throughout the computations the values used were, $t = 1, M = 2, Re = 1, Gr = 2, Gc = 1, R = 2, \alpha = 3, Pe = 4, Kr = 2, Sc = 1, g = 0.1, d1 = 0.002, d2 = 0.002, \omega = 0.5$ except where a parameter is varied.

In Fig 1, increase in the magnetic field intensity initially increases the velocity field at first, thereafter the velocity begins to decrease as it increases. The effect of magnetic field is found to have a zero effect on the velocity field as we far away from the plate. In fig 2, it is noticed that, increased in the frequency of excitation lead to an increase in the velocity profile. In Fig 3, while all other parameters are held constant, increase in the thermal grashof number raise in the velocity field. In Fig 4, increases in the modified grashof number contribute to decrease in the velocity field. In Fig 5, increase in the permeability coefficient of the porous medium against the porosity of the porous medium decreases the fluid velocity. In Fig 6, increase in the slip parameter has the tendency to reduce the friction forces which reduces the fluid velocity. In Fig 7, increase in the radiation parameter increases the temperature distribution because large values of radiation parameter oppose the conduction over radiation, thereby increases the buoyancy force and increases the thickness of the thermal boundary layer. In Fig 8, increase in the heat source parameter significantly increases the thermal buoyancy effects which raise fluid temperature. In Fig 9, increase in temperature variation parameter initially increases the temperature and thereafter decreases it as we move away from the plate. In Fig 10, increase in the Schmidt number increases the species concentration (mass transfer).In fig 11, increase in the concentration variation parameter decreases the fluid concentration. In Fig 12, increase in the chemical radiation parameter decreases the concentration profile.

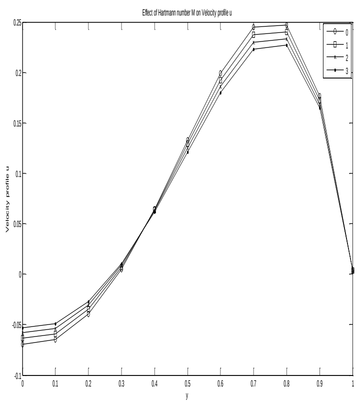

Fig.1. Velocity profile for different values of M

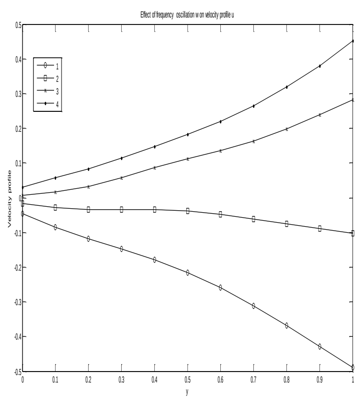

Fig.2. Velocity profiles for different values of ω.

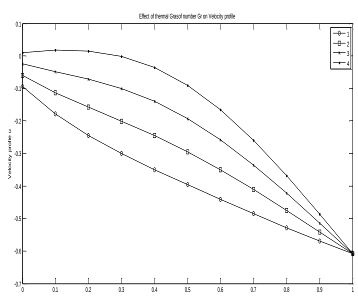

Fig.3. Velocity profiles for different value Gr

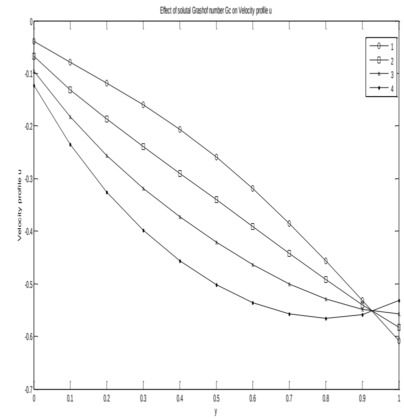

Fig.4. Velocity profiles for different values of Gc.

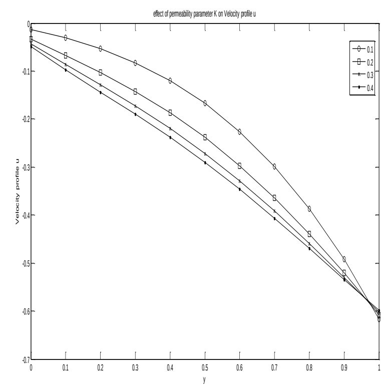

Fig.5. Velocity profiles for different values of K

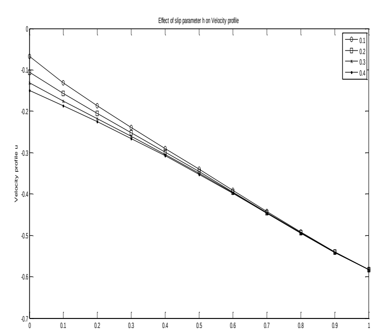

Fig.6. Velocity profiles for different values of γ

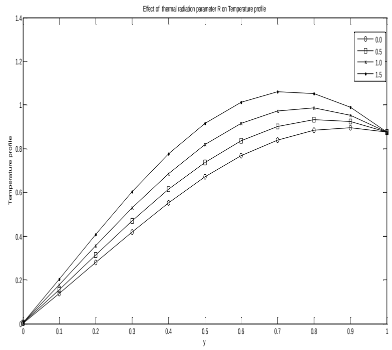

Fig.7. Temperature profiles for different values of R

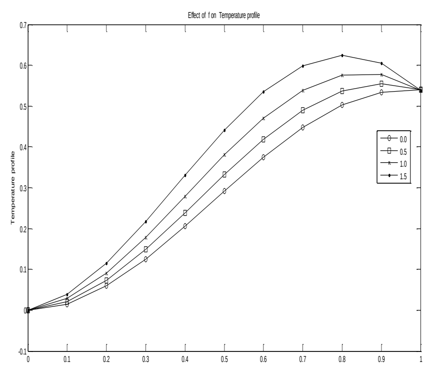

Fig 8. Temperature profiles for different values of α.

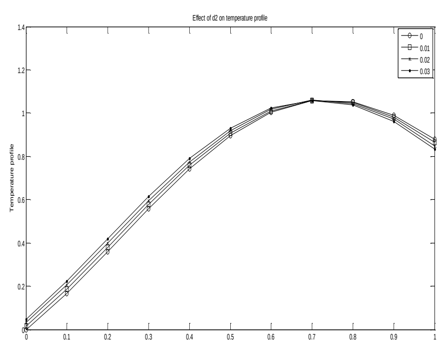

Fig.9. temperature profiles for different values of $d2$.

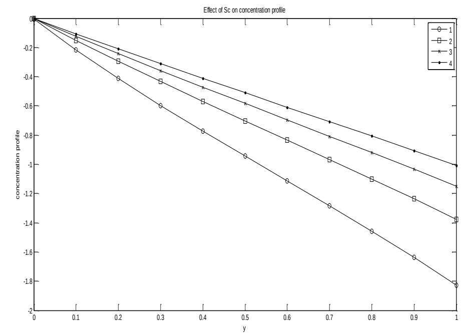

Fig.10. Concentration profiles for different values of Sc

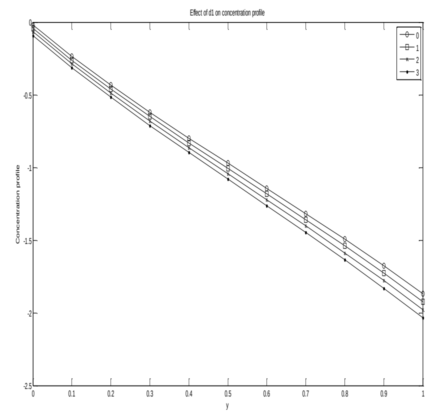

Fig.11. Concentration profiles for different values of $d1$.

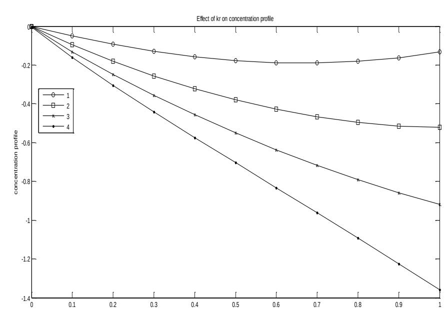

Fig.12. Concentration profiles for different values of Kr.

CONCLUSION AND RECOMMENDATION

The problem of unsteady MHD mixed convictive oscillatory flow of an electrically conducting optically thin fluid through a planer channel filled with saturated porous medium has been examined. The effect of buoyancy, heat source, thermal radiation and chemical reaction of the fluid were taken into considerations with slip boundary condition, varying temperature and concentration. It was discovered that the velocity increases with increase in ω, and Gr. While the velocity decreases with increase in M, Gm, K and γ. The temperature increases with increase in R, α and $d2$. The concentration increases with increase in Sc, while it decreases with increase in $d1$ and K_r.

It is concluded that the results obtained in a chemically reacting fluid on MHD oscillatory slip flow in a planar channel with varying temperature and concentration in the presence of suction/injection have a great applications in different systems such as reciprocating engines, pulse combustors and chemical reactors.

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