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EFFECT OF HEAT AND MASS TRANSFER ON UNSTEADY MHD POISEUILLE FLOW BETWEEN TWO INFINITE PARALLEL POROUS PLATES IN AN INCLINED MAGNETIC FIELD.

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ABSTRACT

The effect of heat and mass transfer on unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field has been investigated, where the lower plate is considered porous. The governing equations of the flow field are solved by perturbation technique and the expression for the velocity u, temperature θ and concentration c were obtained. The effect of parameters such as Hartmann number Ha, Grashof number (Gr and Gc), Radiation N, Prandtl number Pr, Schmidt number Sc and Chemical parameter Kc were studied. The results show that at high Hartmann number Ha, the velocity decreased. Velocity increased due to effect of thermal Grashof number Gr and solutal Grashof number Gc. An increase in Prandtl number Pr decreased temperature. Species concentration reduced with increase in chemical parameter Kc and Schmidt number Sc.

Keywords: unsteady, MHD, poiseuille flow, porous plate, Heat Transfer, Mass Transfer.

1. INTRODUCTION

Magnetohydrodynamic (MHD) is the study of the dynamics of electrically conducting fluids under the influence of a magnetic field. Fluid such as mercury, molten iron and ionized gases often called plasma by physicist of which the solar atmosphere is an example, are but a few electrically conducting fluids.

The applications of the effect of heat and mass transfer on unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field are visible in several fields of engineering technology.

The motion of an electrically conducting fluid placed in a constant magnetic field induces current that creates a force on the fluid. The current generated has been used in the designed MHD generators for electricity generation, MHD devices, nuclear engineering and the possibility of thermonuclear power that has created an immense practical used for understanding the dynamics of electrically conducting fluid. The effect of magnetic field in viscous incompressible flow of electrically conducting fluid is of use in extrusion of plastics and food.

Hannes Alfven (1942), a Swedish electrical engineer first initiated the study of MHD. Shercliff (1956) considered the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Sparrow and Cess (1961) observed that free convection heat transfer to liquid metals may be significantly affected by the presence of magnetic field. Drake (1965) considered flow in a channel due to periodic pressure gradient and solved the resulting equation by separation of variables methods. Singh and Ram (1978) studied Laminar flow of an electrically conducting fluid through a channel in the presence of a transverse magnetic field under the influence of a periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. More to this, Ram et al (1984) analyzed hall effects on heat and mass transfer flow through porous media. Soundelgekar and Abdulla Ali (1986) studied the flow of viscous incompressible electrically conducting isothermal plate. Singh (1993) considered the steady MHD fluid flow between two parallel plates. John Mooney and Nick Stokes (1997) considered the effects of thermal radiation and free convection flow past a moving vertical plate. Al-Hadhrami (2003) discussed flow through horizontal channels of porous material and obtained velocity expressions in terms of the Reynolds number. Ganesh (2007) studied unsteady MHD Stokes flow of a viscous fluid between two parallel porous plates. Stamenkovic et al (2010) investigated MHD flow of two immiscible and electrically conducting fluids between isothermal, insulated moving plates in the presence of applied electric and magnetic fields. They matched the solution at the interface and it was found that decrease in magnetic field inclination angle flattens out the velocity and temperature profiles. Rajput and Sahu (2011) studied the effect of a uniform transverse magnetic field in the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel porous plates with constant temperature and variable mass diffusion. Mayonge et al (2012) studied steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field and discovered that high magnetic field strength decreases the velocity. Heat transfer effects on rotating MHD coquette flow in a channel partially filled by a porous medium with hall current has been discussed by Singh and Rastogi (2012). Choudhary and DebH (2012) studied heat and mass transfer for viscoelastic MHD boundary layer flow past a vertical flat plate. Sandeep and Sugunamma (2013) analyzed the effect of an inclined magnetic field in unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Joseph et al (2014) studied the unsteady MHD coquette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. They found out that when the magnetic field is high, it reduces the energy loss through the plate. Large Nusselt number corresponds to more active convection and high prandtl number decreases the temperature distribution. The unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer has been studied by Idowu et al (2014).

In this paper, we considered one dimensional poiseuille flow of an electrically conducting fluid between two infinite parallel porous plates under the influence of magnetic field with heat and mass transfer.

2. PROBLEM FORMULATION

The concept of magneto hydrodynamics phenomenon can simply be described as follows: consider an electrically conducting fluid moving with velocity V. at right angles to this flow, we apply a magnetic field, the field strength of which is represented by the vector B. we assume that the fluid has attained unsteady state conditions. That is, flow variables are dependent of the time t. Because of the interaction of the two fields, an electric field vector denoted E is induced at right angles to both V and B. This electric field is given by

$$E = V \times B \qquad (1)$$

If we assume that the conducting fluid is isotropic/exhibits adiabatic flow in spite of the magnetic field, then we denote the electrical conductivity of the fluid by a scalar σ. By Ohm's law[14], the density of the current induced in the conducting fluid denoted J is given by

$$J = \sigma E \qquad (2)$$

Or $J = \sigma(V \times B)$ (3)

Simultaneously occurring with the induced current is the Lorentz force F given by

$$F = J \times B \qquad (4)$$

This force occurs because, as an electric generator, the conducting fluid cuts the lines of the magnetic field. The vector F is the vector cross product of both J and B and is a vector perpendicular to the plane of both J and B. This induced force is parallel to V but in opposite direction. Laminar flow through a channel under uniform transverse magnetic field is important because of the use of MHD generator, MHD pump and electromagnetic flow meter.

We now consider an electrically conducting viscous, unsteady, incompressible fluid moving between two infinite parallel plates both kept at a constant distance $2h$ between them. Both plates of the channel are fixed with no motion. This is plane poiseuille flow. The equations of motion are the continuity equation

$$\nabla . V = 0 \qquad (5)$$

and the Navier-Stokes equations

$$\rho\left[\left(\frac{\partial}{\partial t} + V.\nabla\right)\right] V = f_B - \nabla P + \mu \nabla^2 V \qquad (6)$$

Where ρ is the fluid density, f_B is the body force per unit mass of the fluids, μ is the fluid viscosity and P is the pressure acting on the fluid. If we assume a one dimensional flow so that we choose the axis of the channel formed by the two plates as the x-axis and assume that the flow is in this direction. Observe that $\bar{u}, \bar{v}$ and $\bar{w}$ are the velocity components in $\bar{x}, \bar{y}$ and $\bar{z}$ directions respectively. Then this implies $\bar{v} = \bar{w} = 0$ and $\bar{u} \neq 0$, then the continuity equation is satisfied.

From this we infer that $\bar{u}$ is independent of $\bar{x}$. This makes the nonlinear term $[(V.\nabla)V]$ in the Navier-Stokes equation vanish. We neglect body forces f_B which are mainly due to gravity in the Navier-Stokes equations and replace them with the Lorentz force and from the assumption that the flow is one dimensional, it means that the governing equation for this flow is

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{F_x}{\rho} \quad (7)$$

Where $\upsilon = \frac{\mu}{\rho}$ is the kinematics viscosity and F_x is the component of the magnetic force in the direction of x-axis. Assuming unidirectional flow so that $\bar{v} = \bar{w} = 0$ and $B_x = B_z = 0$ since magnetic field is along y-direction so that $V = i\bar{u}$ and $B = B_o j$. Where B_o is the magnetic field strength.

Now,

$$F_x = \sigma[(i\bar{u} \times jB_o)] \times jB_o \quad (8)$$

So that we have

$$\frac{F_x}{\rho} = -\frac{\sigma}{\rho} B_o^2 \bar{u} \quad (9)$$

Then (7) becomes

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho} B_o^2 \bar{u} \quad (10)$$

From (10), when angle of inclination is introduced, we have

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho}B_o^2\bar{u}sin^2(\alpha) \qquad (11)$$

Where α is the angle between V and B. Equation (11) is general in the sense that both fields can be assessed at any angle α for $0 \leq \alpha \leq \pi$.

Because of the porosity of the lower plate, the characteristic velocity v_o is taken as a constant so as to maintain the same pattern of flow against suction and injection of the fluid in which it is moving perpendicular to the fluid flow. The origin is taken at the Centre of the channel and $\bar{x}, \bar{y}$ coordinate axis are parallel and perpendicular to the channel walls respectively. The governing equations, that is; the momentum equation, the energy equation and the concentration equation are as follows:

The momentum equation is given as

$$\rho\frac{\partial \bar{u}}{\partial \bar{t}} = -v_o\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{P}}{\partial \bar{x}} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho}B_o^2\bar{u}sin^2(\alpha) + g\beta(\bar{T} - \bar{T}_\infty) + g\beta^*(\bar{C} - \bar{C}_\infty) \qquad (12)$$

since the flow is isentropic, the energy equation is given as

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{k}{\rho Cp}\frac{\partial^2 \bar{T}}{\partial y'^2} + \frac{1}{\rho Cp}\frac{\partial q}{\partial y} \qquad (13)$$

Where k is the thermal conductivity of the fluid, ρ is the density, c_p is the specific heat constant pressure and $\bar{T}$ is the temperature.

The concentration equation is given as

$$\frac{\partial \bar{C}}{\partial \bar{t}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - Kc'(\bar{C} - \bar{C}_\infty) \qquad (14)$$

The q in (13) is called the heat flux. It is given by,

$$\frac{\partial q}{\partial y} = 4\alpha^2(\bar{T}_\infty - \bar{T}) \qquad (15)$$

The boundary conditions are

$$\bar{U}(y,t) = 0, \bar{T} = \bar{T}_\infty; \text{ at } \bar{t} = 0, \bar{U}(-L,\bar{t}) = 0,$$

$$\bar{U}(L,\bar{t}) = \frac{v}{L}, \bar{T} = \bar{T}_w; \text{ at } \bar{t} > 0, \bar{C} = \bar{C}_\infty \text{ at } \bar{t} = 0\,, \bar{C} = \bar{C}_w; \text{ at } \bar{t} > 0 \quad (16)$$

In order to solve equations (12), (13) and (14) subject to the boundary conditions (16), we introduce the following dimensionless parameters:

$$\bar{u} = \frac{uv}{L}, \bar{t} = \frac{tL^2}{v}, \bar{y} = yL, \bar{P} = p\rho\frac{v^2}{L^2}, \bar{x} = xL,\ \theta = \frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty} \Longrightarrow \bar{T} - \bar{T}_\infty = \theta(\bar{T}_w - \bar{T}_\infty) \Longrightarrow \bar{T} = \theta(\bar{T}_w - \bar{T}_\infty) + \bar{T}_\infty,\ Gr = \frac{\rho L^2 g\beta(\bar{T}_w-\bar{T}_\infty)}{\mu v},\ C = \frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty} \Longrightarrow \bar{C} - \bar{C}_\infty = C(\bar{C}_w - \bar{C}_\infty) \Rightarrow\ \bar{C} = C(\bar{C}_w - \bar{C}_\infty) + \bar{C}_\infty,\ Gc = \frac{\rho L^2 g\beta^*(\bar{C}_w-\bar{C}_\infty)}{\mu v},\ Pr = \frac{\mu Cp}{k} \Longrightarrow Cp = \frac{Prk}{\mu}, N^2 = \frac{4\alpha^2 L^2}{k} \Longrightarrow \alpha^2 = \frac{N^2 k}{4L^2}, \bar{x} = xL, \bar{y} = yL, \frac{\partial q}{\partial y} = 4\alpha^2(\bar{T}_\infty - \bar{T})$$

$$,\ Sc = \frac{V}{D} \Longrightarrow D = \frac{V}{Sc}$$

Equations (12), (13) and (14) now become

$$\frac{\partial u}{\partial t} = A\frac{\partial u}{\partial y} - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M^2 u + Gr\theta + GcC \quad (17)$$

Where $M = m^* sin\alpha$ and $m^* = LB_o\sqrt{\frac{\sigma}{\mu}} = Ha, A = \frac{-Re}{\rho}$. Since it is poiseuille flow, $\frac{\partial p}{\partial x} \neq 0$.

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta \quad (18)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - \bar{K}_c \mathrm{C} \quad (19)$$

The boundary conditions in dimensionless form are

$U(-1,t) = 0, \theta(-1,t) = 0, C(-1,t) = 0\ at\ t = 0$

$U(1,t) = 1, \theta(1,t) = 1, C(1,t) = 1$ -------------- (20)

3. METHOD OF SOLUTION/SOLUTION OF THE PROBLEM

The momentum equation, energy equation and concentration equation can be reduced to the set of ordinary differential equations, which are solved analytically. This can be done by representing the velocity, temperature and species concentration of the fluid in the perturbation series as follows

$$U(y,t) = U_o(y) + \varepsilon U_1(y)e^{i\omega t} + 0(\varepsilon^2) \quad (21)$$

$$\theta(y,t) = \theta_o(y) + \varepsilon\theta_1(y)e^{i\omega t} + 0(\varepsilon^2) \quad (22)$$

$$C(y,t) = C_o(y) + \varepsilon C_1(y)e^{i\omega t} + 0(\varepsilon^2) \quad (23)$$

Substituting equations (21), (22) and (23) into equations (17), (18) and (19). Equating the coefficients of harmonic and non-harmonic term and neglecting the coefficients of higher order of ε^2, we get:

$$U_o''(y) + AU_o'(y) - M^2U_o(y) = -Q - Gr\theta_o(y) - GcC_o(y) \quad (24)$$

$$U_1''(y) + AU_1'(y) - bU_1(y) = -Gr\theta_1(y) - GcC_1(y) \quad (25)$$

Where $b = M^2 + i\omega$, $Q = \frac{-\partial p}{\partial x}$ (constant)

$$\theta_o''(y) + N^2\theta_o(y) = 0 \quad (26)$$

$$\theta_1''(y) - a_1\theta_1(y) = 0 \quad (27)$$

Where $a_1 = i\omega Pr - N^2$

$$C_o''(y) - ScKcC_o(y) = 0 \quad (28)$$

$$C_1''(y) - ScLC_1(y) = 0 \quad (29)$$

The corresponding boundary conditions become

$$U_o(-1,t) = 0, \theta_o(-1,t) = 0, C_o(-1,t) = 0$$

$$U_o(1,t) = 1, \theta_o(1,t) = 1, C_o(1,t) = 1$$

$$U_1(-1,t) = 0, \theta_1(-1,t) = 0, C_1(-1,t) = 0$$

$$U_1(1,t) = 1, \theta_1(1,t) = 1, C_1(1,t) = 1 \quad (30)$$

We now solve equations (24) – (29) under the relevant boundary conditions for the mean flow and unsteady flow separately.

The mean flows are governed by the equations (24), (26) and (28) where U_o, θ_o and C_o are called the mean velocity, mean temperature and mean concentration respectively. The unsteady flows are governed by equations (25), (27) and (29) where U_1, θ_1 and C_1 are the unsteady components.

These equations are solved analytically under the relevant boundary conditions (30) as follows;

Solving equations (24), (26) and (28) subject to the corresponding relevant boundary conditions in (30), we obtain the mean velocity, mean temperature and mean concentration as

$$U_o(y) = C_5 e^{m_5 y} + C_6 e^{m_6 y} + K_1 + K_2 cosNy + K_3 sinNy + K_4 e^{m_3 y} + K_5 e^{m_4 y} \quad (31)$$

$$\theta_o(y) = c_1 cosNy + c_2 sinNy \quad (32)$$

$$C_o(y) = c_3 e^{m_3 y} + c_4 e^{m_4 y} \quad (33)$$

Similarly, solving equations (25), (27) and (29) under the relevant boundary conditions in (30), the unsteady velocity, unsteady temperature and unsteady concentration becomes

$$U_1(y) = C_{11} e^{m_{11} y} + C_{12} e^{m_{12} y} + K_6 e^{m_7 y} + K_7 e^{m_8 y} + K_8 e^{m_9 y} + K_9 e^{m_{10} y} \quad (34)$$

$$\theta_1(y) = C_7 e^{m_7 y} + C_8 e^{m_8 y} \quad (35)$$

$$C_1(y) = C_9 e^{m_9 y} + C_{10} e^{m_{10} y} \quad (36)$$

Therefore, the solutions for the velocity, temperature and species concentration profiles are

$$U(y,t) = C_5 e^{m_5 y} + C_6 e^{m_6 y} + K_1 + K_2 cosNy + K_3 sinNy + K_4 e^{m_3 y} + K_5 e^{m_4 y} + \varepsilon[C_{11} e^{m_{11} y} + C_{12} e^{m_{12} y} + K_6 e^{m_7 y} + K_7 e^{m_8 y} + K_8 e^{m_9 y} + K_9 e^{m_{10} y}] e^{iwt} \quad (37)$$

$$\theta(y,t) = c_1 cosNy + c_2 sinNy + \varepsilon[C_7 e^{m_7 y} + C_8 e^{m_8 y}]e^{iwt} \quad (38)$$

$$C(y,t) = c_3 e^{m_3 y} + c_4 e^{m_4 y} + \varepsilon[C_9 e^{m_9 y} + C_{10} e^{m_{10} y}]e^{iwt} \quad (39)$$

4. DISCUSSION OF RESULTS

To discuss the effect of heat and mass transfer on unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. The velocity profile u, the temperature distribution θ and the species concentration C are shown graphically against y using matlab for different values of the following parameters such as Hartmann number Ha, thermal Grashof number Gr, modified Grahof number Gc, Radiation parameter N, Prandtl number Pr, Schmidt number $Sc$and chemical parameter Kc.

Figures 1,2 and 3 present the effect of Hartmann number Ha on velocity u. It is inferred from these figures that an increase in the Hartmann number decreases the fluid velocity.

Figures 4, 5 and 6 show the effect of thermal Grashof number Gr on velocity u. It is observed that an increase in thermal Grashof number Gr and the angle of inclination on velocity profile u increases the velocity.

Figures 7, 8 and 9 depict the effect of modified Grashof number Gc on velocity u. It is shown that the velocity increases as the modified Grashof number Gc increases.

Figure 10 describes the effect of Prandtl number Pr on temperature distribution θ. It is simulated from the figure that an increase in Prandtl number Pr leads to the decrease in temperature.

Figures 11 and 12 illustrate the effect of chemical parameter Kc and Schmidt number Sc on the species concentration. It is seen that an increase in the chemical parameter Kc and Schmidt number Sc decreases the species concentration.

Table 1 depicts variation of skin frictions $\tau_1 and\ \tau_2$, Nusselt numbers $Nu_1 and\ Nu_2$ and Sherwood numbers $Sh_1\ and\ Sh_2$ with time t. It is observed that, the time is constant and does not affect the values.

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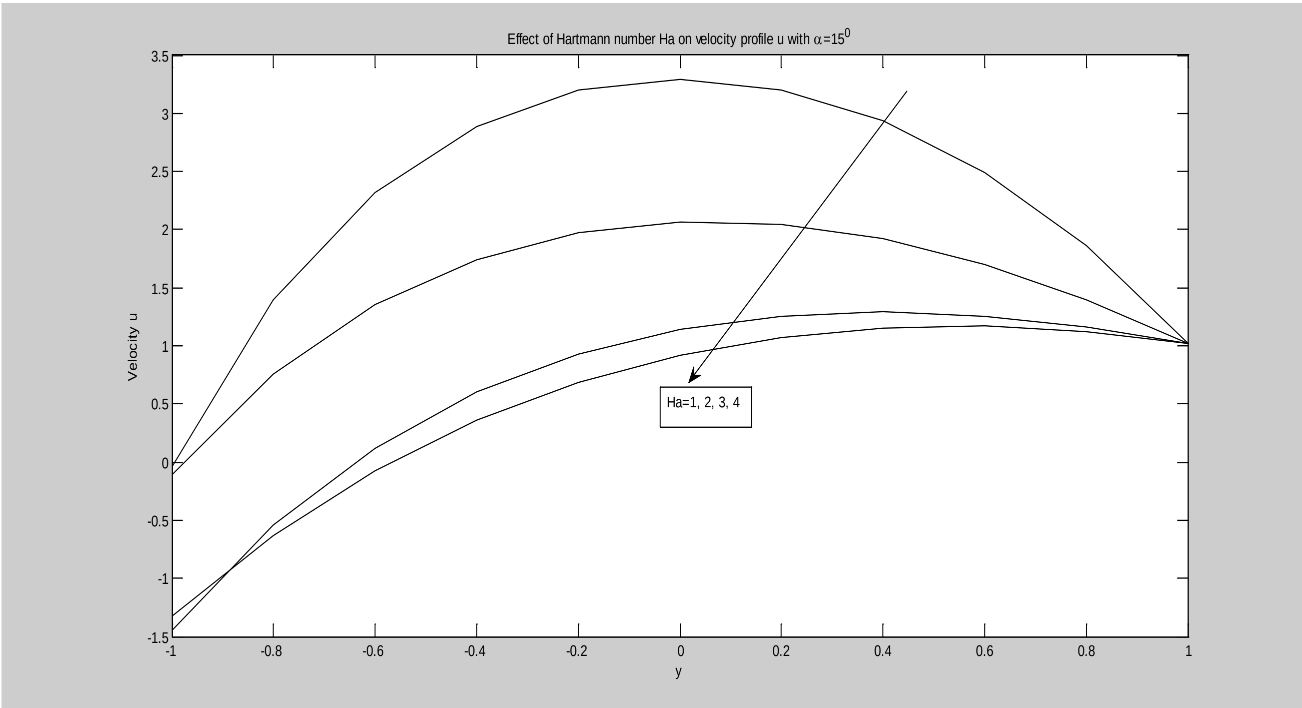

Figure 1: Effect of Hartmann number Ha on velocity profile u with $\alpha = 15^0, B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Gc = 1, N = 1$ and $\omega = 1$.

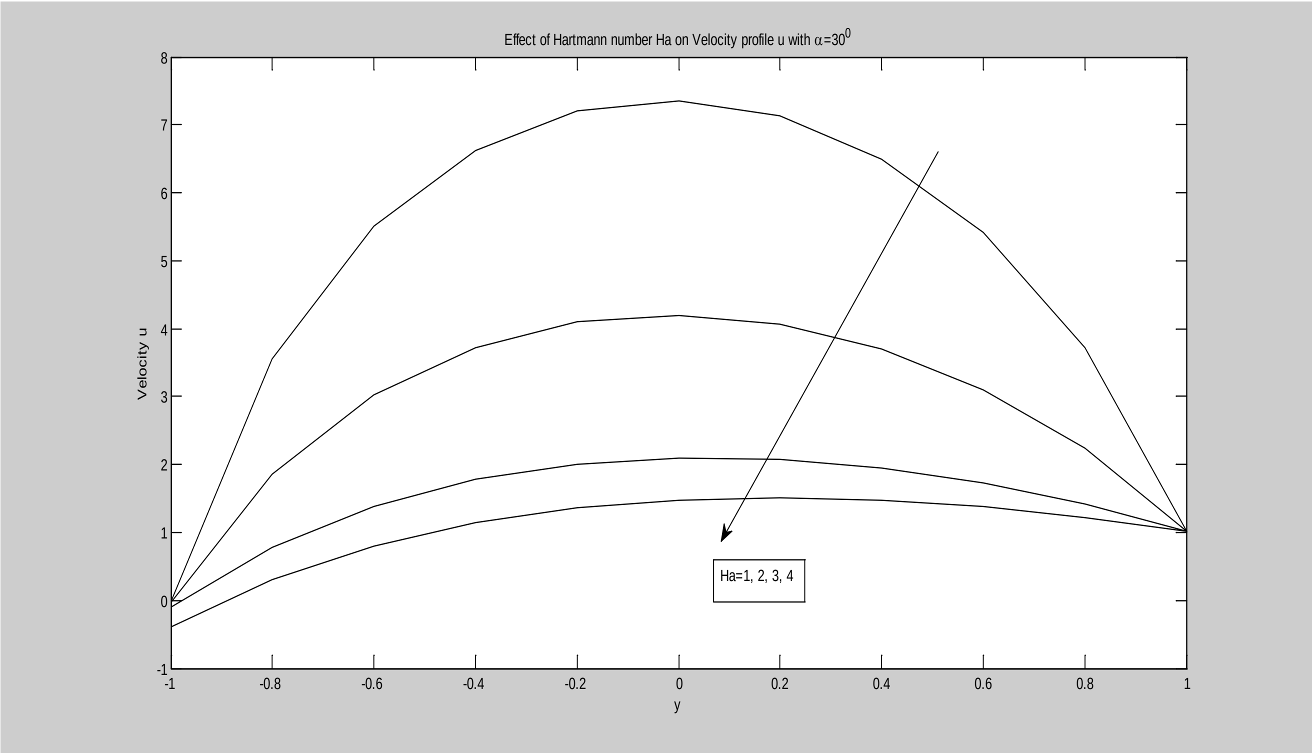

Figure 2: Effect of Hartmann number Ha on velocity profile u with $\alpha = 30^0, B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Gc = 1, N = 1 \; and \; \omega = 1.$

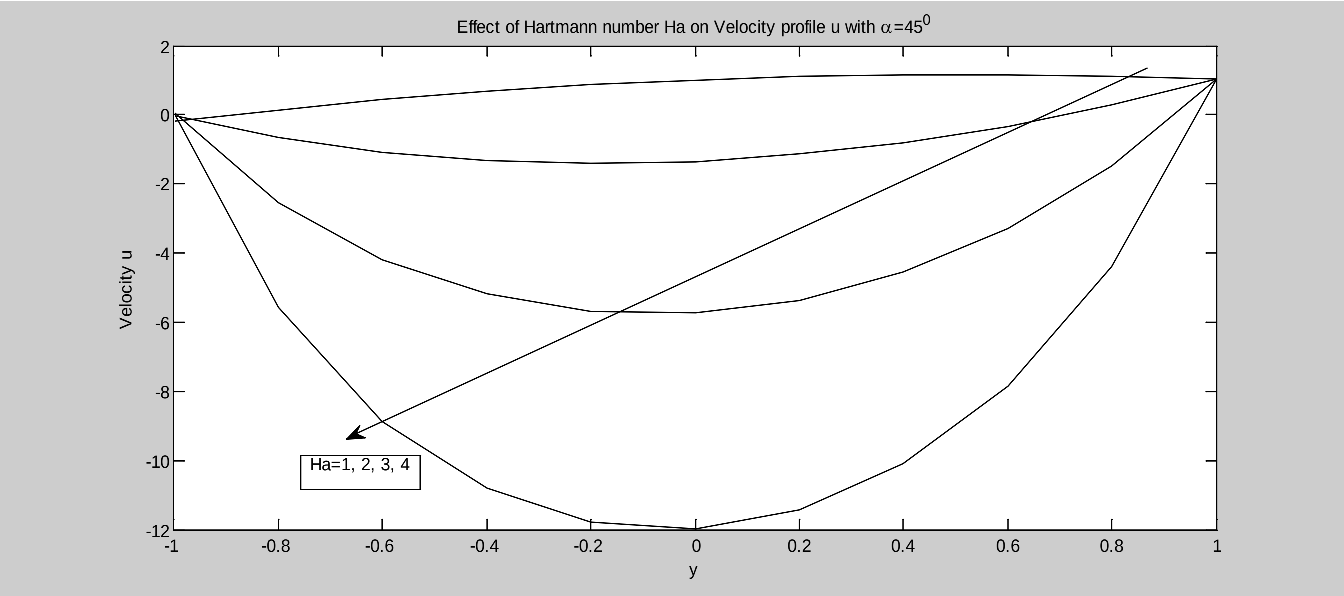

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Figure 3: Effect of Hartmann number Ha on velocity profile u with $\alpha = 45^0, B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Gc = 1, N = 1$ and $\omega = 1$.

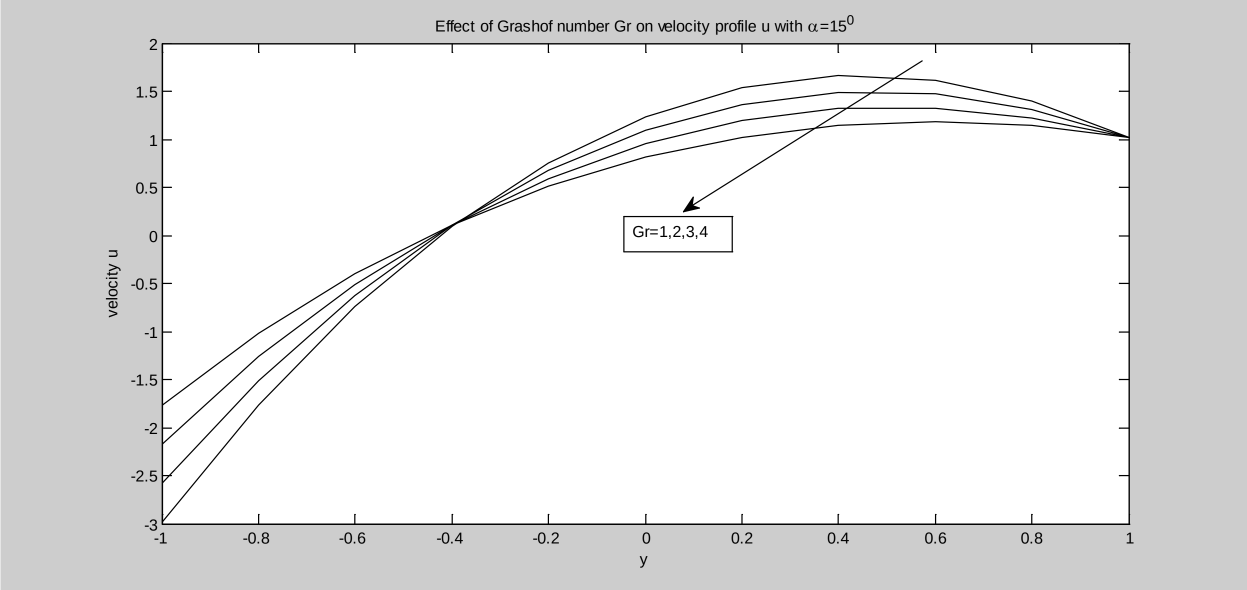

Figure 4: Effect of Grashof number Gr on velocity profile u with $\alpha = 15^0, B = 1, t = 0.5, \varepsilon = 0.02, Gc = 1, Ha = 1, N = 1$ and $\omega = 1$.

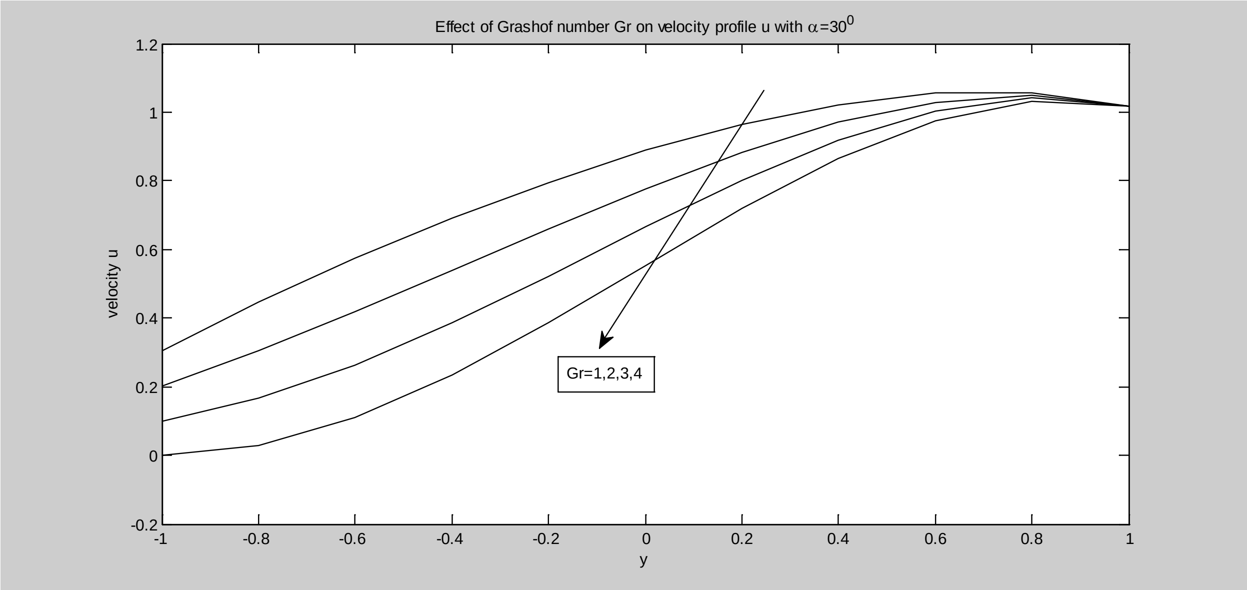

Figure 5: Effect of Grashof number Gr on velocity profile u with $\alpha = 30^0, B = 1, t = 0.5, \varepsilon = 0.02, Gc = 1, Ha = 1, N = 1$ and $\omega = 1$.

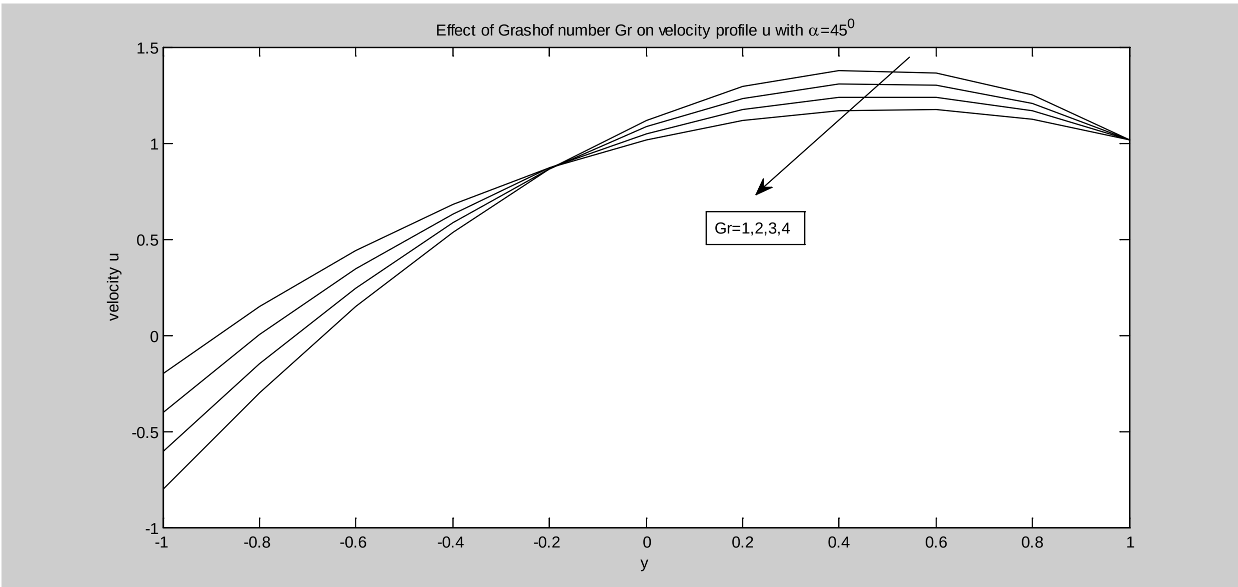

Figure 6: Effect of Grashof number Gr on velocity profile u with $\alpha = 45^0, B = 1, t = 0.5, \varepsilon = 0.02, Gc = 1, Ha = 1, N = 1\ and\ \omega = 1$.

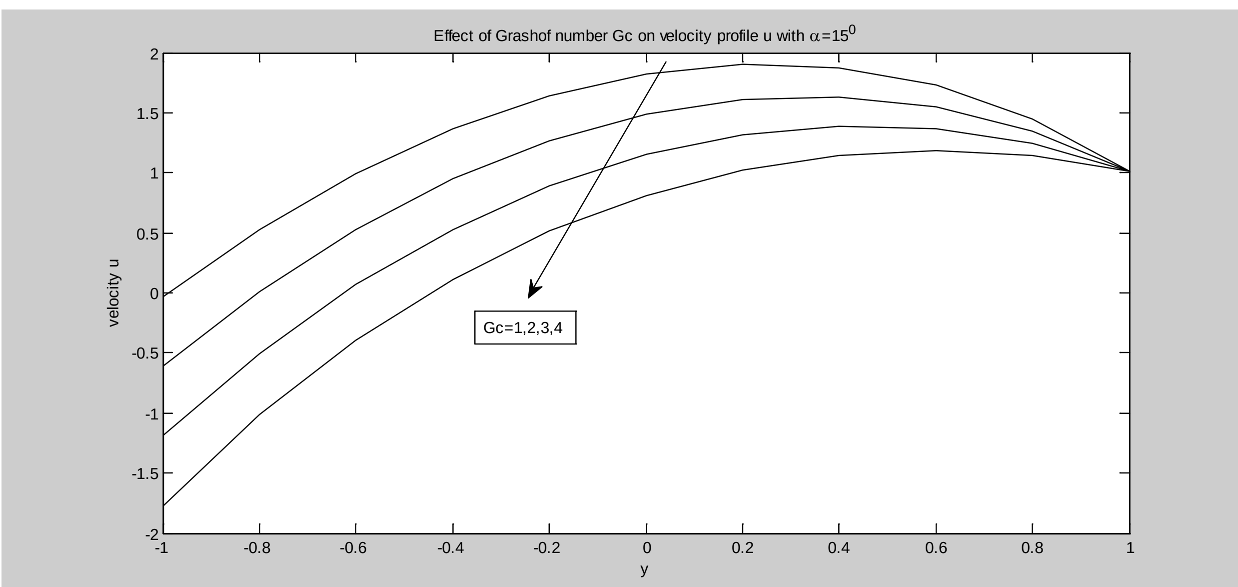

Figure 7: Effect of Grashof number Gc on velocity profile u with $\alpha = 15^0, B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Ha = 1, N = 1\ and\ \omega = 1$.

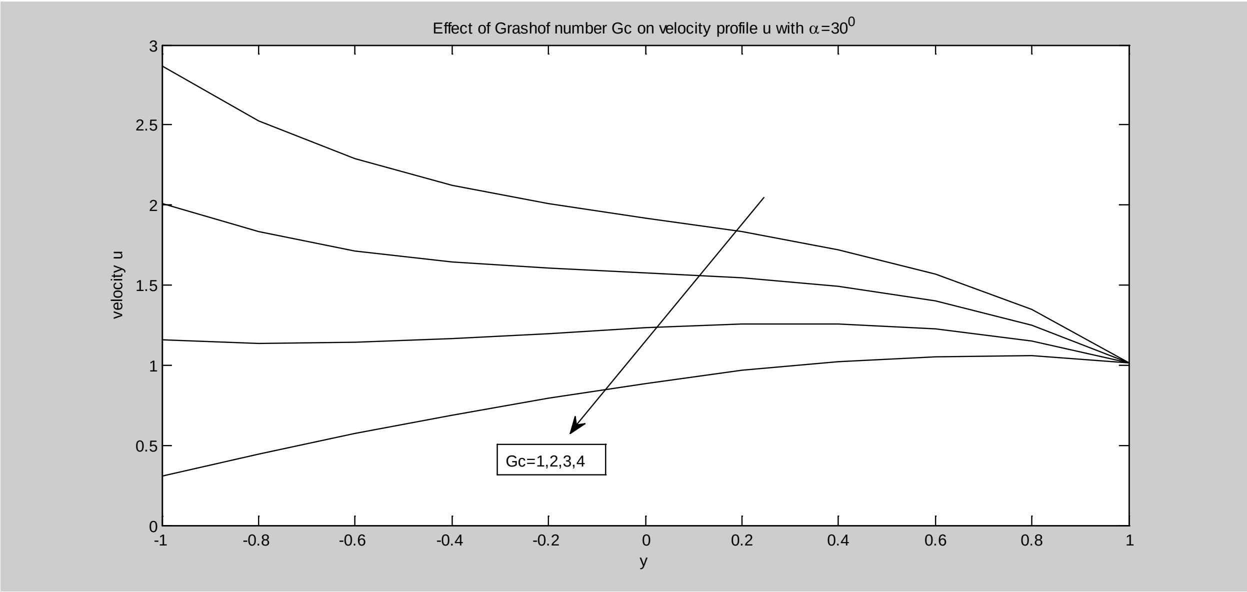

Figure 8: Effect of Grashof number Gc on velocity profile u with $\alpha = 30^0, B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Ha = 1, N = 1$ and $\omega = 1$.

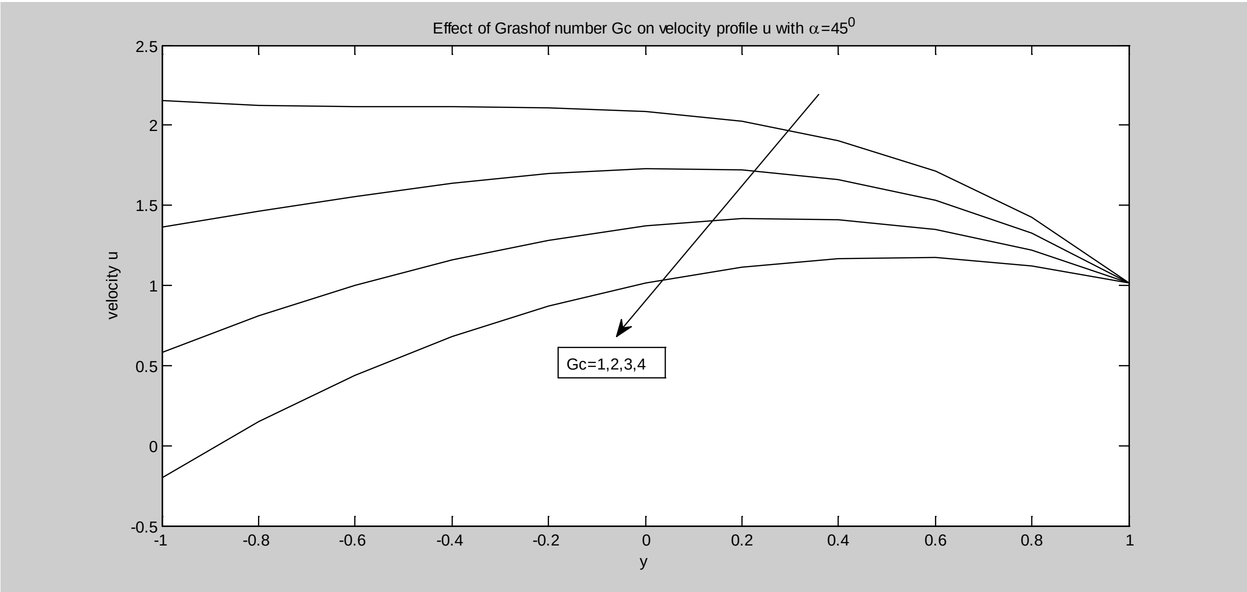

Figure 9: Effect of Grashof number Gc on velocity profile u with $\alpha = 45^0, B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Ha = 1, N = 1$ and $\omega = 1$.

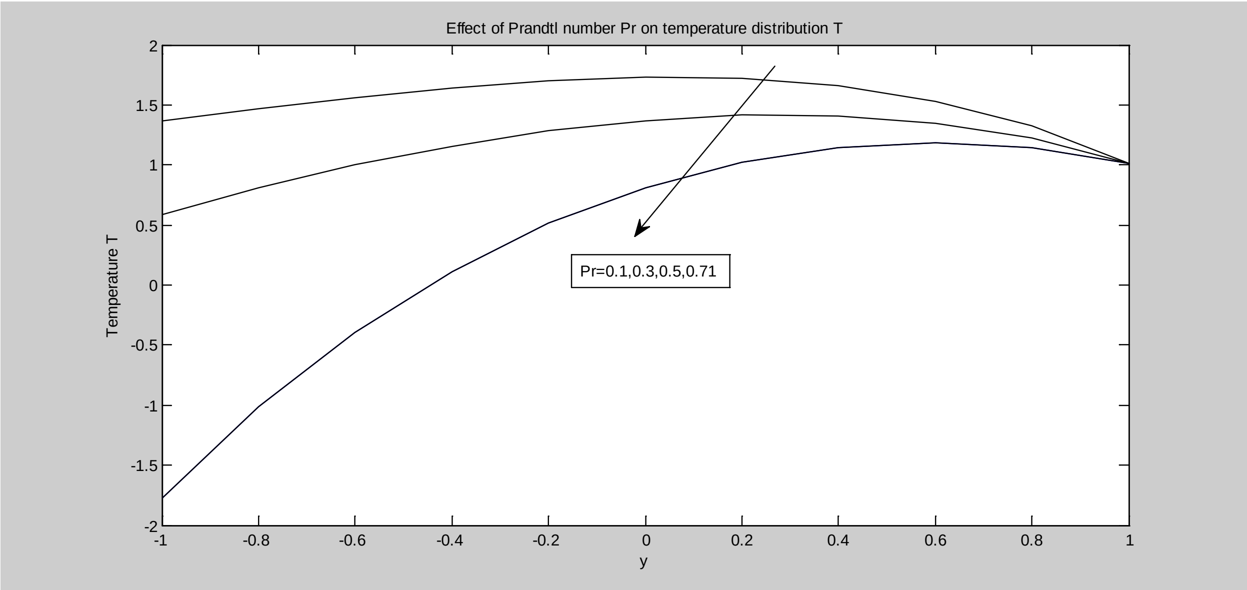

Figure 10: Effect of Prandtl number Pr on temperature distribution θ with $B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Ha = 1, N = 1 \; and \; \omega = 1$.

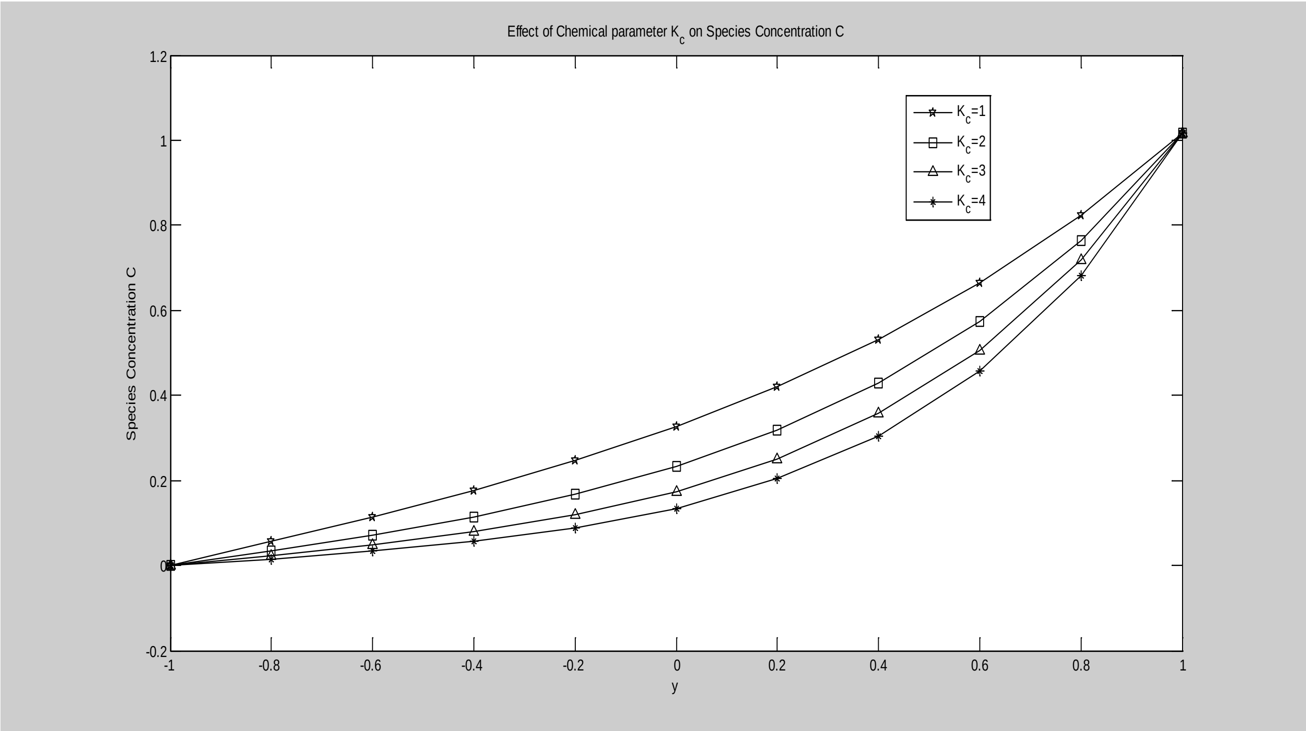

Figure 11: Effect of Chemical parameter Kc on Species Concentration C with $B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Sc = 1, N = 1 \; and \; \omega = 1$.

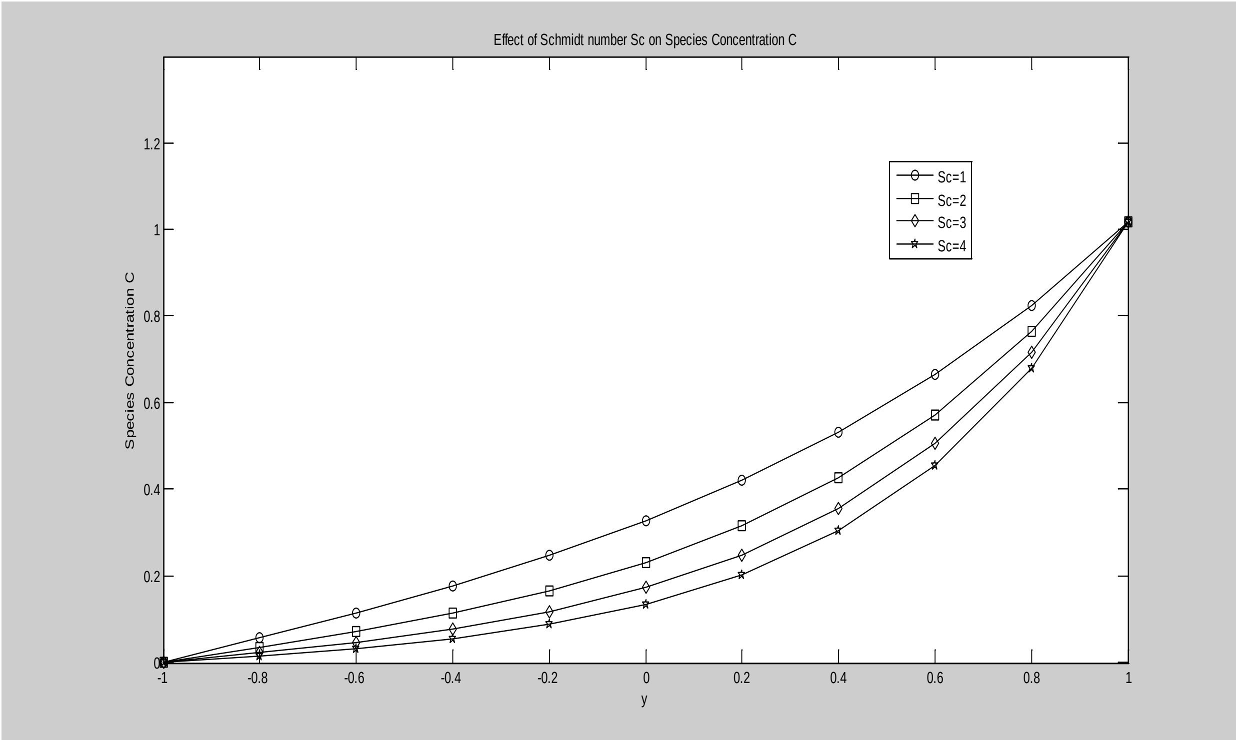

Figure 12: Effect of Schmidt number Sc on Species Concentration C with Gr=1,Gc=1, $B = 1, t = 0.5, \varepsilon = 0.02, Gr = 1, Kc = 1, N = 1$ and $\omega = 1.$

Table 1: Variation of skin frictions $\tau_1 and\ \tau_2$, Nusselt numbers $Nu_1 and\ Nu_2$ and Sherwood numbers Sh_1 and Sh_2 with time t.

t	τ_1	τ_2	Nu_1	Nu_2	Sh_1	Sh_2
0.0	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.1	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.2	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.3	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373

0.4	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.5	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.6	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.7	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.8	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
0.9	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373
1.0	5.9958	-1.0028	1.0998	-0.4577	0.2757	1.0373

5. SUMMARY AND CONCLUSION

In this section, we studied the effect of heat and mass transfer on unsteady MHD poiseuille flow between two inifinite parallel porous plates in an inclined magnetic field.

The governing equations, that is, the momentum, energy and species concentration equations have been written in dimensionless form using the dimensionless parameters.

Perturbation method was employed to evaluate and solve the velocity profile u, temperature distribution θ, the species concentration C, Skin frictions τ_1 and τ_2, Nusselt numbers Nu_1 and Nu_2 and Sherwood numbers Sh_1 and Sh_2.

The investigation of this research work leads to the following conclusions:

- ✓ At high Hartmann number Ha, the velocity decreased.
- ✓ Velocity increased due to effect of thermal Grashof number Gr and solutal Grashof number Gc.
- ✓ An increase in Prandtl number Pr decreased temperature.
- ✓ Species concentration reduced with increase in chemical parameter Kc and Schmidt number Sc.

This work can be applied in electric power generator, extrusion of plastics in the manufacture of rayon and nylon, extrusion of food in the manufacture of macaroni etc.

Appendix

CONSTANTS

$m_1 = iN, m_2 = -iN, m_3 = \sqrt{ScKc}, m_4 = -\sqrt{ScKc},\ \ m_5 = \frac{-A+\sqrt{A^2+4M^2}}{2},\qquad m_6 = \frac{-A-\sqrt{A^2+4M^2}}{2}, m_7 = \sqrt{a_1}, m_8 = -\sqrt{a_1};\ a_1 = i\omega Pr - N^2,$

$m_9 = \sqrt{ScL}, m_{10} = -\sqrt{ScL}; L = Kc - i\omega,$

$m_{11} = \frac{-A+\sqrt{A^2+4b}}{2}, m_{12} = \frac{-A-\sqrt{A^2+4b}}{2},$

$T_1 = K_1 + K_2 \cos N - K_3 \sin N + K_4 e^{-m_3} + k_5 e^{-m_4},$

$T_2 = K_1 + K_2 \cos N + K_3 \sin N + K_4 e^{m_3} + k_5 e^{m_4},$

$T_3 = K_6 e^{-m_7} + k_7 e^{-m_8} + K_8 e^{-m_9} + k_9 e^{-m_{10}},$

$T_4 = K_6 e^{m_7} + k_7 e^{m_8} + K_8 e^{m_9} + k_9 e^{m_{10}},$

$K_1 = \frac{Q}{M^2}, K_2 = \frac{-Gr\left(C_2 AN - C_1(M^2+N^2)\right)}{((M^2+N^2)^2 - A^2N^2)}, K_3 = \frac{Gr(C_2(M^2+N^2) - C_1 AN)}{\left(A^2N^2 + (M^2+N^2)\right)}, K_4 = \frac{-GcC_3}{\left(m_3^2 + Am_3 - M^2\right)}$

$K_5 = \frac{-Gc\,C_4}{\left(m_4^2 + Am_4 - M^2\right)}, K_6 = \frac{-GrC_7}{m_7^2 + Am_7 - b}, K_7 = \frac{-GrC_8}{m_8^2 + Am_8 - b}, K_8 = \frac{-GcC_9}{m_9^2 + Am_9 - b},$

$K_9 = \frac{-GcC_{10}}{m_{10}^2 + Am_{10} - b},$

$C_1 = \frac{1}{2\cos N}, C_2 = \frac{1}{2\sin N}, C_3 = \frac{1 - C_4 e^{m_4}}{e^{m_3}}, C_4 = \frac{-e^{-m_3}}{(e^{m_3 - m_4} - e^{m_4 - m_3})},$

$C_5 = \frac{1 - T_2 - C_6 e^{m_6}}{e^{m_5}}, C_6 = \frac{-T_1 e^{m_5} - e^{-m_5} - T_2 e^{-m_5}}{(e^{m_5 - m_6} - e^{m_6 - m_5})},\ C_7 = \frac{1 - C_8 e^{m_8}}{e^{m_7}},$

$C_8 = \frac{-e^{-m_7}}{(e^{m_7 - m_8} - e^{m_8 - m_7})}, C_9 = \frac{1 - C_{10} e^{m_{10}}}{e^{m_9}}, C_{10} = \frac{-e^{-m_9}}{(e^{m_9 - m_{10}} - e^{m_{10} - m_9})},$

$$C_{11} = \frac{1-T_4-C_{12}e^{m_{12}}}{e^{m_{11}}},\ C_{12} = \frac{-T_3e^{m_{11}}-e^{-m_{11}}-T_4e^{-m_{11}}}{e^{m_{11}-m_{12}}-e^{m_{12}-m_{11}}},$$

$$\tau_1 = m_5C_5e^{-m_5} + m_6C_6e^{-m_6} + NK_2sinNy + NK_3cosNy + m_3K_4e^{-m_3} + m_4K_5e^{-m_4} + \varepsilon(m_{11}C_{11}e^{-m_{11}} + m_{12}C_{12}e^{-m_{12}} + m_7K_6e^{-m_7} + m_8K_7e^{-m_8} + m_9K_8e^{-m_9} + m_{10}K_9e^{-m_{10}})e^{i\omega t},$$

$$\tau_2 = m_5C_5e^{m_5} + m_6C_6e^{m_6} - NK_2sinNy + NK_3cosNy + m_3K_4e^{m_3} + m_4K_5e^{m_4} + \varepsilon(m_{11}C_{11}e^{m_{11}} + m_{12}C_{12}e^{m_{12}} + m_7K_6e^{m_7} + m_8K_7e^{m_8} + m_9K_8e^{m_9} + m_{10}K_9e^{m_{10}})e^{i\omega t},$$

$$Nu_1 = NC_1sinN + NC_2cosN + \varepsilon(m_7C_7e^{-m_7} + m_8C_8e^{-m_8})e^{i\omega t},$$

$$Nu_2 = -NC_1sinN + NC_2cosN + \varepsilon(m_7C_7e^{m_7} + m_8C_8e^{m_8})e^{i\omega t},$$

$$Sh_1 = m_3C_3e^{-m_3} + m_4C_4e^{-m_4} + \varepsilon(m_9C_9e^{-m_9} + m_{10}C_{10}e^{-m_{10}})e^{i\omega t},$$

$$Sh_2 = m_3C_3e^{m_3} + m_4C_4e^{m_4} + \varepsilon(m_9C_9e^{m_9} + m_{10}C_{10}e^{m_{10}})e^{i\omega t}$$

References

[1] A.K. Al-Hadhrami, L. Elliot, M.D. Ingham, X. Wen, (2003): flow through horizontal channels of porous materials, inter. J. of Energy Research **27**, 875-889.

[2] A. Raptis, C. Perkidis, (1999): Radiation and Free Convection flow past a moving plate, Int. J. of Applied Mechanics and Engineering, **4**(4): 817-821.

[3] A.S. Idowu and J.O. Olabode (2014): Unsteady MHD Poiseuille Flow between Two Infinite Parallel Plates in an Inclined Magnetic Field with Heat Transfer. IOSR Journal of Mathematics **10**(3): 47-53.

[4] C.B. Singh (1993): MHD steady flow of liquid between two parallel plates, In: Proc. Of First Conference of Kenya Mathematical Society, 24-26.

[5] C.B. Singh, P.C. Ram (1978): Unsteady MHD Fluid Flow through a channel: Journal of Scientific Research, **28**(2).

[6] D.G. Drake (1965): Flow in a channel due to a periodic pressure gradient, Quart. Journal of Mech. And Appl. Maths., **18**(1).

[7] Dileep Singh Chauchan and Priyanka Rastogi (2012): Heat Transfer effects on rotating MHD coquette flow in a channel partially filled by a porous medium with hall current. Journal of Appl. Sci. Eng. **15**(3): 281-290.

[8] E.M. Sparrow, R.D. Cess (1961): The effect of a magnetic field on free convection heat transfer, Int. J. heat mass Tran., **3**(4): 267-274.

[9] H. Alfven (1942): Existence of electromagnetic-hydrodynamic waves, Nature, **150**(3805), 405-406.

[10] J.A. Sherliff (1956): Entry of conducting and non-conducting fluids in pipes, Journal of Mathematics Proc. Of the Cambridge Philosophical Soc., **52**, 573-583.

[11] John Mooney, Nick Stokes (1997): Time varying MHD flows with free surfaces, Inter. Conf. on CFD in Mineral Metal Processing and Power Generation CSIRO.

[12] Joseph K.M, Daniel S and Joseph G.M (2014): Unsteady MHD coquette flow between Two Infinite Parallel Plates in an Inclined Magnetic Field with Heat transfer, Inter. J. Math. Stat. Inv. **2**(3): 103-110.

[13] N. Sandeep and V. Sugunamma (2013): Effect of an Inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Int. Journal of Appl. Maths. Modelling, **1**(1): 16-33.

[14] P.A. Davidson (2001): An introduction to MHD, First Edition, Cambridge University Press, UK.

[15] P.C. Ram, C.B. Singh, U. Singh (1984): Hall effects on Heat and Mass Transfer flow through porous medium, Astrophysics Space Science, 100: 45-51.

[16] S. Ganesh, S. Krishnambal (2007): Unsteady MHD Stokes flow of viscous fluid between two parallel porous plates, Journal of Applied Sciences, 7, 374-379.

[17] Stamenkovic M. Zivojin, Dragiza D. Nikodijevic, Bratislav D. Blagojevic, Slobodan R. Savic (2010): MHD flow and Heat Transfer of two immiscible fluids between moving plates, Transaction of the Canadian Society for Mechanical Engineering, 4(34): 351-372.

[18] U.S. Rajput, P.K. Sahu (2011): Transient free convection MHD flow between two long vertical parallel plates with constant Temperature and variable mass diffusion, Int. Journal of Math. Analysis, 34(5): 1665-1671.

[19] V.M. Soundelgekar, Mohammed Abdulla-Ali (1986): Free convection effects on MHD flow past an impulsively started infinite vertical isothermal plate, Reg. J. Energy, Heat and Mass Transfer **8**(2): 119-125.

[20] W.A. Manyonge, D.W. Kiema, C.C.W. Iyaya (2012): Steady MHD Poiseuille Flow between two infinite parallel porous plates in an inclined magnetic field, Inter. Journal of Pure and Applied Maths., **76**(5): 661-668.